

Fast Approximation for Diagnosing Convective Cloud Top Heights

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■ Why is Cb top height important?



**SEVERE
TURBULENCE**



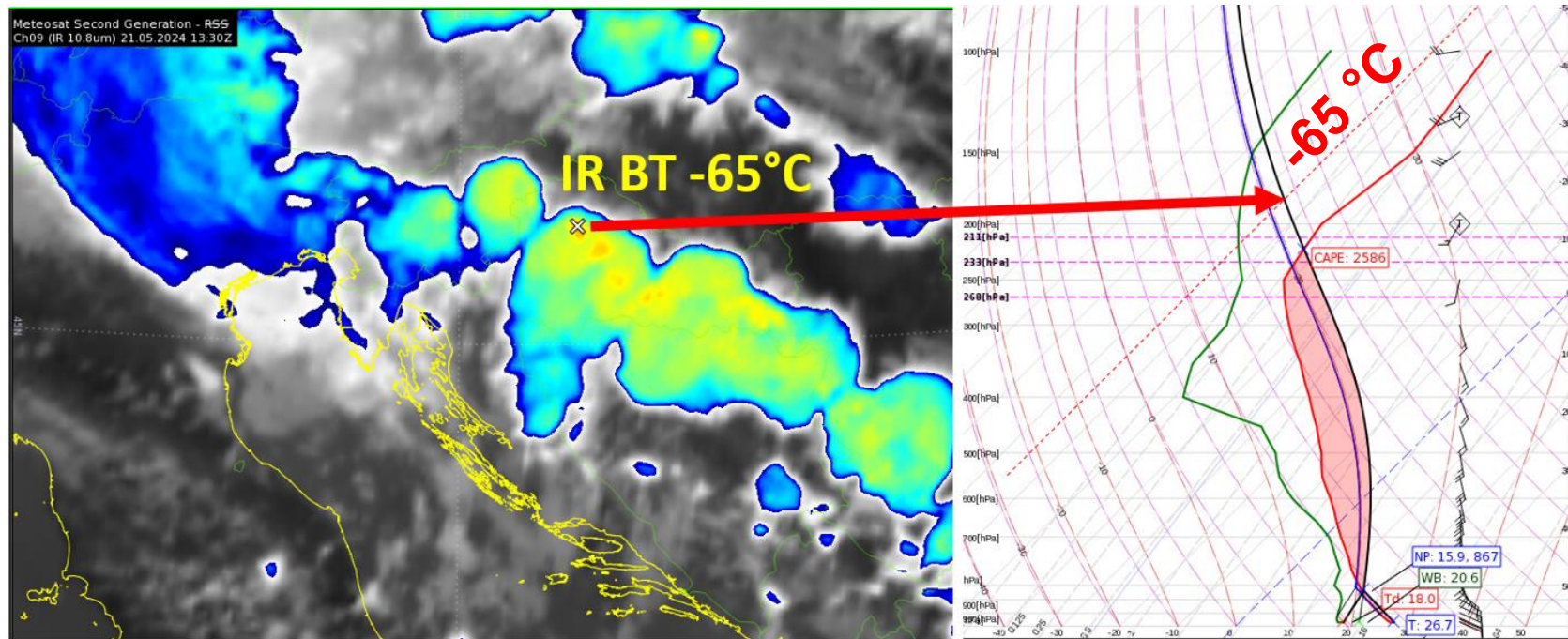
HAIL



**SEVERE
ICING**



How to diagnose Cb top height?



Cb top pressure => standard atmosphere altitude in hectofeet = **Flight Level [FL]**



■ Moist adiabat iterative calculation

$$e_s = e_o \exp \left[\frac{17.67 \left(\frac{273.15 \text{ K}}{T - 273.15 \text{ K}} \right)}{T - 273.15 \text{ K}} \right]$$
$$r_s = \epsilon e_s / (P - e_s).$$

$$b = [1 + (r/\epsilon)] / [1 + (r/c)] \approx 1 + 0.24 r$$

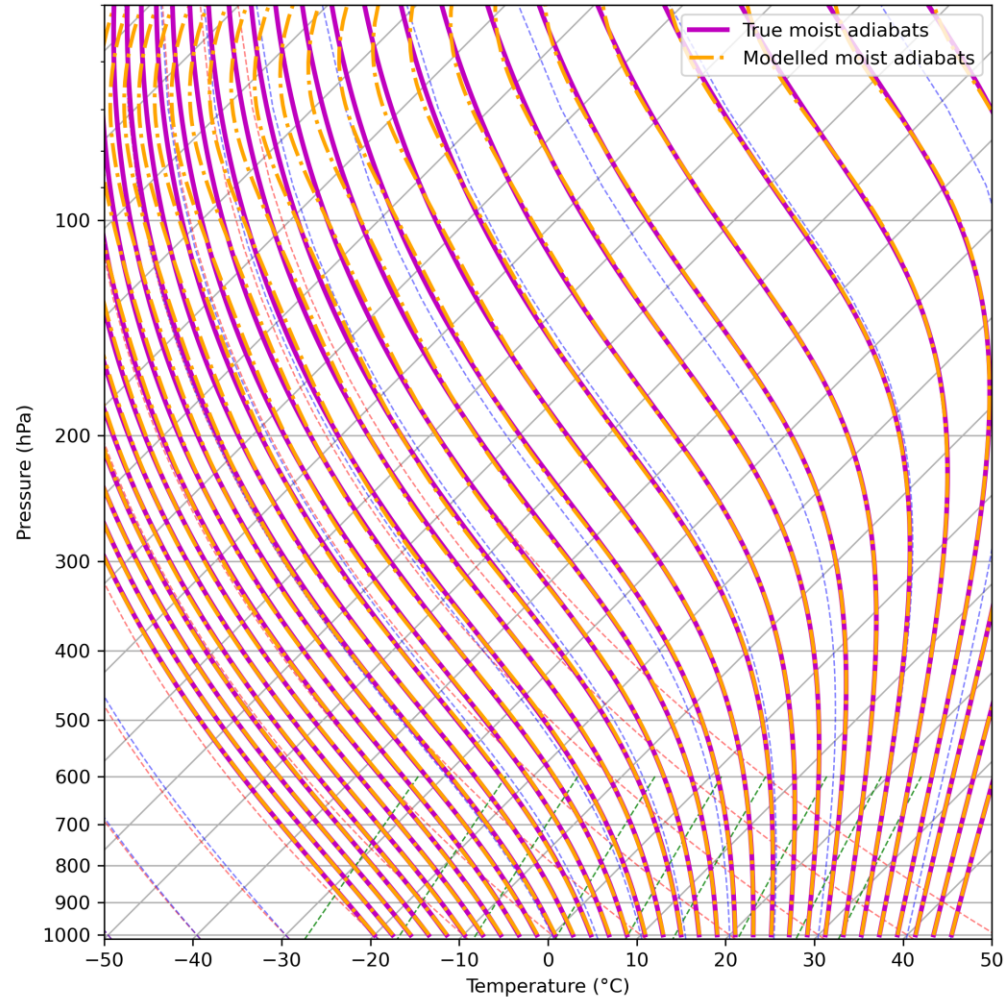
or use the approximation $b \approx 1$ *Bakhshaii and Stull 2013*

$$\frac{\partial T}{\partial P} = \left(\frac{b}{P} \right) \frac{R_d T + L_v r_s}{C_{pd} + \frac{L_v^2 r_s \epsilon}{R}}$$

