Hailstone Inertial Adjustment to Storm Wind Fields and Implications for Growth

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Introduction

Hailstones have large inertia, delaying their adjustment to rapidly changing wind fields. This suggests (i) trajectories do not follow the horizontal wind, and (ii) growth by collection of cloud droplets may occur in *both* the vertical *and* horizontal.

Theory

A dense, spherical particle's acceleration, neglecting accelerations owing to pressure gradients, the virtual mass force, and the Basset ("history term") force, is governed by **aerodynamic drag** and **gravity** (e.g., Stout et al. 1995; Shapiro 2005):

$$\frac{d\vec{\mathbf{v}}_{\mathbf{H}}}{dt} = \frac{3}{4}\frac{\rho_{air}}{\rho_H}\frac{C_D}{D_H}\left|\vec{\mathbf{v}}_{\mathbf{rel}}\right|\vec{\mathbf{v}}_{\mathbf{rel}} - g\left(\frac{\rho_H - \rho_{air}}{\rho_H}\right)\hat{\mathbf{k}}$$

$\vec{\mathbf{v}}_{\mathbf{H}}$: hailstone 3D motion vector; ρ_{air}, ρ_H: air and hailstone density; C_D : drag coefficient

D_H: hailstone diameter; $\vec{\mathbf{v}}_{\mathbf{rel}} = \vec{\mathbf{v}}_{\mathbf{air}} - \vec{\mathbf{v}}_{\mathbf{H}}$: relative velocity between air velocity ($\vec{\mathbf{v}}_{\mathbf{air}}$) and $\vec{\mathbf{v}}_{\mathbf{H}}$

g : gravitational acceleration; $\hat{\mathbf{k}}$: vertical unit vector (positive defined upwards)

Taking the vertical component and assuming force balance gives the terminal fall speed:

$$W_T = \left(\frac{4}{3}\frac{D_H}{C_D}g\frac{\rho_H - \rho_{air}}{\rho_{air}}\right)^{\frac{1}{2}}$$

The *e*-folding timescale for hailstones initially at rest to relax to W_T (**Fig. 1a**) is useful and can be derived (e.g., Bohme 1982; Stout 1995):

$$\tau_H = W_T\left(g\frac{\rho_H - \rho_{air}}{\rho_H}\right)^{-1}$$

Alternatively, we can define an adjustment fall distance ($\delta_H = W_T\tau_H$; **Fig. 1b**).

These scales characterize ***delays in hailstone adjustments to changes in the wind field*** owing to hailstone inertia.

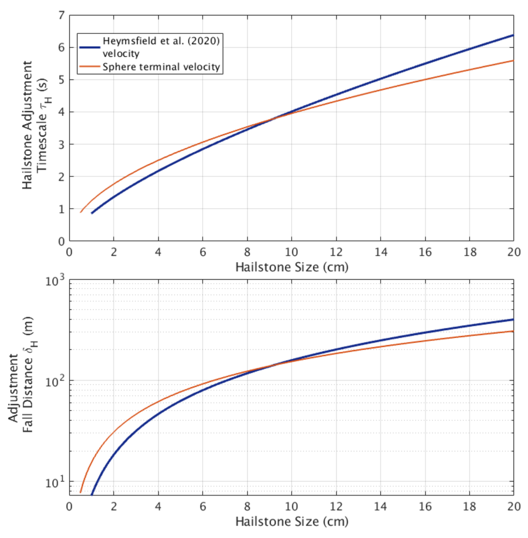

Fig. 1: Relaxation (a) time scale τ_H and (b) fall distance δ_H as a function of hail diameter for W_T shown above and that from Heymsfield et al. (2020).

Idealized Flows

Hailstones experiencing a sudden change in horizontal (u) wind take several τ_H to adjust (**Fig. 2**). During this time, hailstones can fall hundreds of meters with nonzero horizontal relative velocity (**Fig. 3a**), leading to trajectory differences of 10s of meters per km of fall (**Fig. 3b**).

