AN INTEGRATIVE DATA-ADAPTIVE APPROACH FOR GLOBAL SENSITIVITY ANALYSIS

APPLICATION TO SUBSURFACE FLOW AND TRANSPORT

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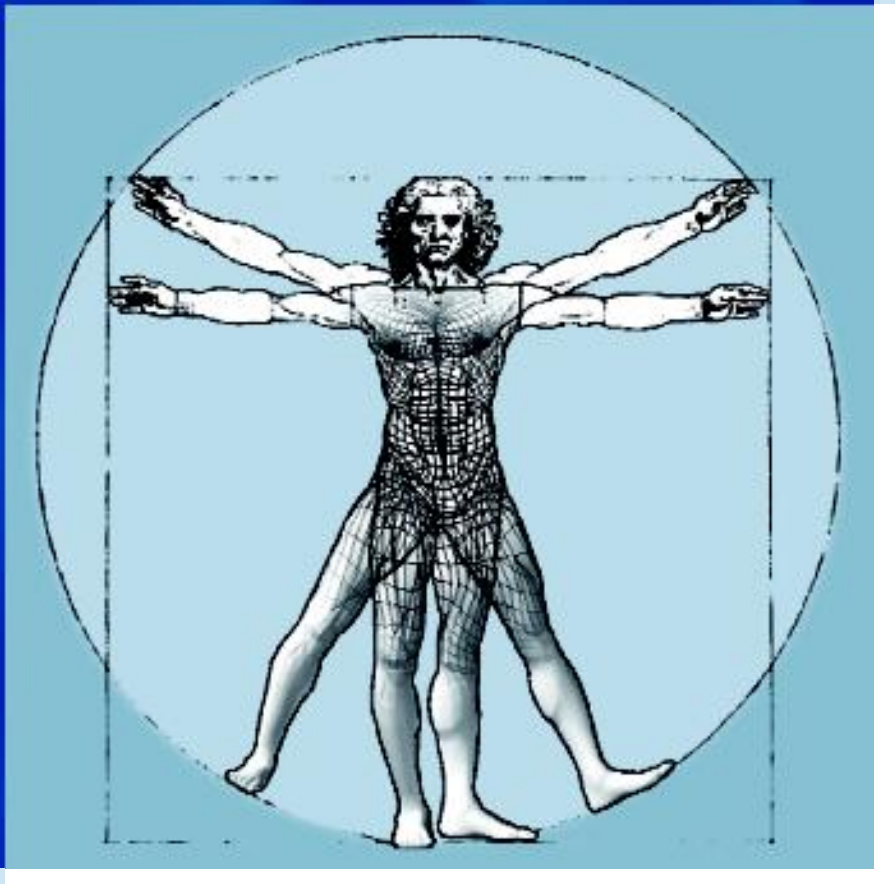


MOTIVATION

Modeling uncertain systems continues to pose high demands. For example predicting flow and transport processes in the subsurface is a challenge since uncertainty in hydraulic properties of the subsurface is ubiquitous. Data sets are limited and costly. For such reasons, stochastic tools are required to support engineering design tasks under uncertainty, and sensitivity analysis with respect to uncertain model parameters yields valuable information.