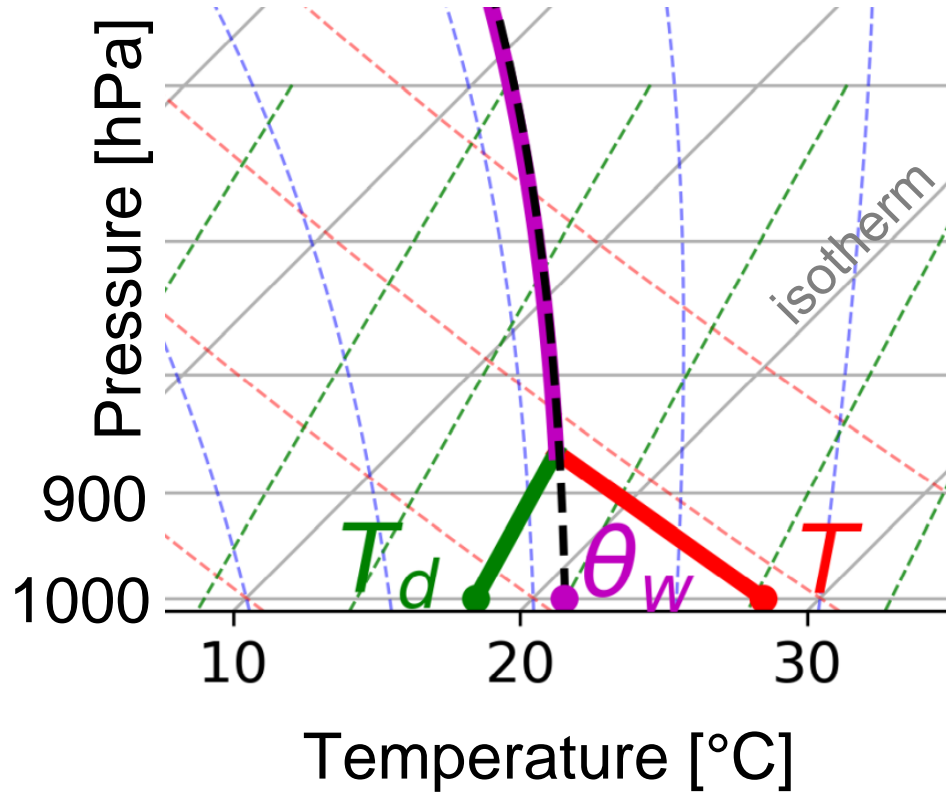
All this needs to be calculated for each small pressure increment ΔP !

Moist adiabats and 5. degree polynomial approximations

$$y(x) = c_5x^5 + c_4x^4 + c_3x^3 + c_2x^2 + c_1x^1 + c_0$$



T and T_d represented as wet bulb potential temperature θ_w



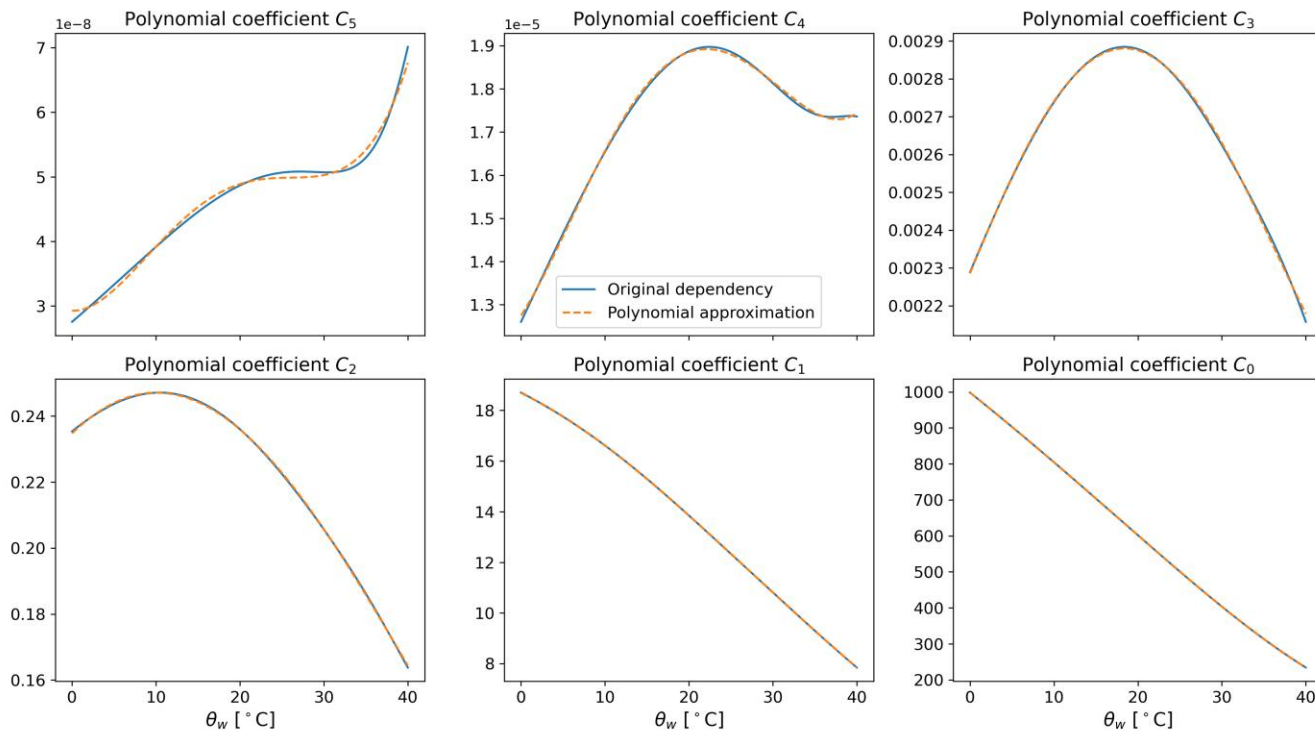
■ Moist adiabat fast approximation

- **Moist adiabat shape changes with wet bulb potential temperature (θ_w) !**
- Every polynomial coefficient (6 of them in total) can be modelled as a function of θ_w

$$p(t) = C_5(\theta_w)t^5 + C_4(\theta_w)t^4 + C_3(\theta_w)t^3 + C_2(\theta_w)t^2 + C_1(\theta_w)t + C_0(\theta_w)$$