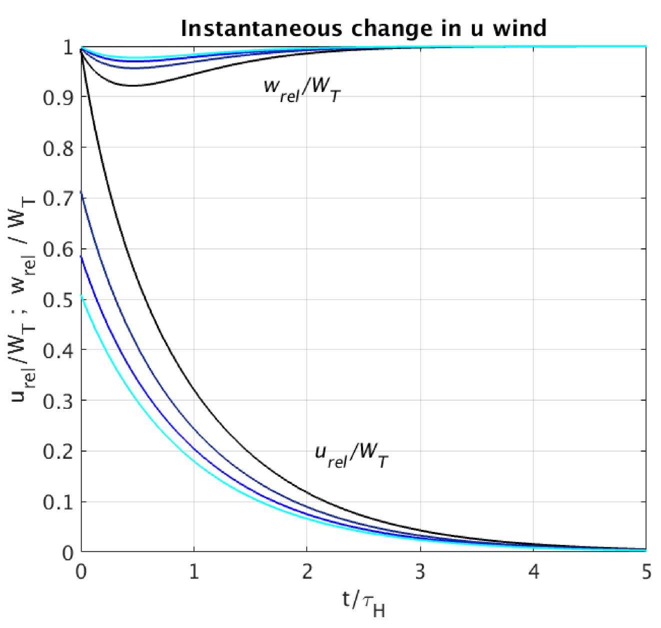

Fig. 2: Normalized horizontal relative velocity (bottom curves) and normalized vertical relative velocity (top curves) as a function of number of relaxation times for an instantaneous change in u wind. Hailstone sizes of 2.5, 5.0, 7.5, and 10.0 cm considered.

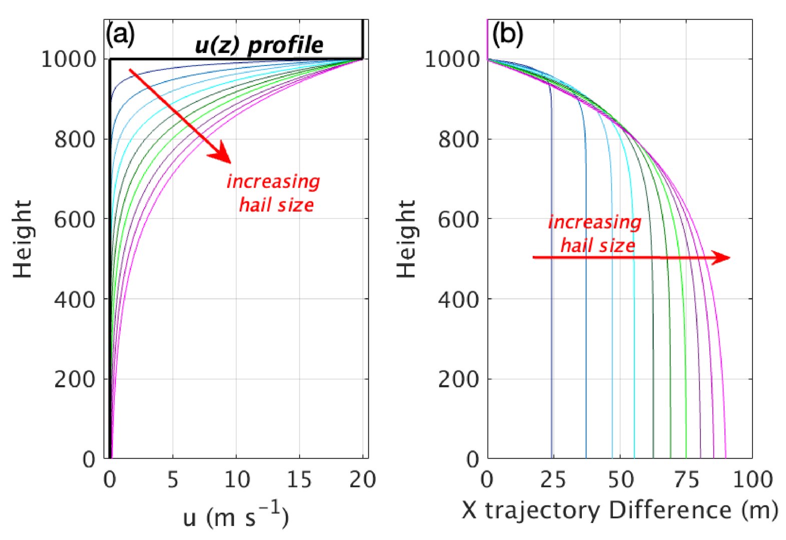

Fig. 3: (a) vertical profile of hailstone horizontal speed for the step-function horizontal wind profile (**thick black line**). (b) Horizontal trajectory distance difference from an instantaneously adjusting hailstone as a function of height. Hailstone sizes indicated by curve colors, as annotated.

As they fall, hailstones' delay in adjusting to oscillating horizontal winds means they are always "catching up": their horizontal oscillation is out of phase with the wind (**Fig. 4**). This leads to an increase in sweep-out volume that increases with u wind oscillation amplitude and with decreasing oscillation wavelength (**Fig. 5**).

