# Quadtree-adaptive tsunami modelling

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## Adaptive solutions of Partial Differential Equations

- Gerris Flow Solver

  `gfs.sf.net`

- Navier–Stokes, Euler, Saint-Venant

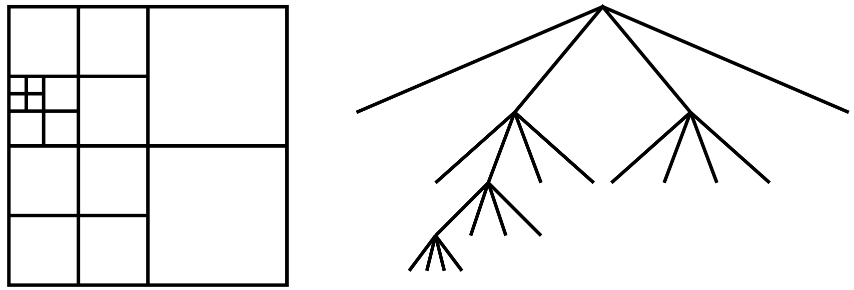
  etc...

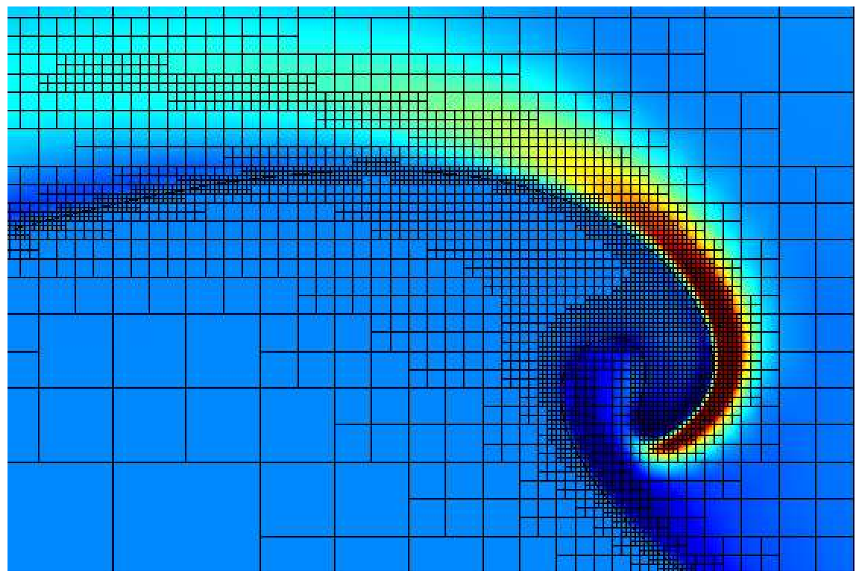
- Adaptive quad/octree discretisation

- Free Software (GPL)

- Parallel with dynamic load-balancing

- Popinet (2003, 2009), *JCP*





## “The curse of dimensionality”

## or is adaptive mesh refinement necessary?

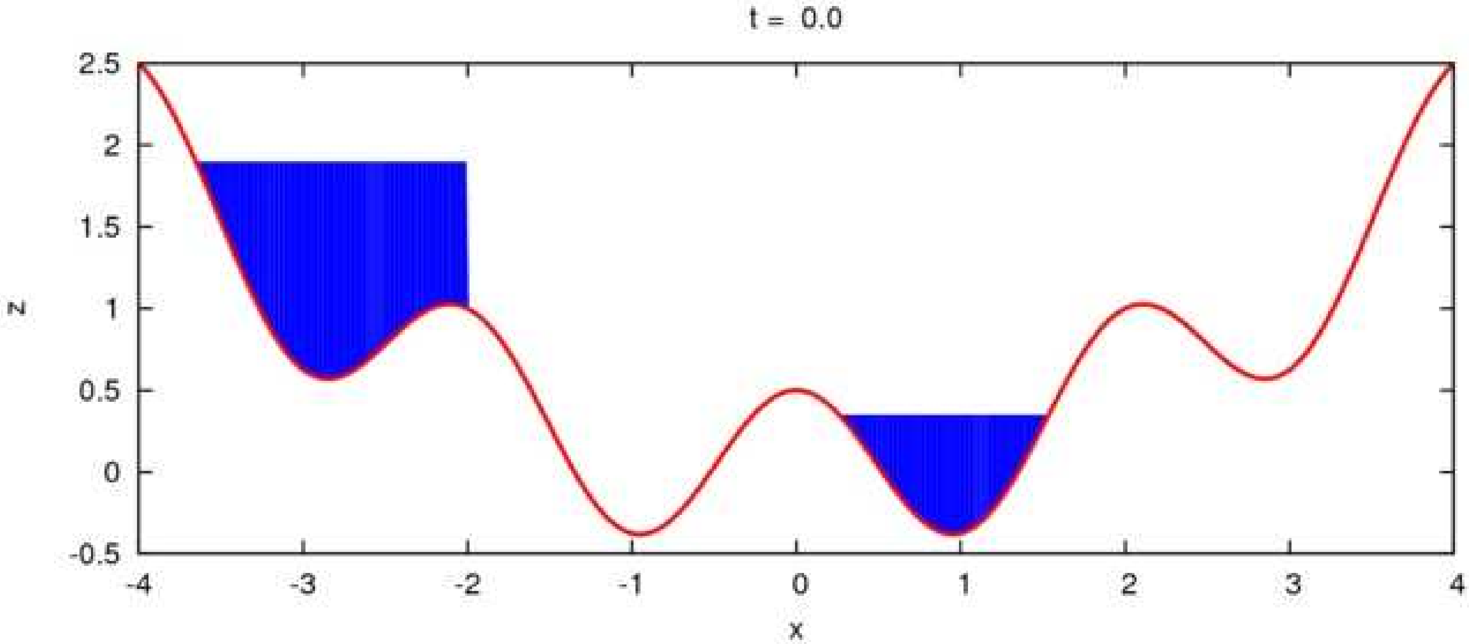
- The universe has (at least) four dimensions
- Using regular Cartesian grids, solution costs scale like