■ Approximation Coefficients vs θ_w

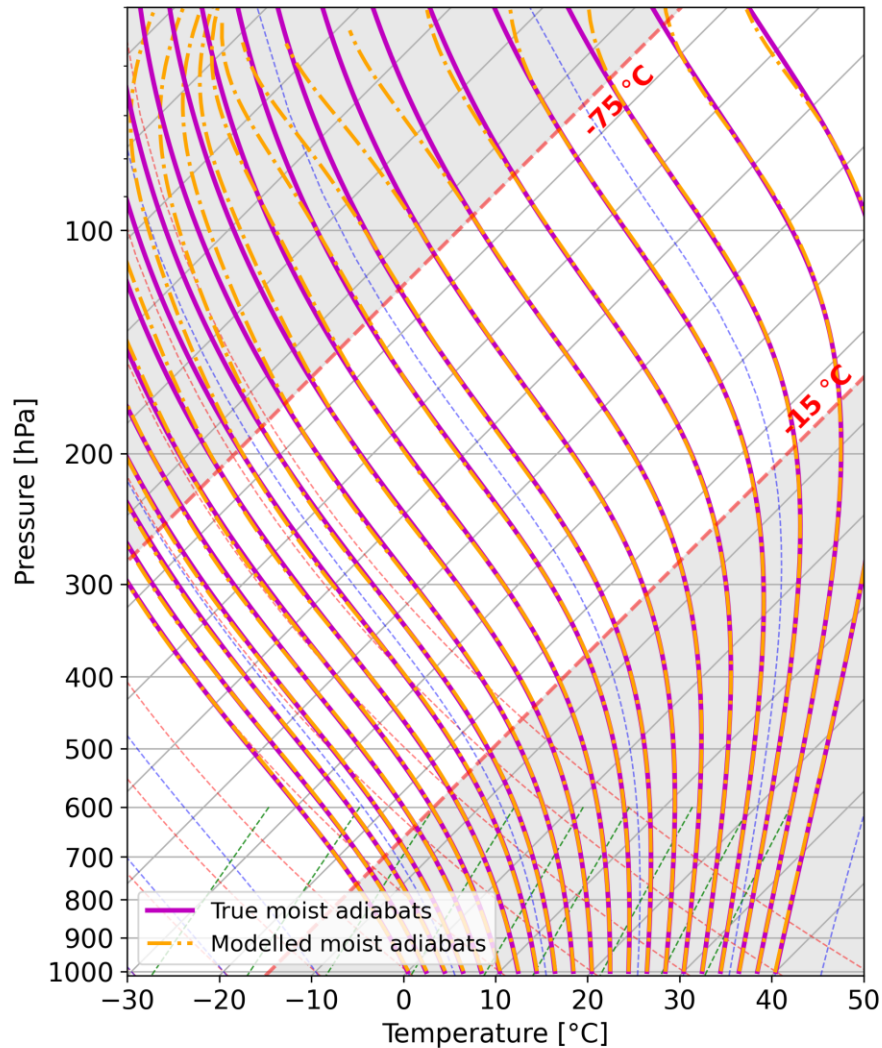
$$p(t) = C_5(\theta_w)t^5 + C_4(\theta_w)t^4 + C_3(\theta_w)t^3 + C_2(\theta_w)t^2 + C_1(\theta_w)t + C_0(\theta_w)$$