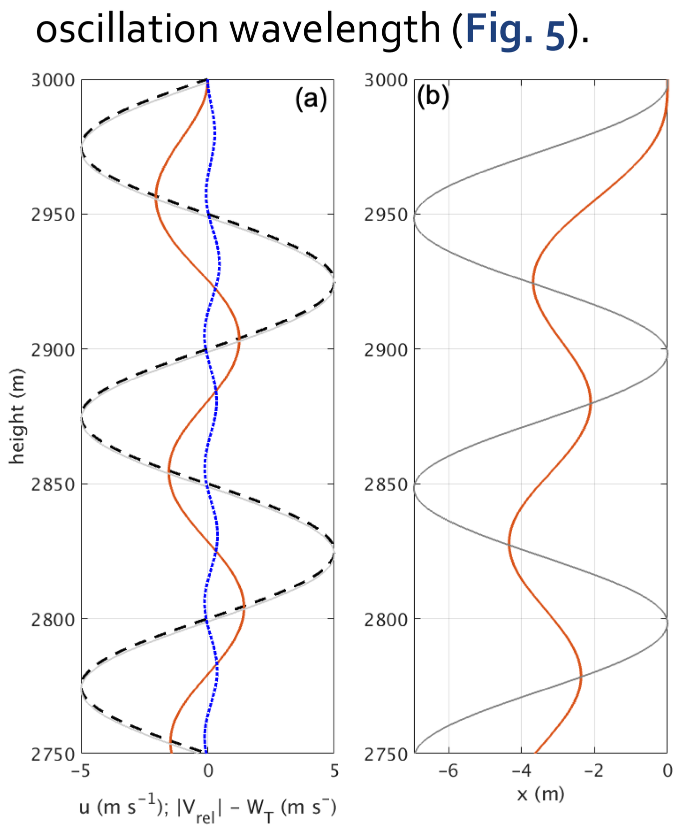

Fig. 4: (a) Vertical u-wind profile and hailstone $|V_{rel}|$-W_T for 2.5-cm and 10-cm stones. (b) Horizontal position as a function of height assuming instantaneous adjustment and for a 2.5-cm stone.

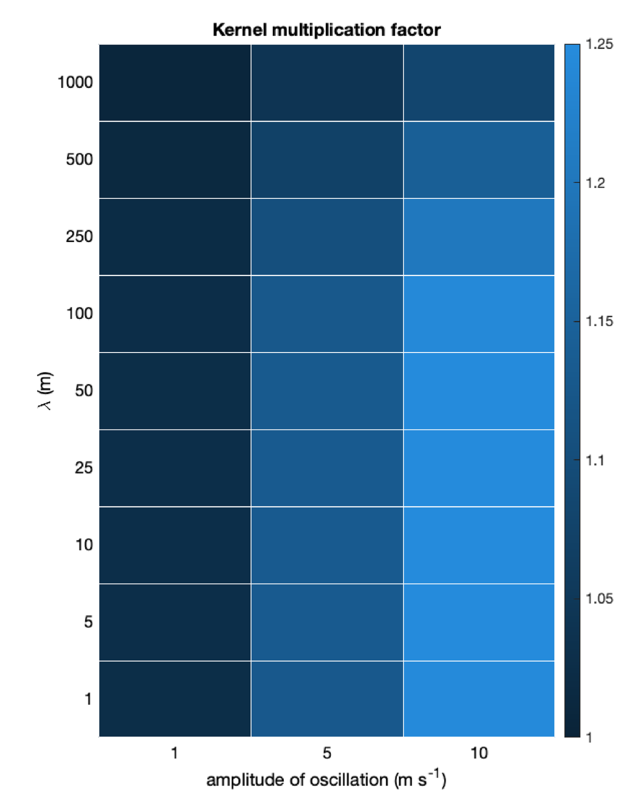

Fig. 5: Sweep-out volume enhancement (multiplicative factor relative to instantaneously adjusting hailstones) as a function of u-wind oscillation amplitude and wavelength.

Implications for Hailstone Growth

Enhanced sweep-out volume implies a potential for enhanced hailstone growth. Hailstone growth by continuous collection of supercooled cloud droplets (LWC) is

$$\frac{dm}{dt} = A_{\oplus}W_T LWC\, E_c$$

where m : hailstone's mass; $A_{\oplus}$: hailstone's cross-sectional area facing the LWC flux; W_T : hailstone terminal fall speed; E_c: collection efficiency

This explicitly assumes the ***only*** relative velocity between the air and hailstone motion is in the vertical, and that the hailstone always falls at W_T. ***Neither is true.***

Key Messages:

Hailstones' inertia prevents their immediate adjustment to changes in airflow; rather, it takes ~seconds. During this adjustment, hailstones can have appreciable velocity relative to the air in both horizontal and vertical directions. This enhances their growth collection kernel. However, the adjustment also alters potential growth pathways, de-emphasizing those in regions of large spatial wind gradients. Hailstones also retain greater kinetic energy near the ground in cases of strong downdrafts/outflow, potentially increasing damage potential.