$$C\Delta^{-4}$$

  with $C$ a constant and $\Delta$ the spatial resolution in each dimension

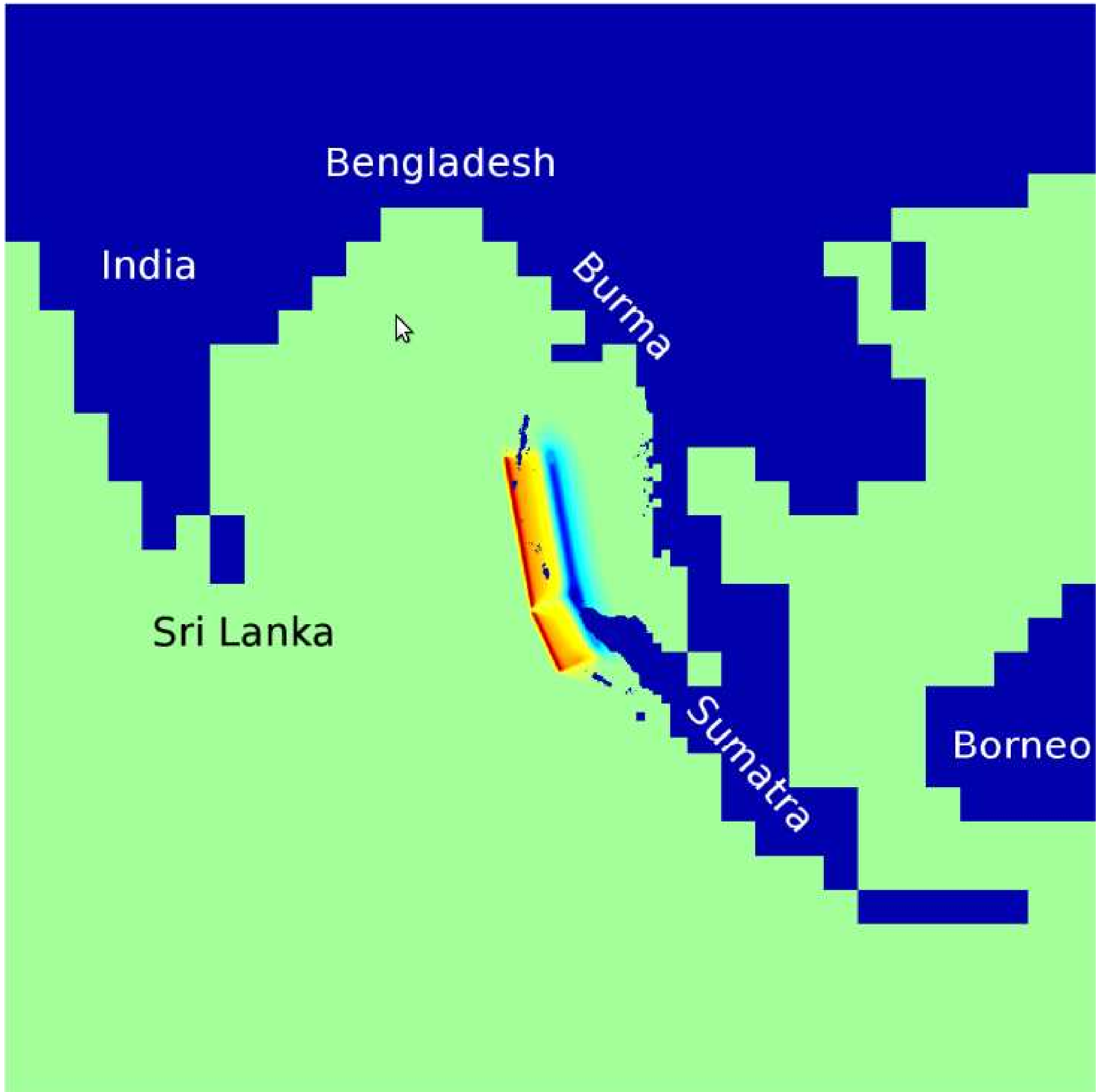
- Just buy bigger computers! A 100-fold increase in computing power will buy you a $\sqrt[4]{100} \approx 3$-fold increase in resolution... (assuming $C$ does not increase)
- Can adaptive methods break the spell?