$$C_i(\theta_w) = \sum_{j=0}^4 a_{ij} \theta_w^j$$

4th degree
approximation

$$p(t, \theta_w) = \sum_{i=0}^5 C_i(\theta_w) t^i$$



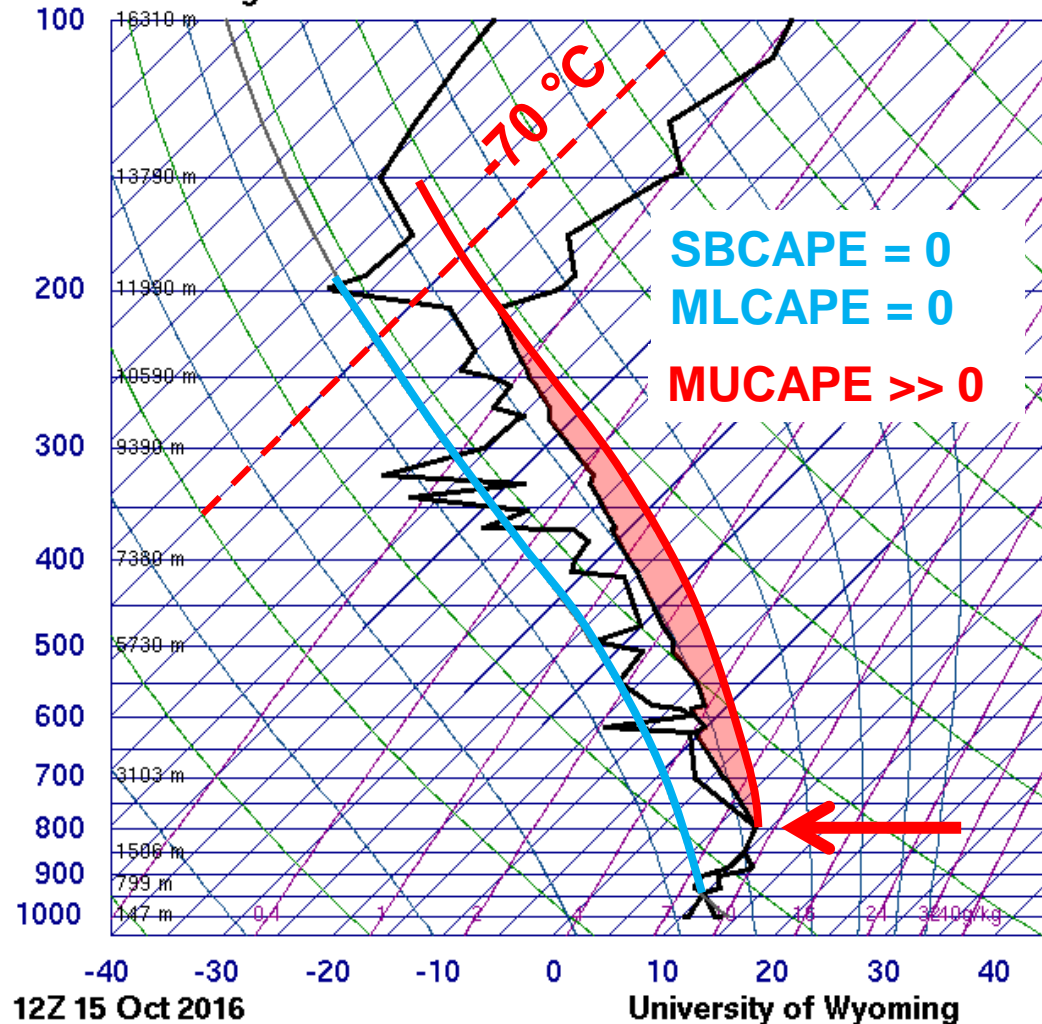
■ CCTH Calculation procedure

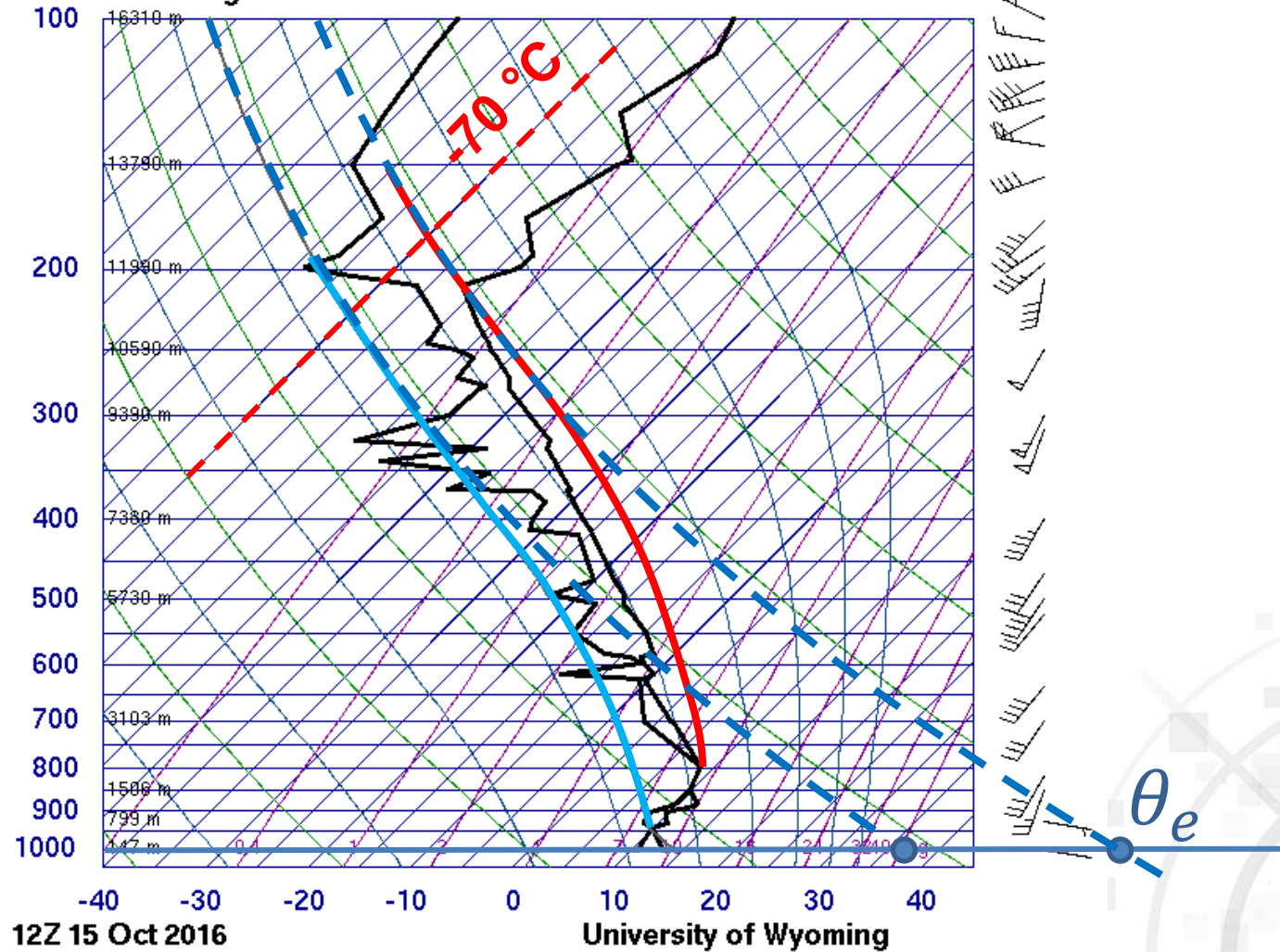
C_i	a_{i0}	a_{i1}	a_{i2}	a_{i3}	a_{i4}
C_0	9.981118e+02	-1.865352e+01	-7.228945e-02	-1.288899e-04	4.152094e-05
C_1	1.868462e+01	-1.584316e-01	-5.652978e-03	7.782649e-05	-1.697159e-07
C_2	2.347380e-01	2.461856e-03	-1.223192e-04	-1.929728e-07	1.532080e-08
C_3	2.285961e-03	5.360970e-05	-2.299950e-07	-6.829626e-08	9.717708e-10
C_4	1.275047e-05	2.984764e-07	1.986974e-08	-1.344314e-09	1.835750e-11
C_5	2.928147e-08	-7.653230e-11	1.876537e-10	-9.439792e-12	1.349002e-13

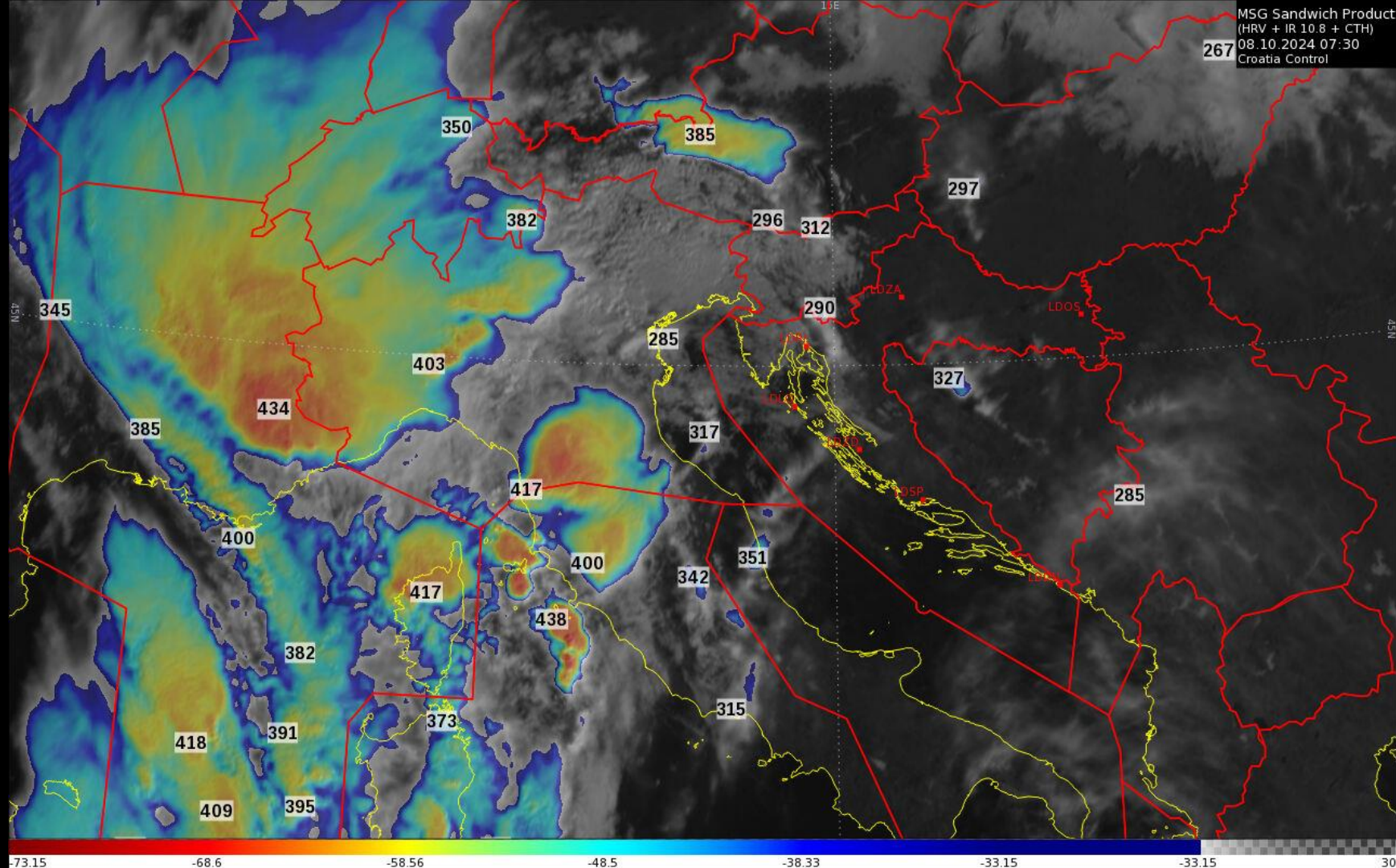
T, T_d [°C], p [hPa] $\Rightarrow \theta_e$ (Bolton's 1980 formulas), $\theta_e \Rightarrow \theta_w$ (Davies-Jones 2008 formula).
Six C_i coefficients are calculated using values from the Table 1. and θ_w [°C] as follows:

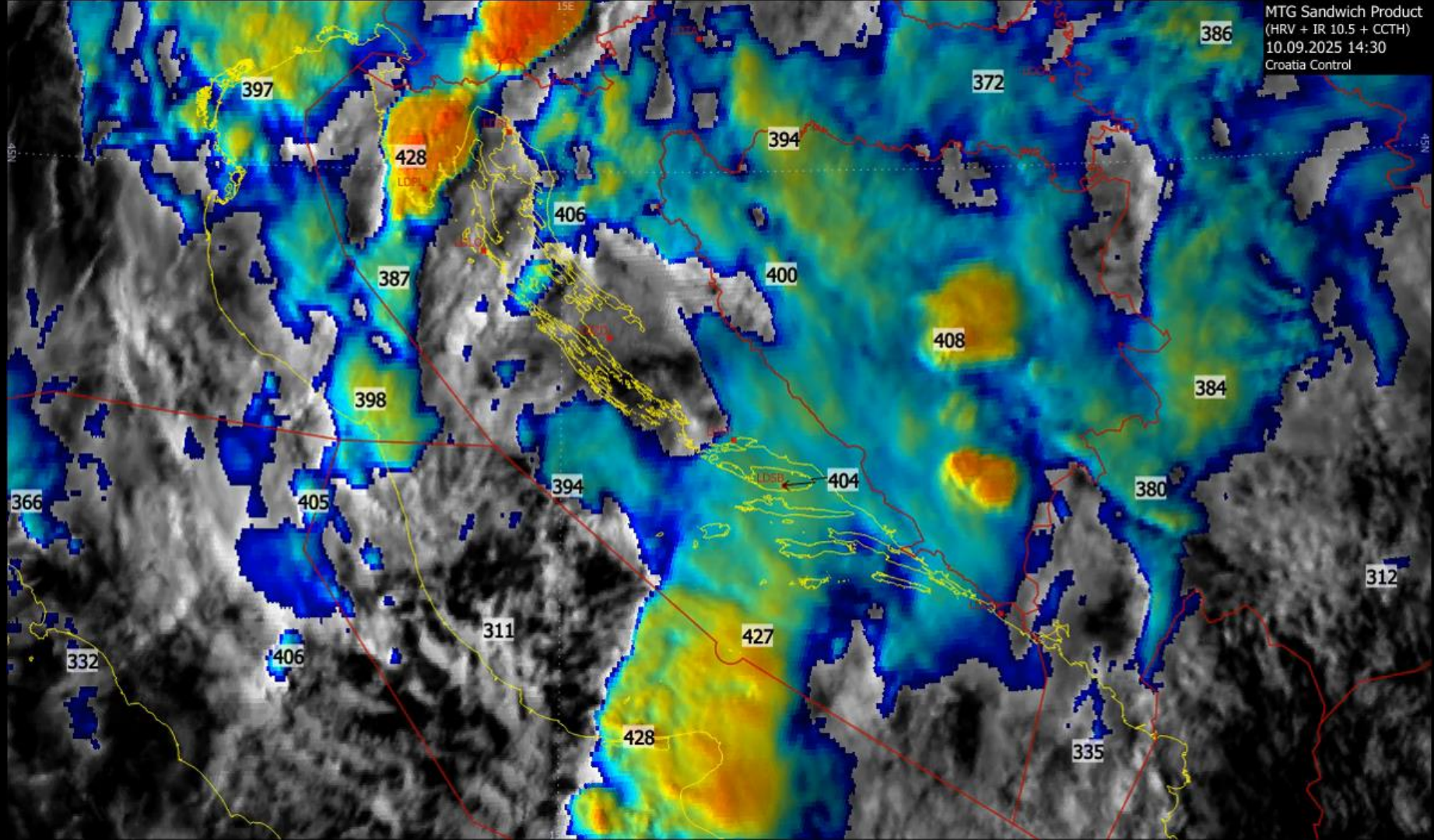
$$C_i(\theta_w) = \sum_{j=0}^4 a_{ij} \theta_w^j \quad \Rightarrow \quad p(t) = \sum_{i=0}^5 C_i(\theta_w) t^i$$

14240 LDDD Zagreb









Comparison with other cloud top height products

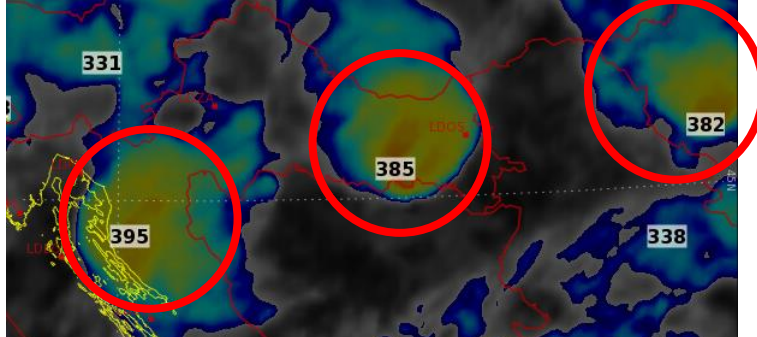
- **CCTH (Convective Cloud Top Height)**

VS

- **NWC SAF CTTH (Cloud Top Temperature and Height)**
- Radar **ECHO TOPS** (Vaisala IRIS Focus product)
- **Radar volume data** (side views)
- **Radar volume data taken as ground truth!**