Growth Implications (continued)

Instead, it ***should be***

$$\frac{dm}{dt} = \oiint \overrightarrow{\mathbf{F}_{\mathrm{LWC}}}\cdot\hat{\mathbf{n}}\,dA, \quad \text{where the LWC flux } \overrightarrow{\mathbf{F}_{\mathrm{LWC}}} = \begin{cases} LWC\,\vec{\mathbf{v}}_{\mathbf{rel}} & \text{for } \vec{\mathbf{v}}_{\mathbf{rel}}\cdot\hat{\mathbf{n}} < \mathbf{0} \\ 0 & \text{for } \vec{\mathbf{v}}_{\mathbf{rel}}\cdot\hat{\mathbf{n}} \geq \mathbf{0} \end{cases}$$

Thus, for spherical hail:

$$\frac{dm}{dt} = A_{\oplus}LWC\,E_c\left(u_{\mathrm{rel}}^2 + v_{\mathrm{rel}}^2 + w_{\mathrm{rel}}^2\right)^{\frac{1}{2}}$$

For nonspherical hail, $A_{\oplus} = f(\vec{\mathbf{v}}_{\mathbf{rel}})$ (e.g., Lin et al. 2024).

Implementation into Full-Physics Model

We implemented inertial adjustment into our Hail Growth Trajectory Model (Kumjian & Lombardo 2020) and ran it on composite fields of the "umax36" storm; embryos from 1 to 10 mm were seeded. Surprisingly, inertial adjustment leads to *reduced* hail sizes overall (**Figs. 6, 7**). This is despite general enhancements to the collection kernel averaged over the trajectories (**Fig. 8**).

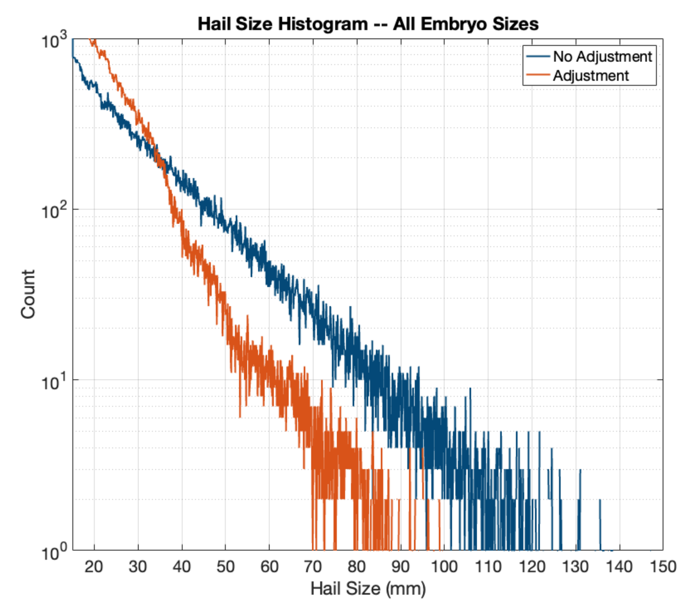

Fig. 6: Histograms of final hail sizes attained for all embryo sizes in simulations with no adjustment and with inertial adjustment.

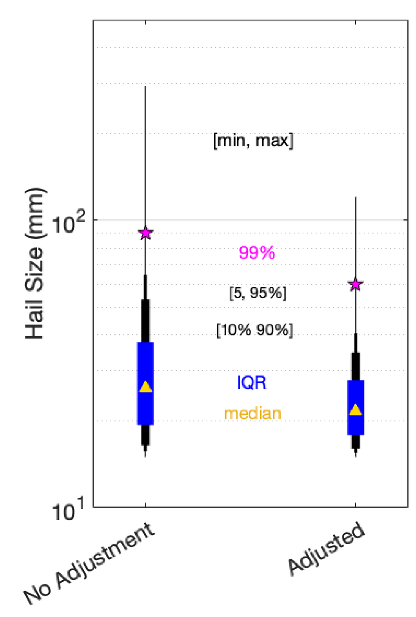

Fig. 7: Box-and-whisker plots summarizing final hail sizes for no inertial adjustment (left), and with inertial adjustment (right). Quantiles conditionally sampled for final sizes >15 mm.