# The Saint-Venant equations



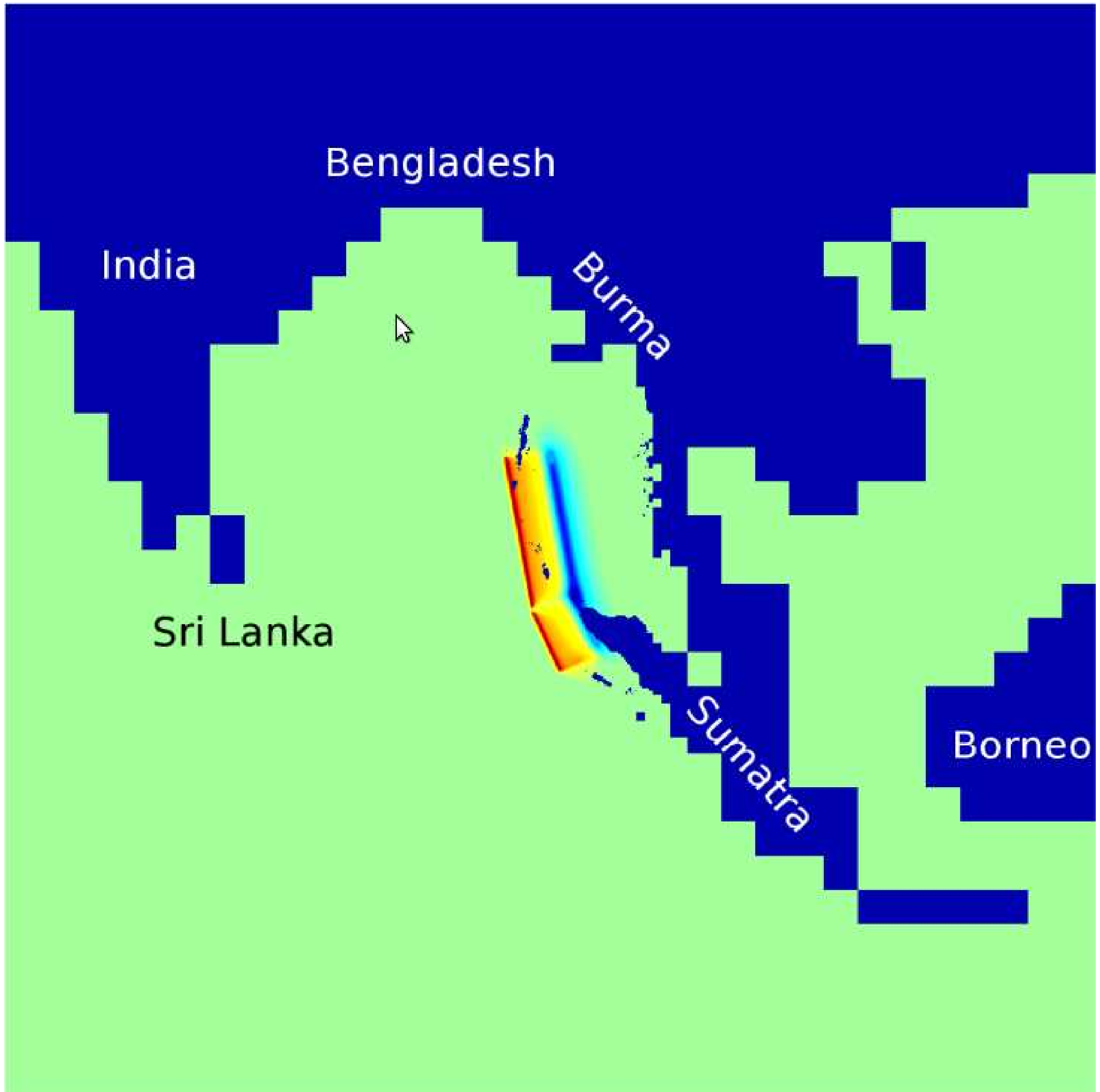

- Godunov-type finite-volume scheme
- HLLC approximate Riemann solver
- Wetting/drying, hydrostatic equilibrium: scheme of Audusse et al (2004)