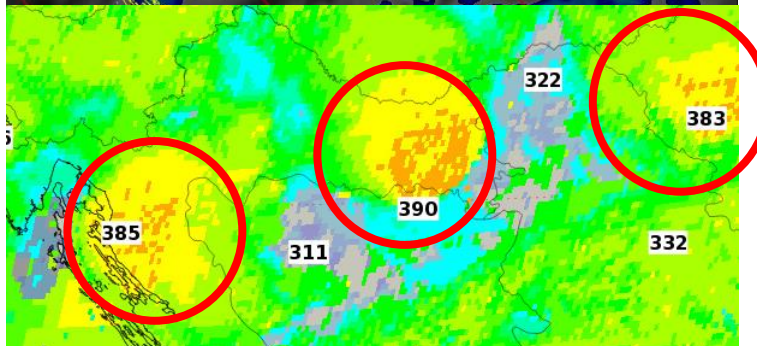
CCTH



NWC

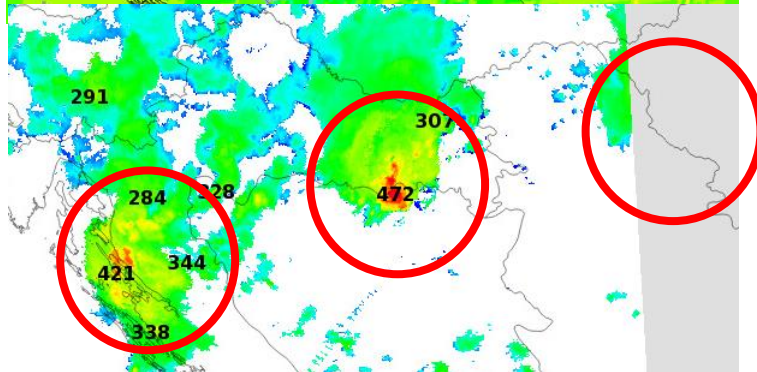
SAF

CTTH



ECHO

TOPS

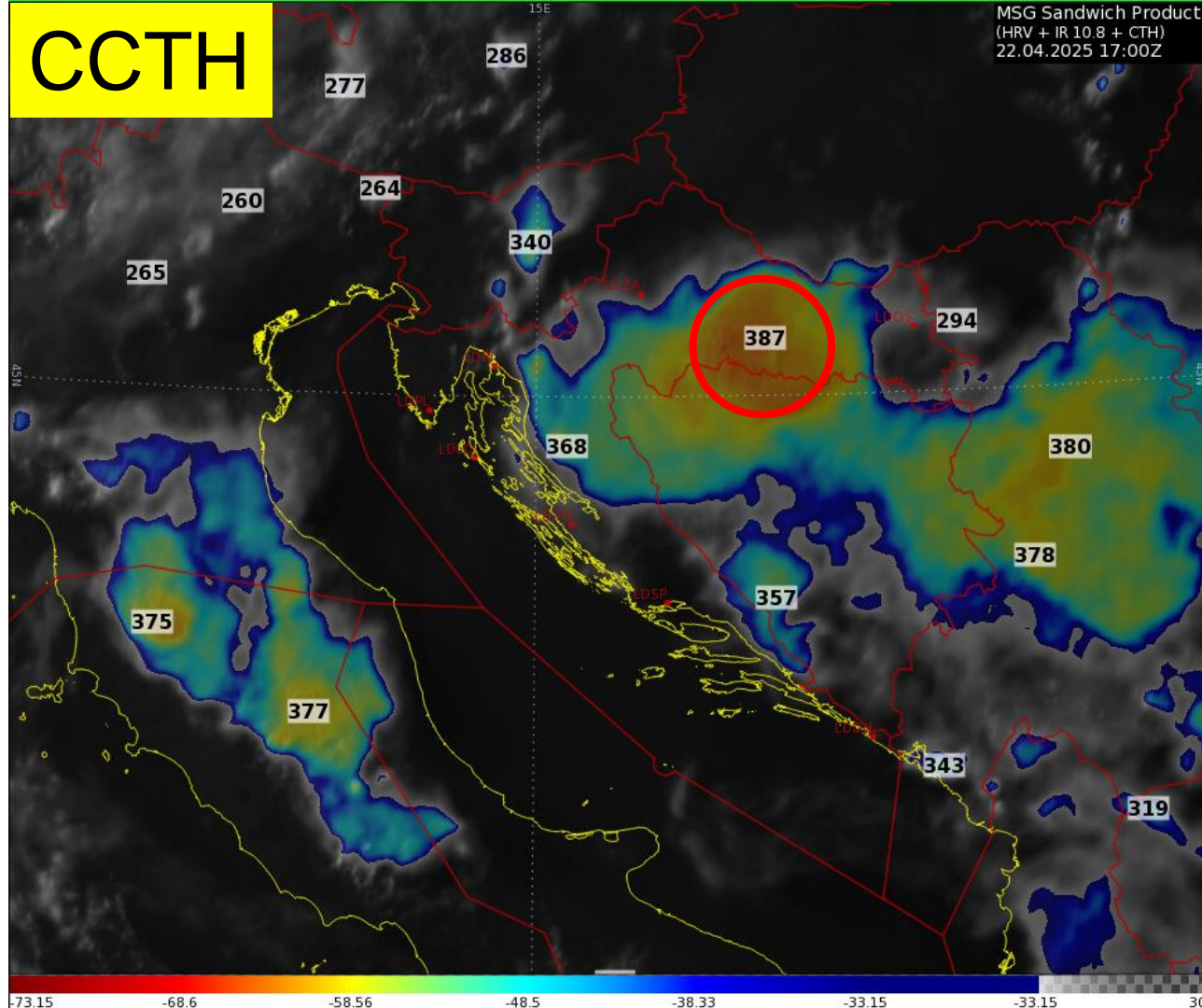


5.5.2025.
18:30 UTC



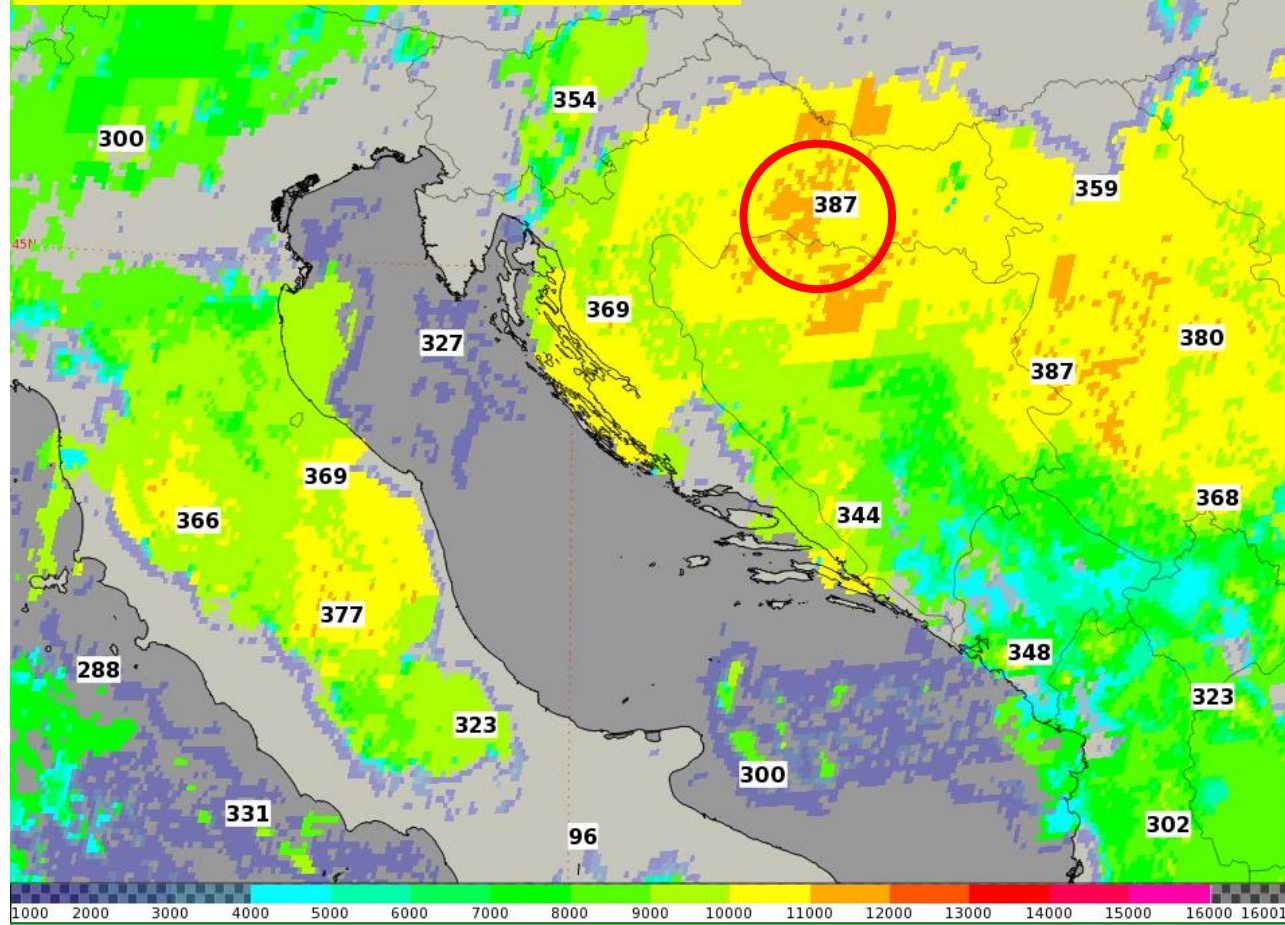
CCTH

MSG Sandwich Product
(HRV + IR 10.8 + CTH)
22.04.2025 17:00Z

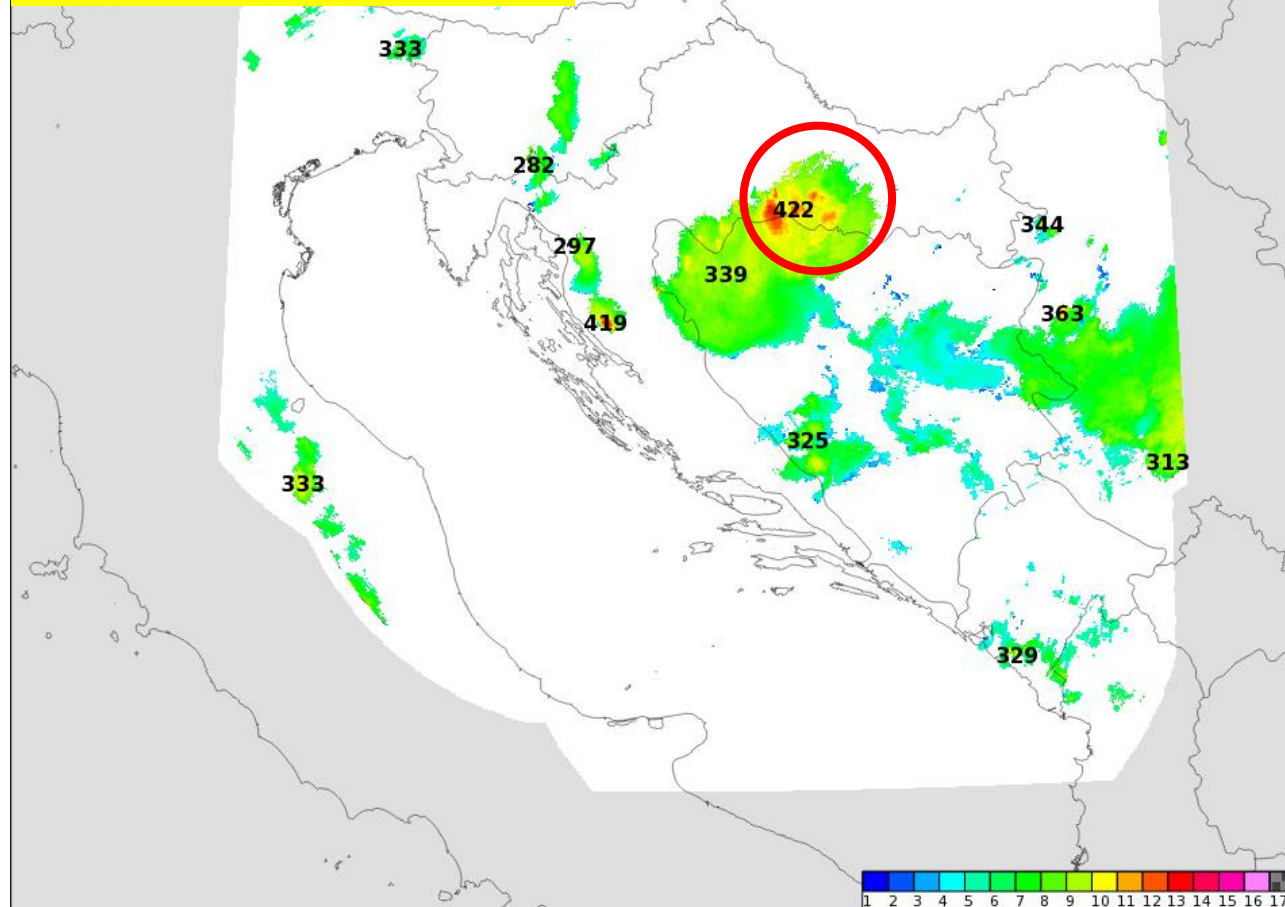