OBJECTIVES

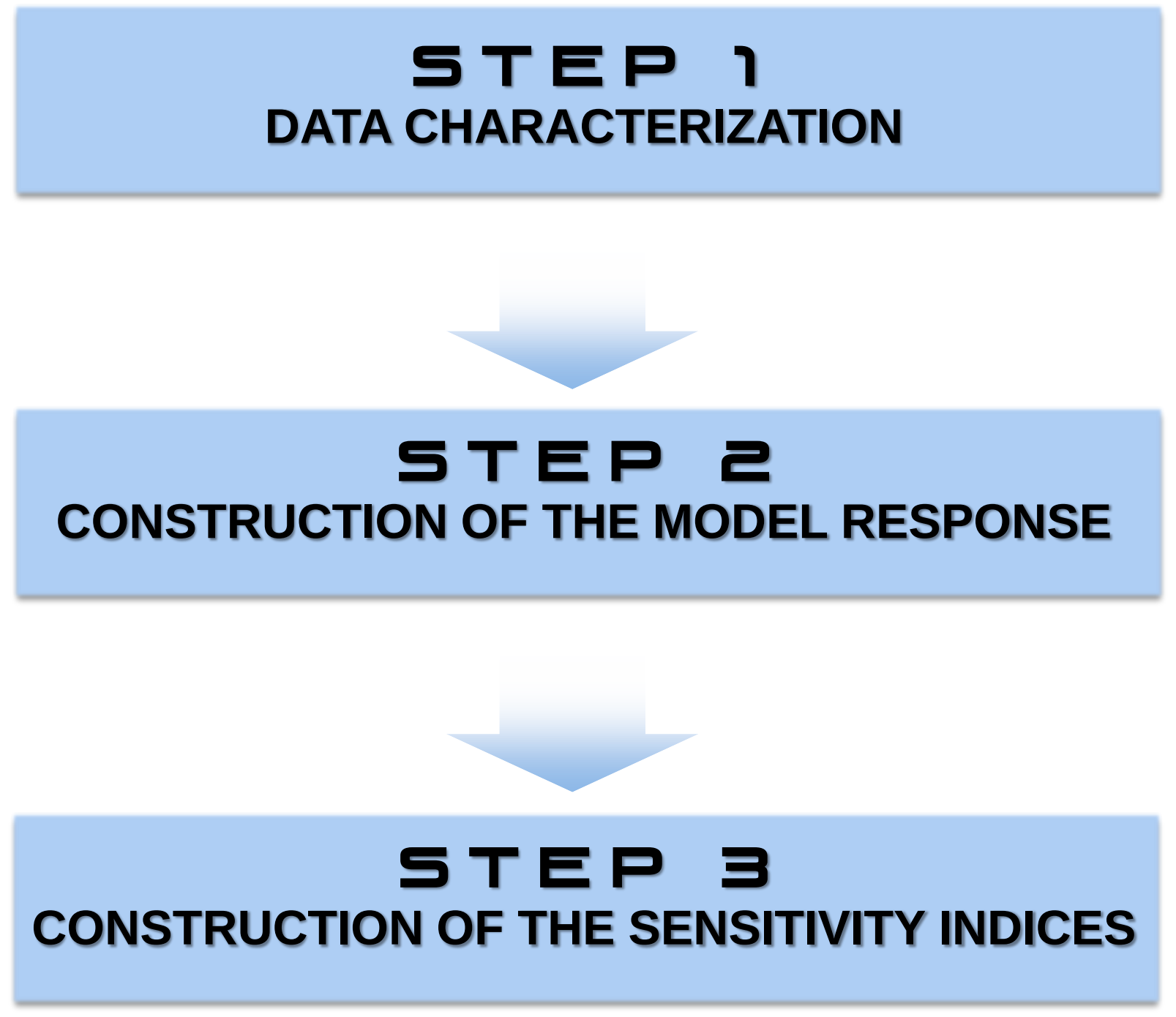
- **Tackle** global sensitivity analysis (GSA) and uncertainty quantification using aPC [1].
- **Make** use of a hybrid-analytical framework towards efficient computations.
- **Provide** method that takes into account different sources of information (raw data, statistical distributions, etc).

METHODOLOGY

We propose a response surface method for global sensitivity analysis (based on the arbitrary Polynomial Chaos Expansion, aPC). We use analytical and hybrid analytical-numerical formulations that further improve computational efficiency.

The key advantage are:

- **reduces the computational** burden associated with Monte Carlo methods or with conventional global sensitivity analysis that also requires many evaluations of a simulation model [5];
- **method incorporates** probabilities or weighting functions for the investigated model parameter and the full range of possible simultaneous outcomes;
- **data-adaptive framework** allows to incorporate this information while requiring only a finite number of statistical moments for the investigated parameters.



STEP 1
DATA CHARACTERIZATION

STEP 2
CONSTRUCTION OF THE MODEL RESPONSE

STEP 3
CONSTRUCTION OF THE SENSITIVITY INDICES

THREE-STEP GSA ALGORITHM

PHYSICAL SCENARIO

We will demonstrate our methodology for a contaminant transport problem in a 3D heterogeneous porous media. A solute is instantaneously released and undergoes purely advective transport. The aquifer has a hydraulic conductivity tensor and effective porosity.

ANALITICAL SOLUTION

For illustration purposes, we consider flow to be incompressible, single-phased, at steady-state, free of boundary effects and with velocity satisfying Darcy law:

$$\mathbf{u}(\mathbf{x}) = -\frac{\mathbf{K}(\mathbf{x})}{n_e(\mathbf{x})}\nabla h$$

where the hydraulic head is determined from the continuity equation:

$$\nabla \cdot [\mathbf{K}(\mathbf{x}) \nabla h(\mathbf{x})] = 0.$$

A tracer with initial concentration is instantaneously released from a rectangular source volume under purely advective transport conditions. Under these conditions, the governing equation for transport is:

$$\frac{\partial C}{\partial t} + u_i \frac{\partial C}{\partial x_i} = 0$$

With the aid of the Lagrangian framework, concentration mean can be expressed as [6]:

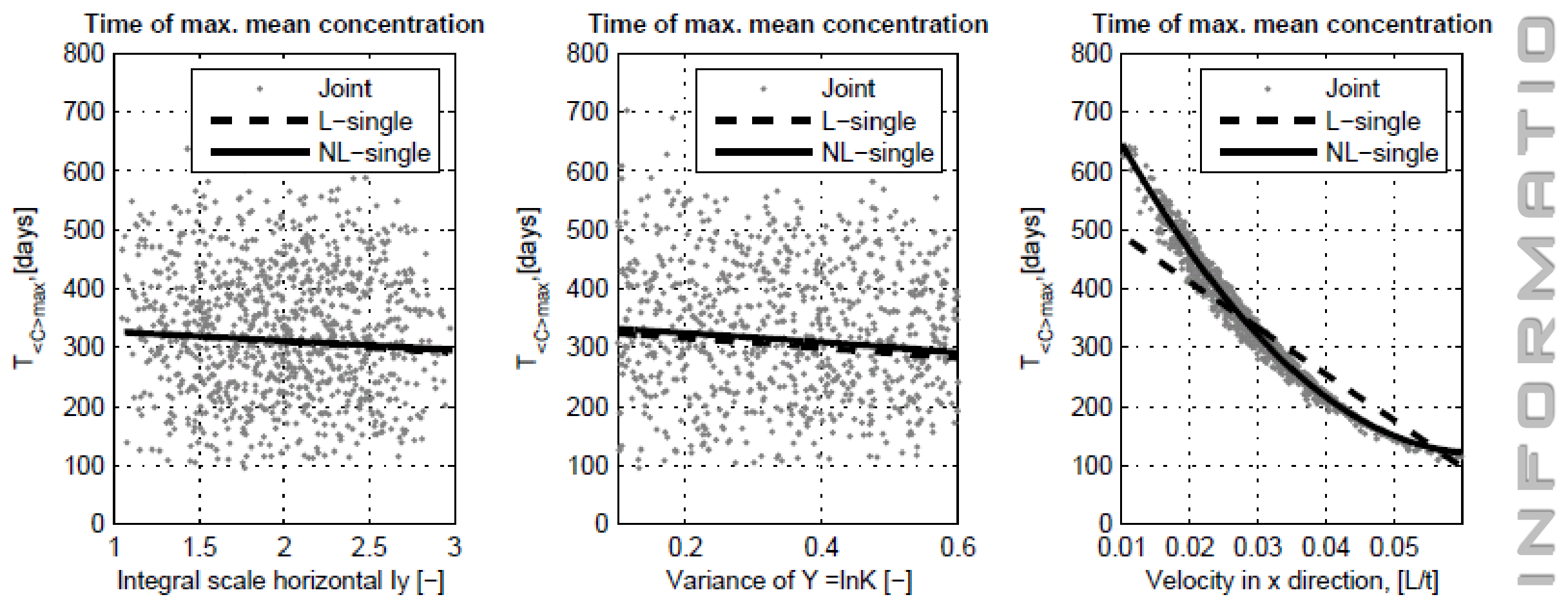
$$\langle C(\mathbf{x},t)\rangle = C_0 \prod_{i=1}^{3} \psi_i(\mathbf{x},t) \quad \psi_i(\mathbf{x},t) = \frac{1}{2}\mathrm{erf}\left[\frac{x_i - U_i t + L_i/2}{\sqrt{2X_{ii}(t;\mathbf{a})}}\right] - \frac{1}{2}\mathrm{erf}\left[\frac{x_i - U_i t - L_i/2}{\sqrt{2X_{ii}(t;\mathbf{a})}}\right]$$