# 2004 Indian ocean tsunami



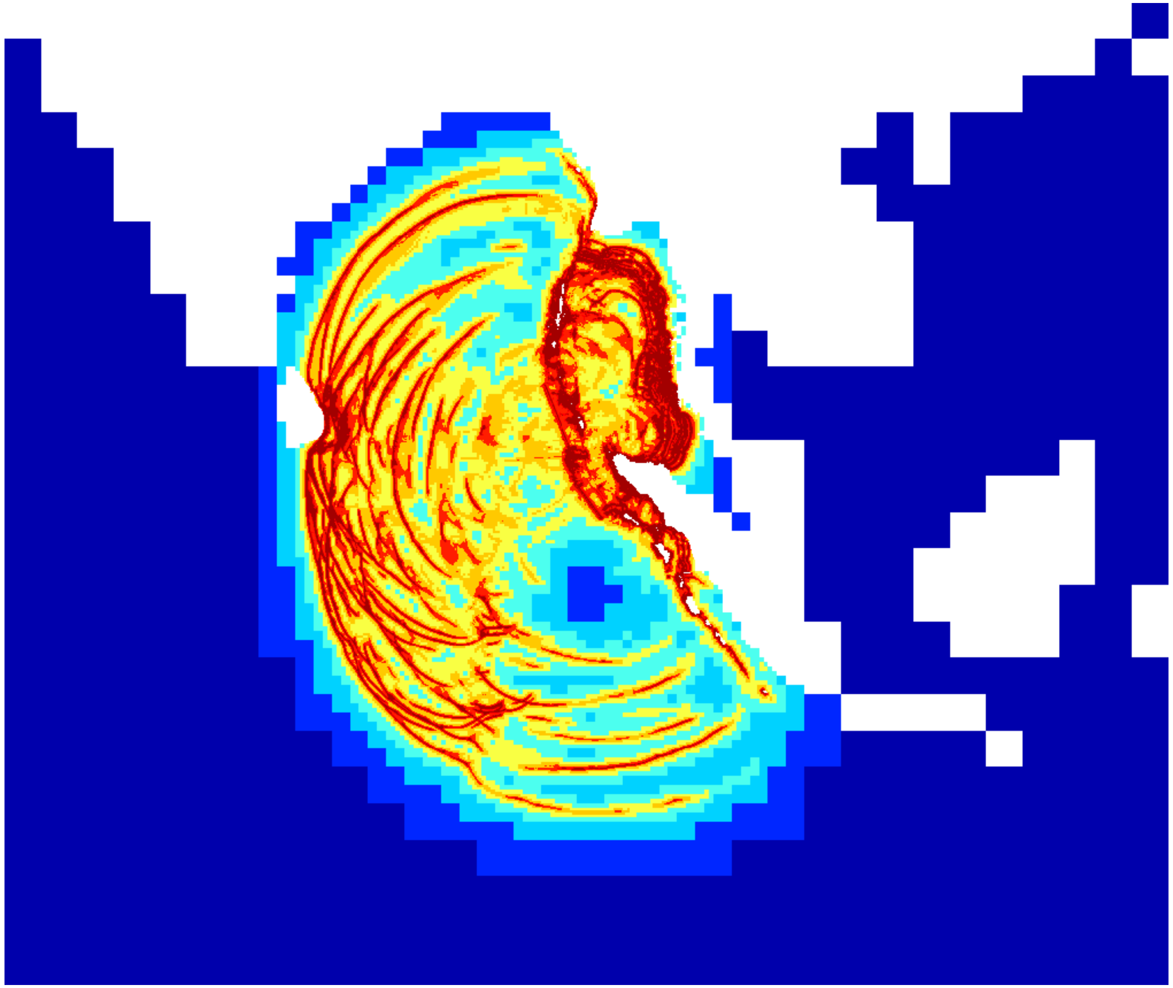

Staggered fault displacement model (5 segments)