CTTH Cloud Top Height: 2025-04-22T17:00:00Z

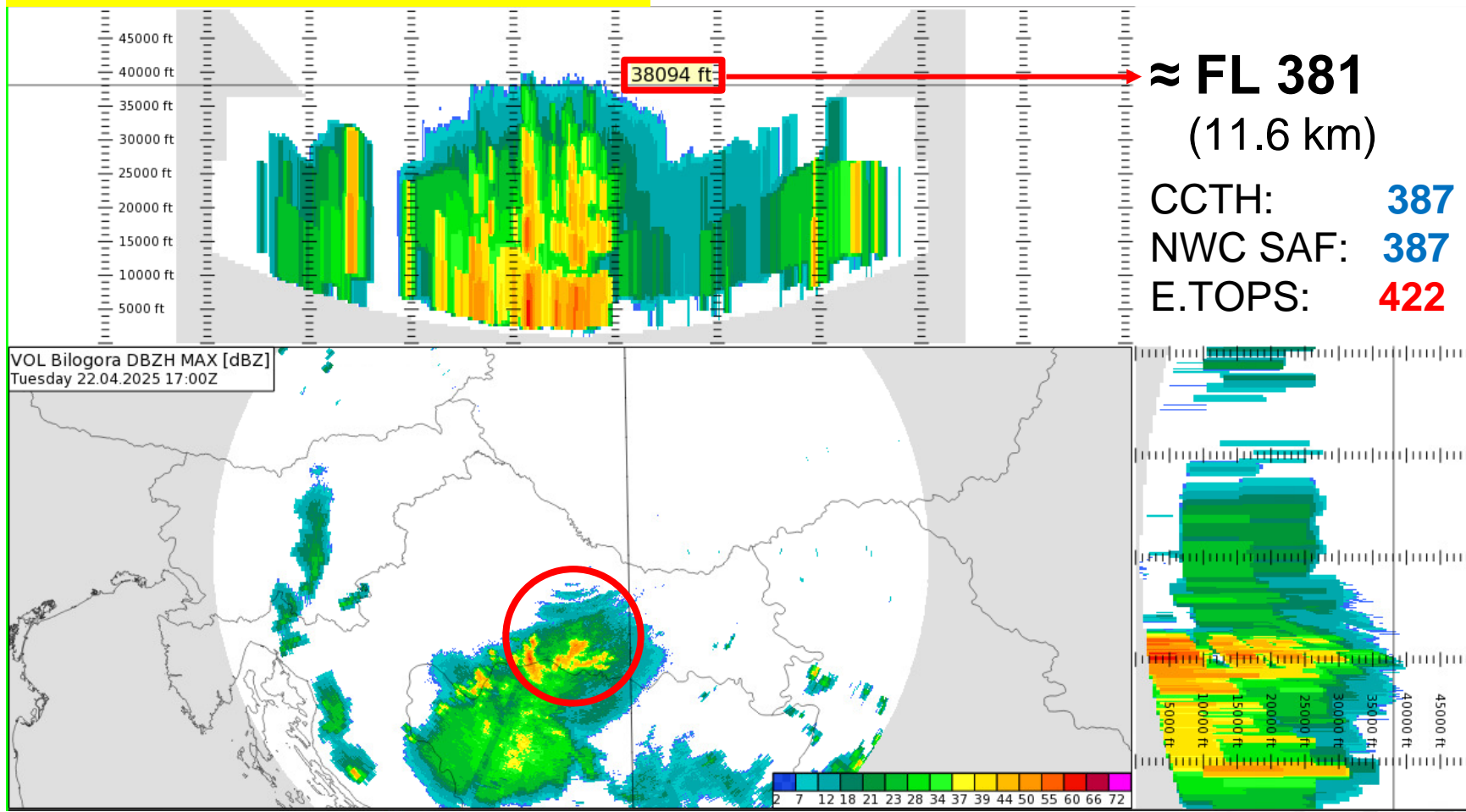
NWC SAF CTTH



ECHO TOPS



3D RADAR DATA

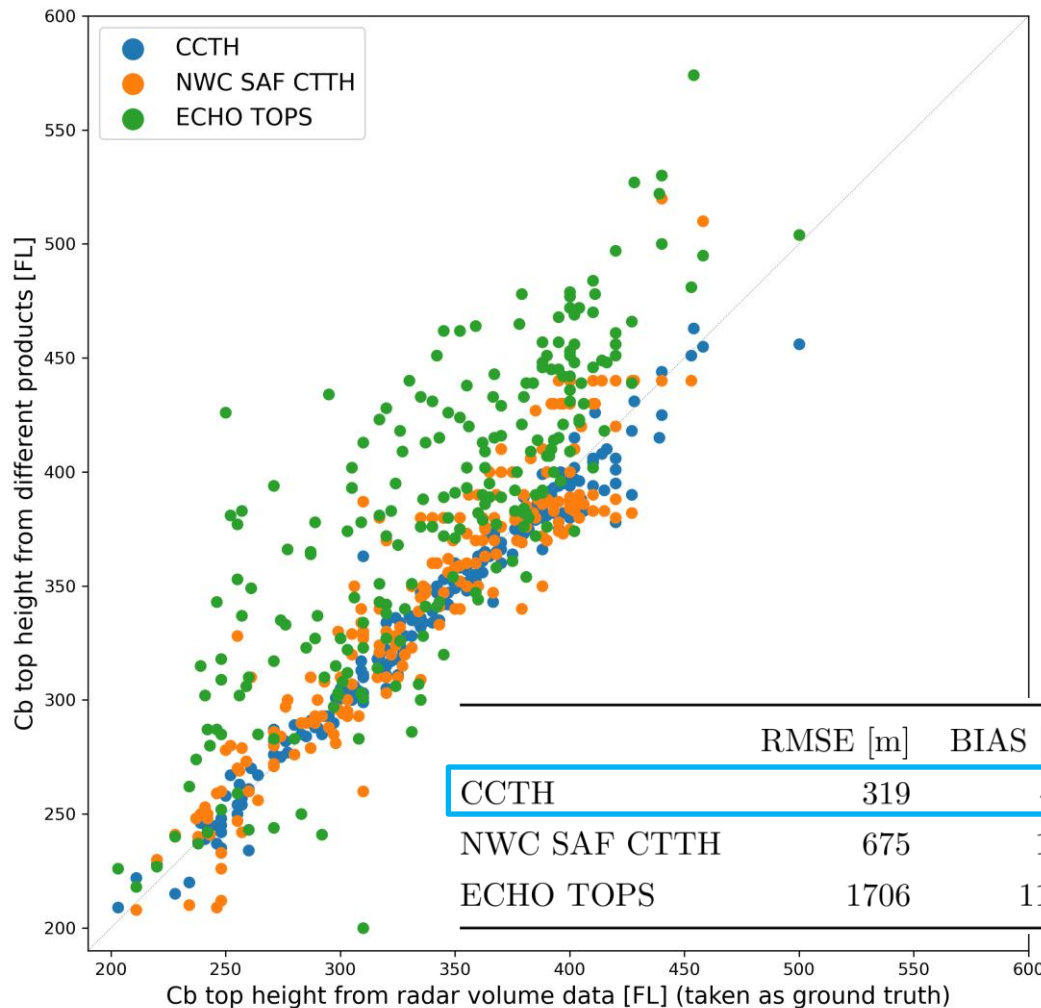


Number of cases: 223

16 km

11 km

6 km



	RMSE [m]	BIAS [m]	MAX ABS ERR [m]	r^2
CCTH	319	-67	1615	0.97
NWC SAF CTTH	675	187	2438	0.87
ECHO TOPS	1706	1195	5364	0.66

■ Summary

- **Novel and very efficient moist adiabat non-iterative approximation** for estimating convective CTH
=> **CCTH product**
- Implemented in operational visualization software (Visual Weather)
- **Ongoing validation shows very good results!**
- <https://github.com/vsoljan/cloud-top-height>