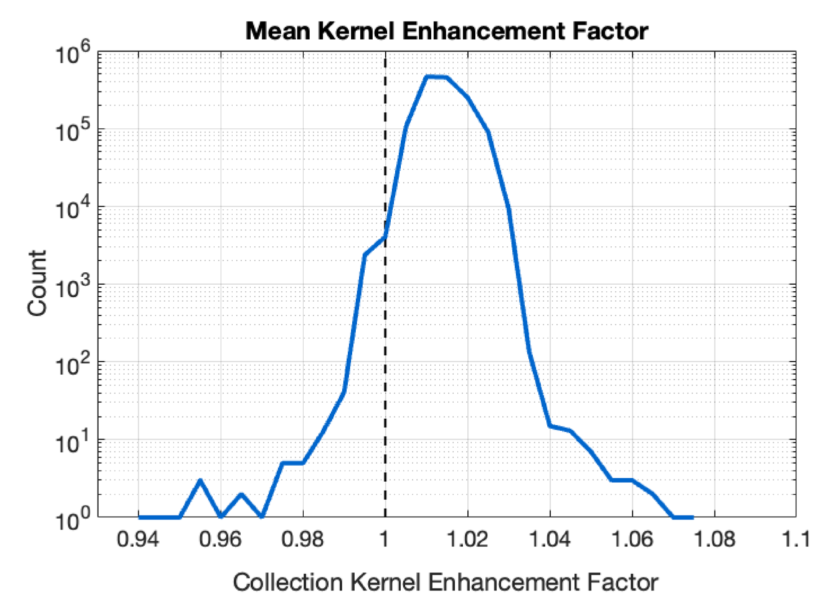

Fig. 8: Histogram of the trajectory-averaged collection kernel enhancement factor for all embryo sizes.

Comparison of embryo maps (**Fig. 9**) reveals that a prime embryo seeding location is shut down when inertial adjustment is implemented.