# 2004 Indian ocean tsunami



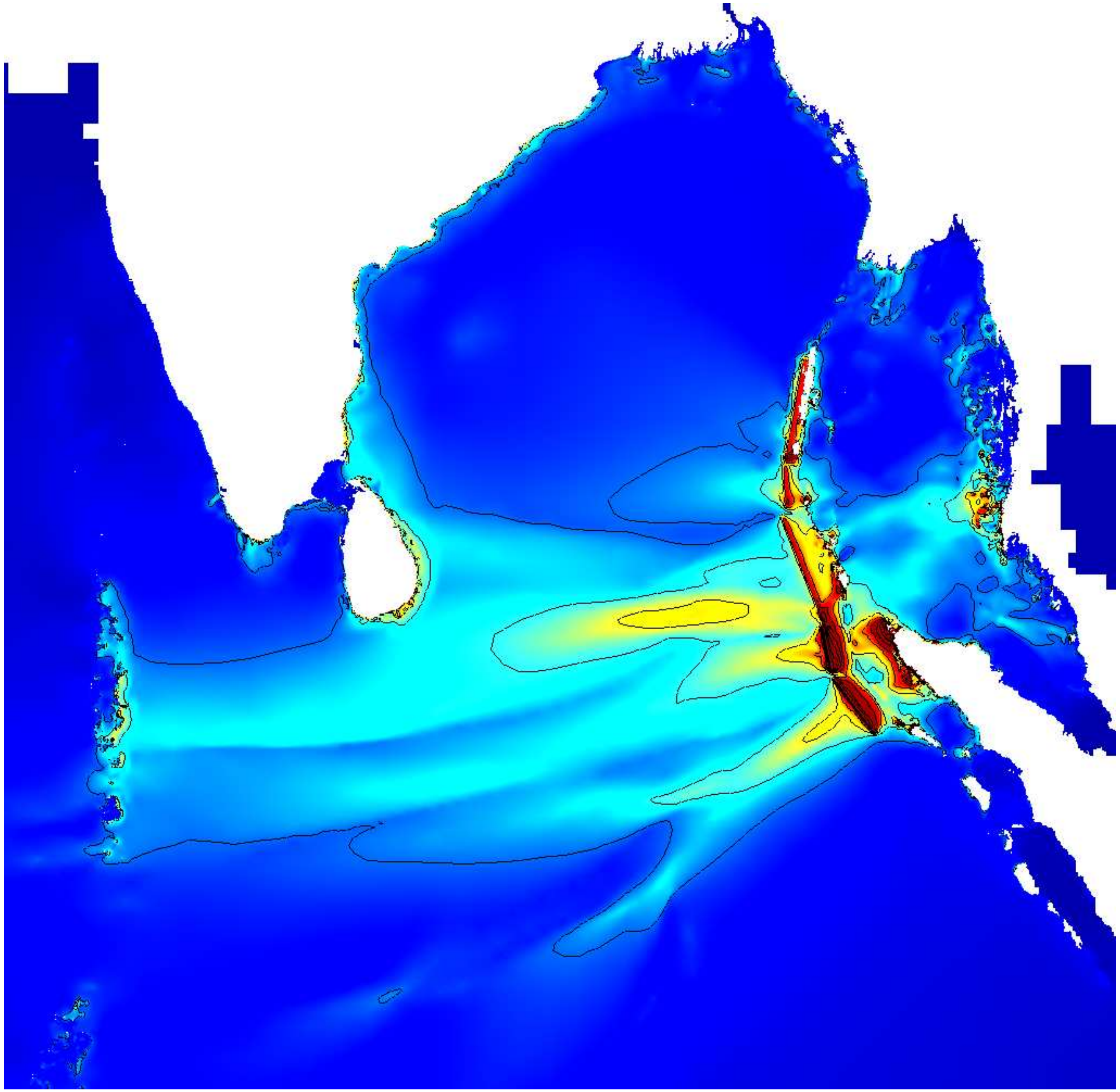

1 km $\leq$ Spatial resolution $\leq$ 150 km

# Adaptivity

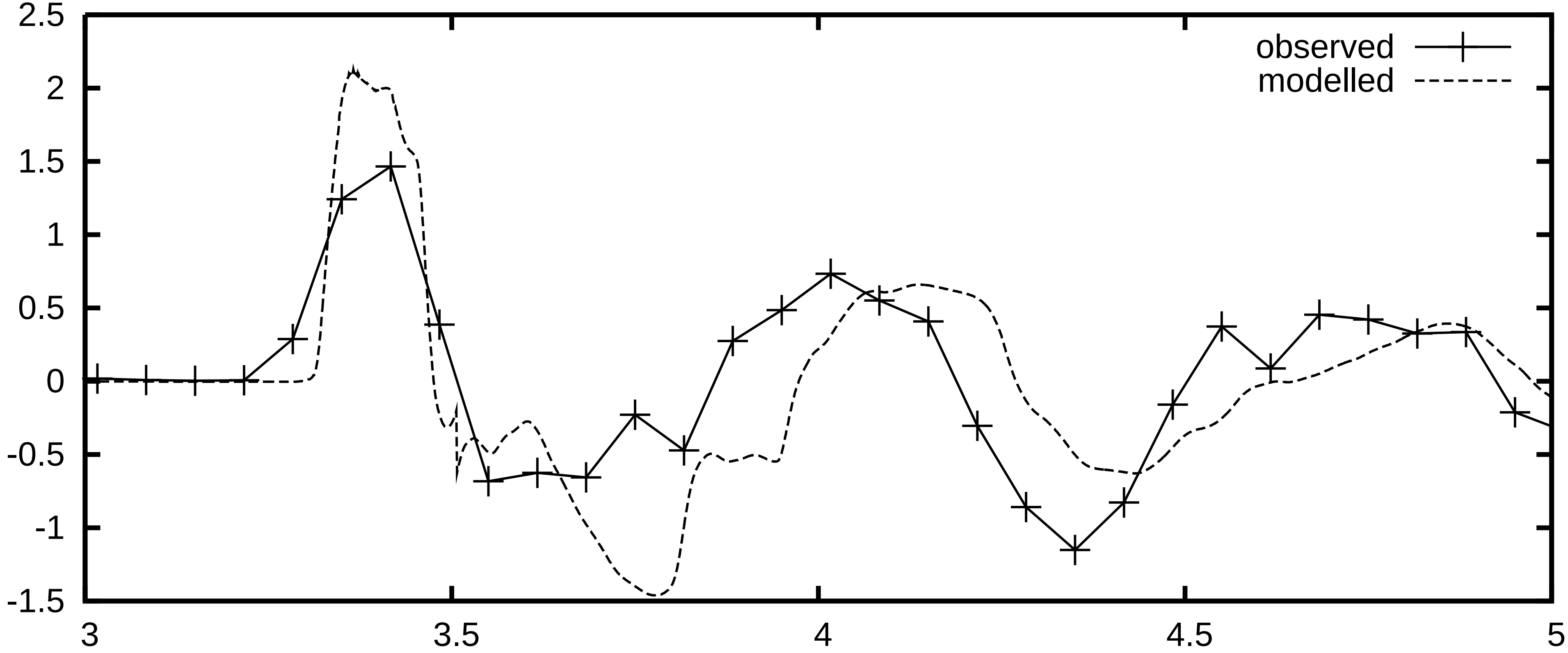


Truncation error of the wave height $<$ 5 cm

## Maximum wave height



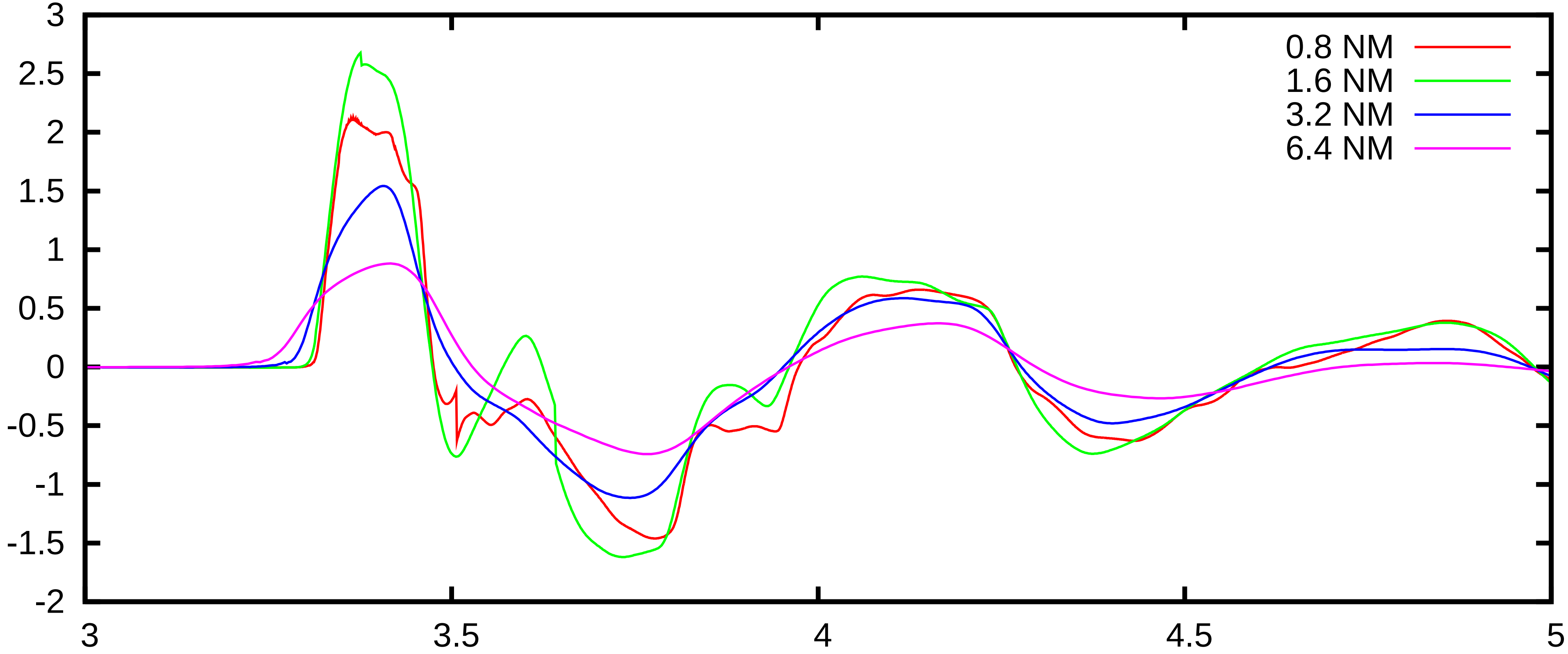