SENSITIVITY ANALYSIS

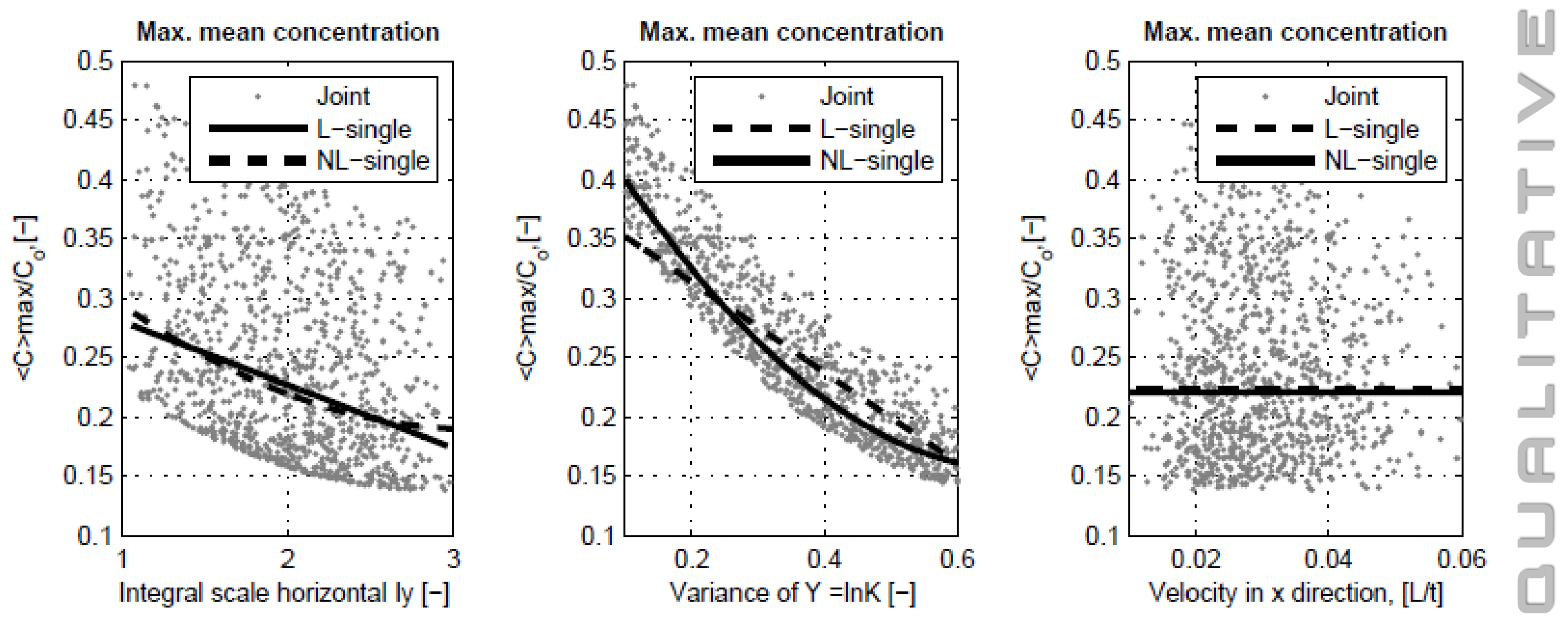
We perform a GSA for two distinct prediction: (1) time of peak mean concentration and (2) the magnitude of the maximum mean concentration. We study the sensitivity towards three uncertain parameters (integral scale, variance of logconductivity and mean longitudinal velocity).

QUALITATIVE INFORMATION

SENSITIVITY OF THE TIME OF THE PEAK MEAN CONCENTRATION



SENSITIVITY OF THE MAGNITUDE OF THE PEAK MEAN CONCENTRATION



QUANTITATIVE INFORMATION

Sobol Index [2] that indicates what fraction of the of total variance of output can be traced back to the joint contributions of the parameters {i,j,..,k}.

Total Index [3] expresses the total contribution to the variance of model output due to the uncertainty of an individual parameter

Weighted Sensitivity Index [4] reflects the slope due to an individual parameter, but averaged over all statistical distribution of all other parameters

SOBOL INDICES FOR TIME CONCENTRATION OF MAXIMAL MEAN CONCENTRATION AND MAXIMAL MEAN

Sobol index	S_1	S_2	S_3	S_{12}	S_{13}	S_{23}
Value for $T_{\langle C\rangle,\max}$	0.004	0.016	0.975	8×10^{-4}	6×10^{-4}	0.003
Rank for $T_{\langle C\rangle,\max}$	3	2	1	6	5	4
Value for $\langle C\rangle_{\max}$	0.111	0.887	6×10^{-9}	0.002	9×10^{-10}	1×10^{-10}
Rank for $\langle C\rangle_{\max}$	2	1	4	3	5	6

TOTAL SENSITIVITY INDICES FOR TIME OF MAXIMAL MEAN CONCENTRATION AND MAXIMAL MEAN CONCENTRATION

Total sensitivity index	S_1^T	S_2^T	S_3^T
Value for $T_{\langle C\rangle,\max}$	0.005	0.021	0.980
Rank for $T_{\langle C\rangle,\max}$	3	2	1
Value for $\langle C\rangle_{\max}$	0.2113	0.889	8×10^{-9}
Rank for $\langle C\rangle_{\max}$	2	1	3

WEIGHTED SENSITIVITY INDICES FOR TIME OF MAXIMAL MEAN CONCENTRATION AND MAXIMAL MEAN CONCENTRATION

Weighted sensitivity index	S_{ω_1}	S_{ω_2}	S_{ω_3}
Value for $T_{\langle C\rangle,\max}$	14.246	66.163	2×10^{4}
Rank for $T_{\langle C\rangle,\max}$	3	2	1
Value for $\langle C\rangle_{\max}$	0.143	0.958	7×10^{-4}
Rank for $\langle C\rangle_{\max}$	2	1	3

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