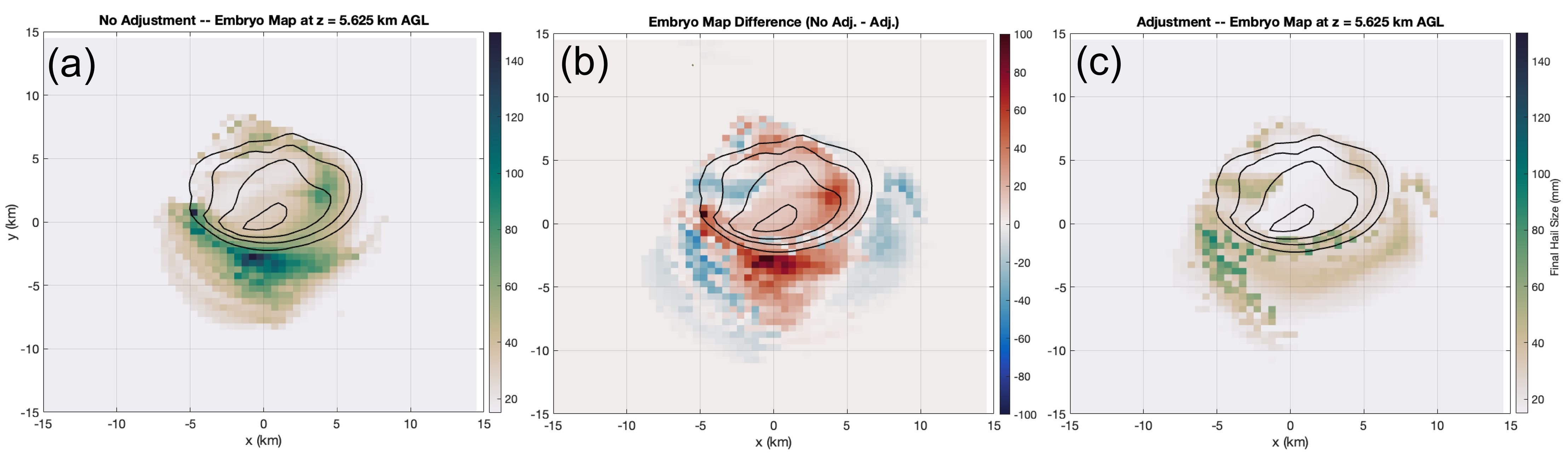

Fig. 9: (a) Embryo map for z = 5.625 km AGL, showing the maximum final hail size (in mm, according to color bar) of embryos seeded at a given location for the simulation with no momentum adjustment. Black contours are the updraft at 10, 20, 30, and 40 m s^{-1}. (b) As in (a), except the difference in maximum final hail size between no adjustment and momentum adjustment is shown. (c) As in (a), except the simulation with momentum adjustment is shown.

Examination of trajectories reveals subtle flow features that sweep embryos into optimal pathways (e.g., regions of enhanced vertical vorticity magnitude, localized deformation enhancements, etc.) in the simulation without momentum adjustment that fail to do so in the simulation with momentum adjustment, resulting in a less-favorable trajectory (**Fig. 10**).

Thus, although **inertial adjustment enhances hailstone collection kernels**, it **can disallow access to certain growth pathways** that involve large spatial wind gradients.

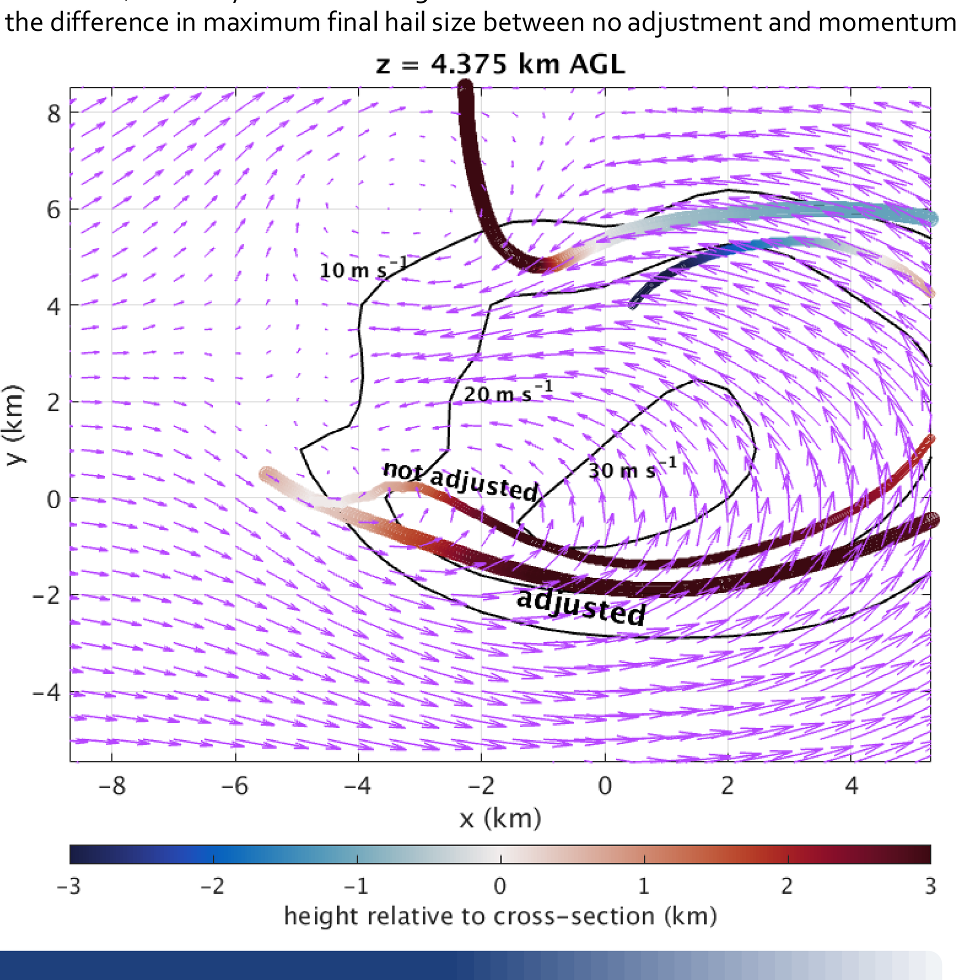

Fig. 10: Horizontal cross-section through the simulated storm at z = 4.375 km AGL showing **updraft contours** and horizontal winds at that altitude. Illustrative hailstone trajectories for the momentum adjustment simulation (**thicker colored line**) and the no momentum adjustment simulation (thinner colored line) initialized at the same location are overlaid. Line color indicates altitude of the trajectory relative to that of the cross section, where blues indicate below and reds indicate above.