## Comparison with field data




Tide gauge at Male, Maldives

Time in hours, wave height in metres

# Effect of spatial resolution

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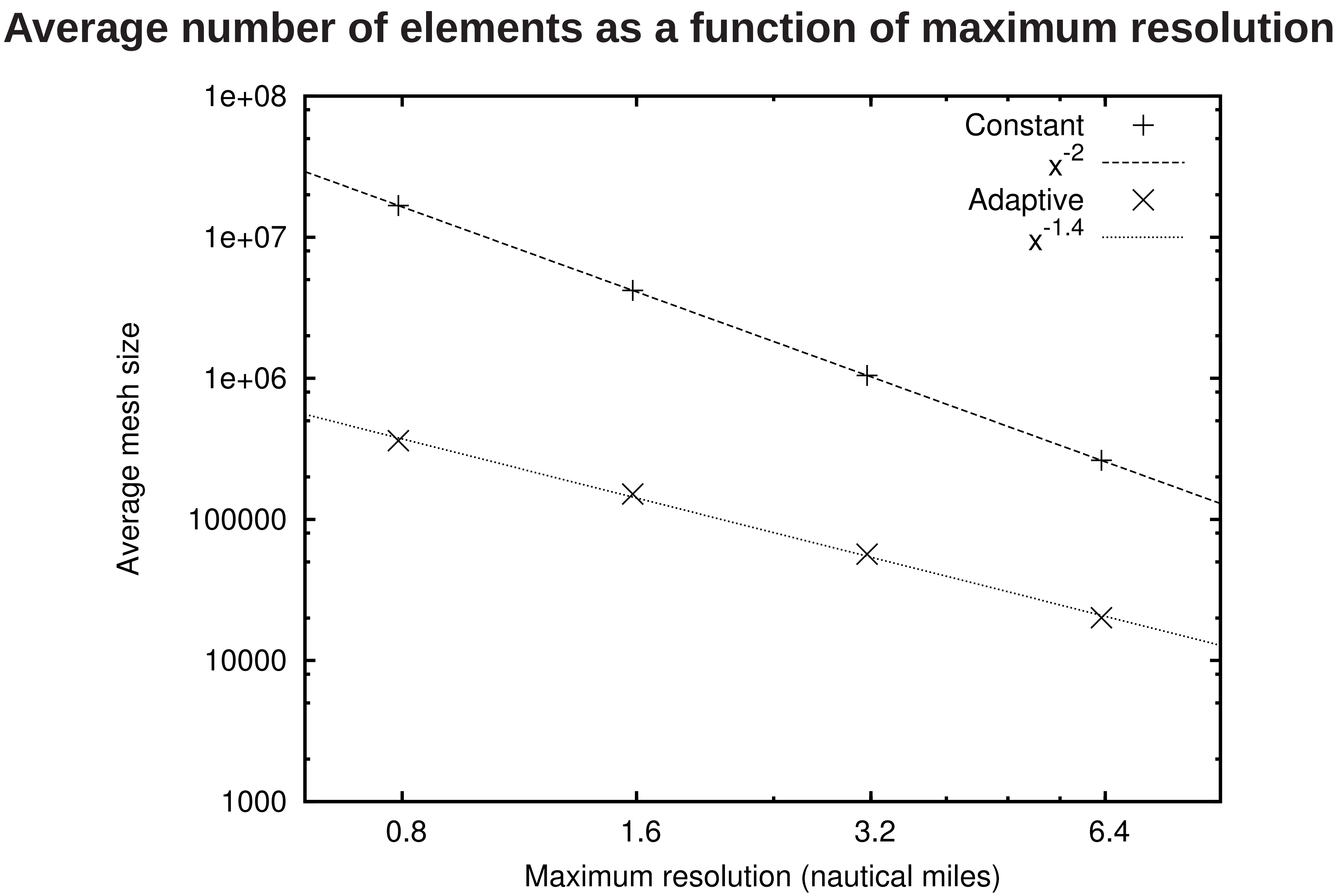
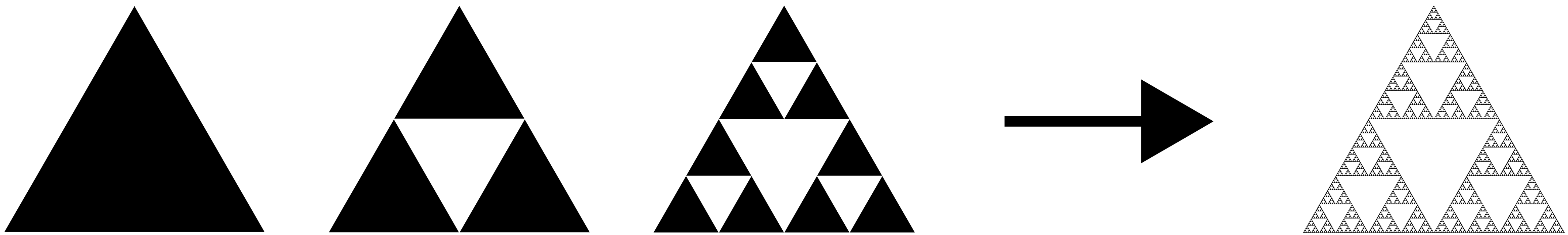

Average number of elements as a function of maximum resolution
1e+08
1e+07
1e+06
100000
10000
1000
Average mesh size
0.8
1.6
3.2
6.4
Maximum resolution (nautical miles)
Constant
x^-2
Adaptive
x^-1.4

## Connection with fractal dimension

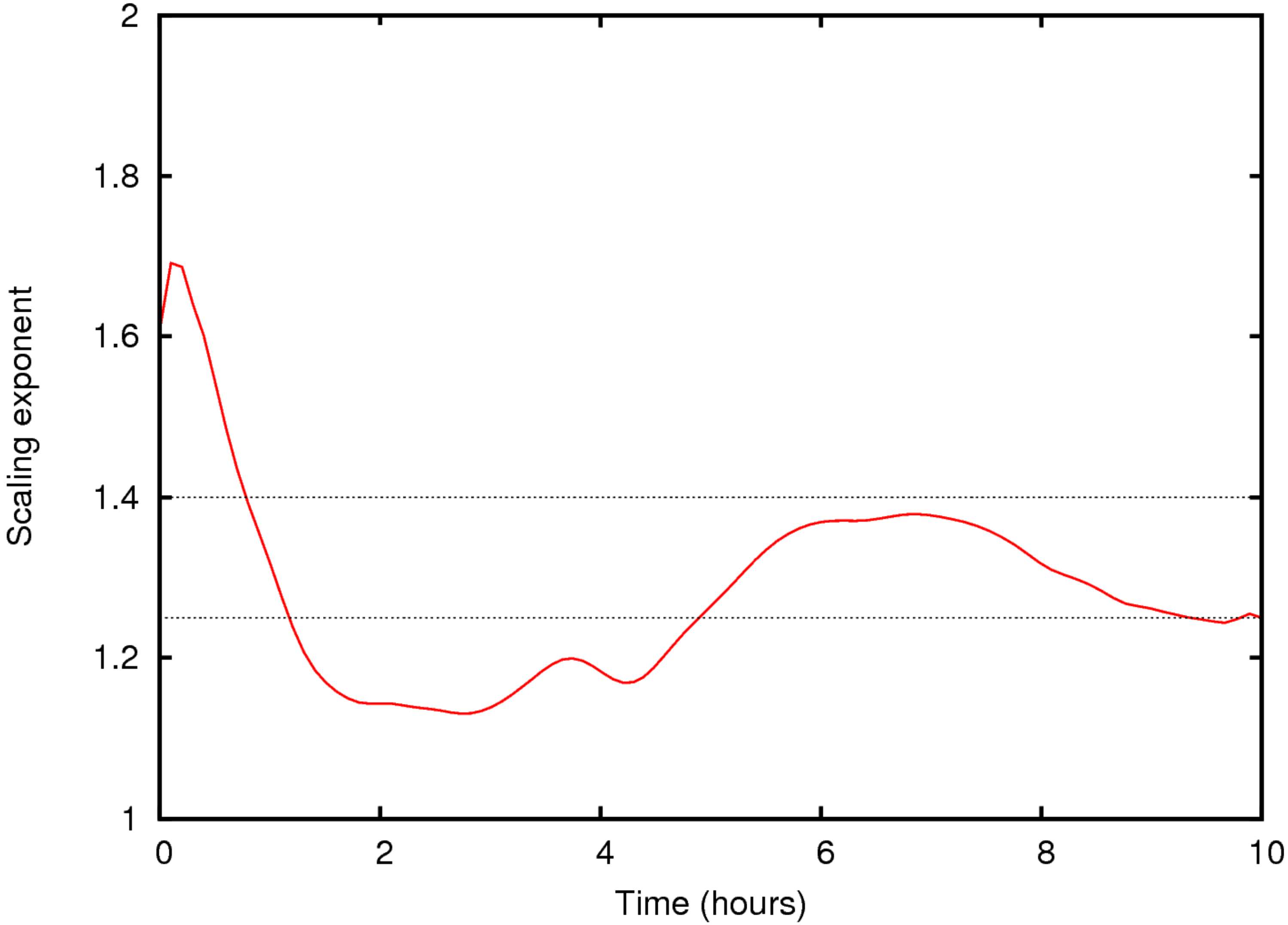
Classical example: the Sierpinski triangle