Implications for Wind-Driven Hail

Future work will investigate how hailstones' inertial adjustment can lead to enhanced kinetic energy at the surface in cases of strong low-level horizontal winds (**Fig. 11**) and/or a downburst (**Fig. 12**). Delayed adjustment means hailstones retain more of their kinetic energy even as the winds decrease owing to the ground's influence.

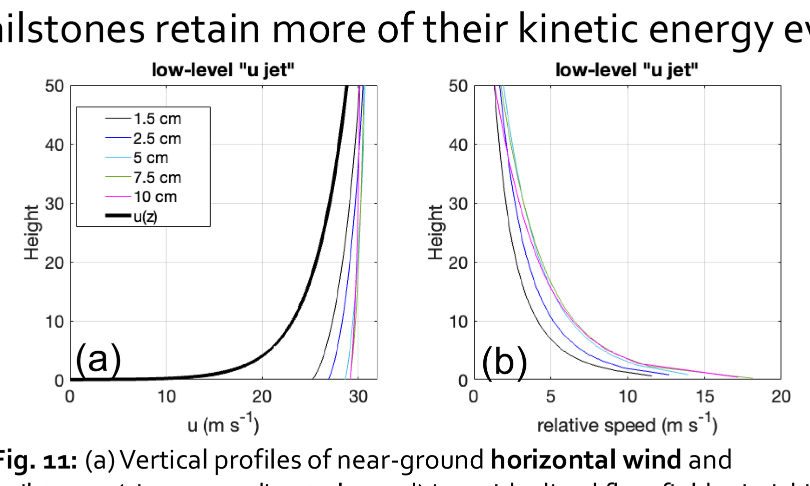

Fig. 11: (a) Vertical profiles of near-ground **horizontal wind** and hailstones (sizes according to legend) in an idealized flow field mimicking strong thunderstorm outflow. (b) As in (a), but showing vertical profiles of horizontal relative speed of the hailstones.

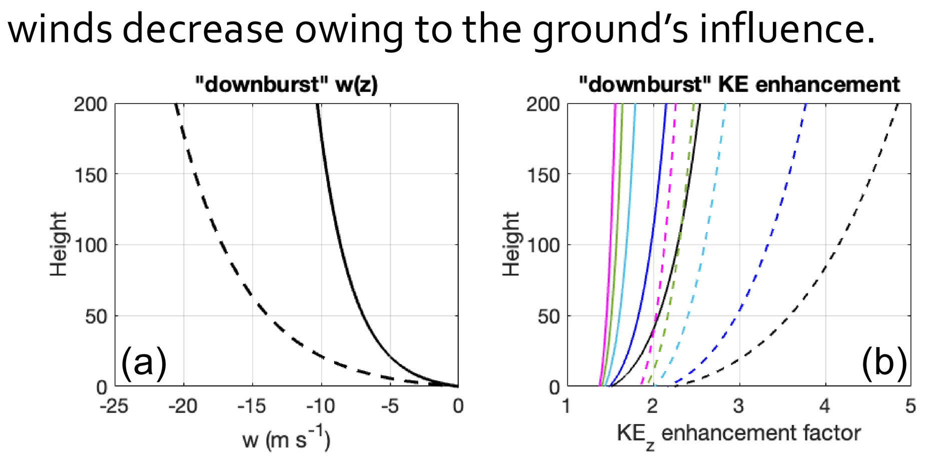

Fig. 12: (a) Vertical profiles of near-ground **vertical wind** for two idealized "downbursts." (b) Vertical profiles of near-ground kinetic energy enhancement factor for hailstones, colored by size as in Fig. 11.