has a fractal (Minkowski–Bouligand or “box-counting” or “information”) dimension of $\approx 1.6$.

In other words, the cost of describing such an object using quadtrees would scale as $\Delta^{-1.6}$ not $\Delta^{-2}$.

# Evolution of the scaling exponent with time




Mandelbrot, *How long is the coast of Britain?*, Science, 1967

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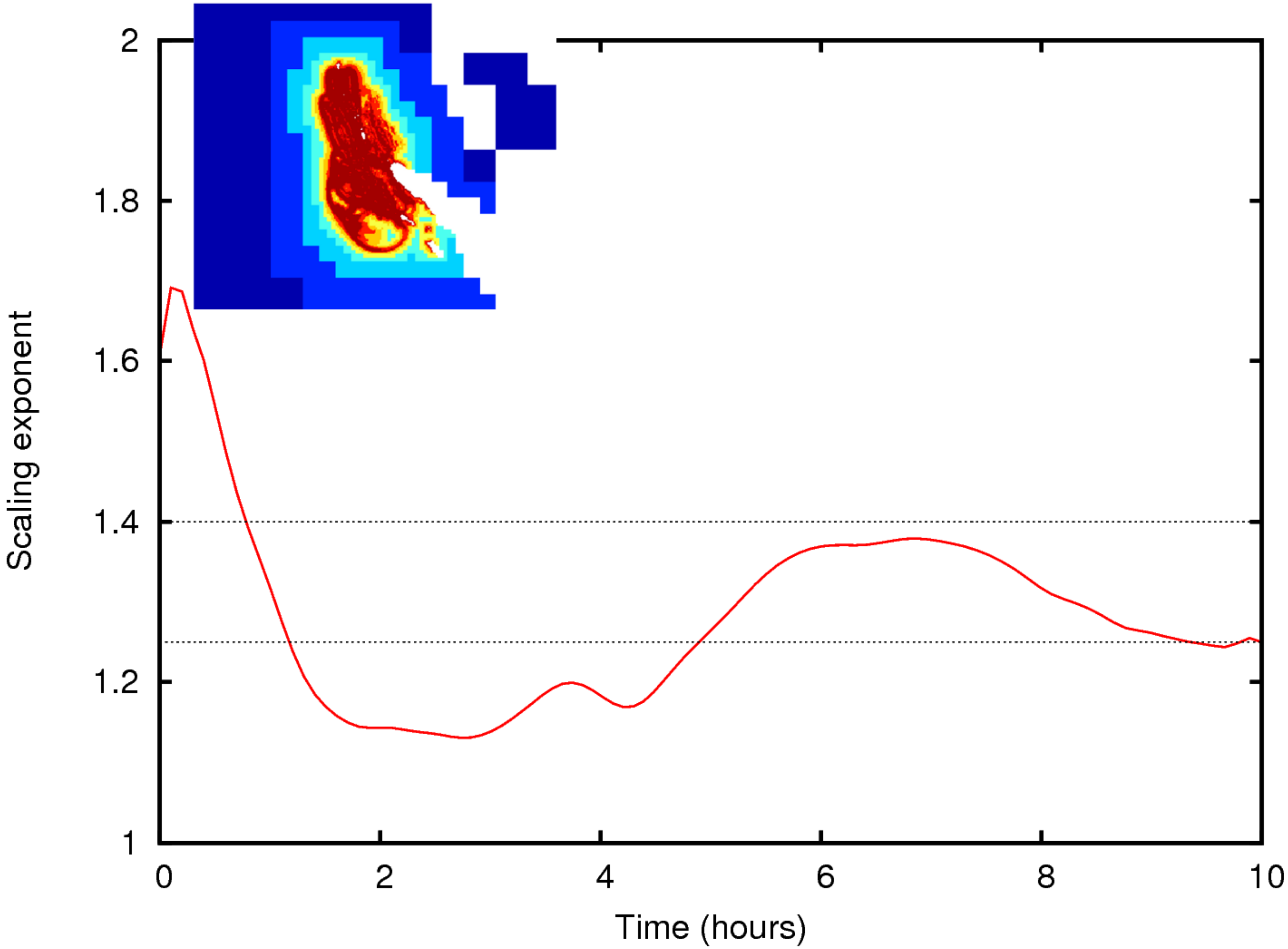


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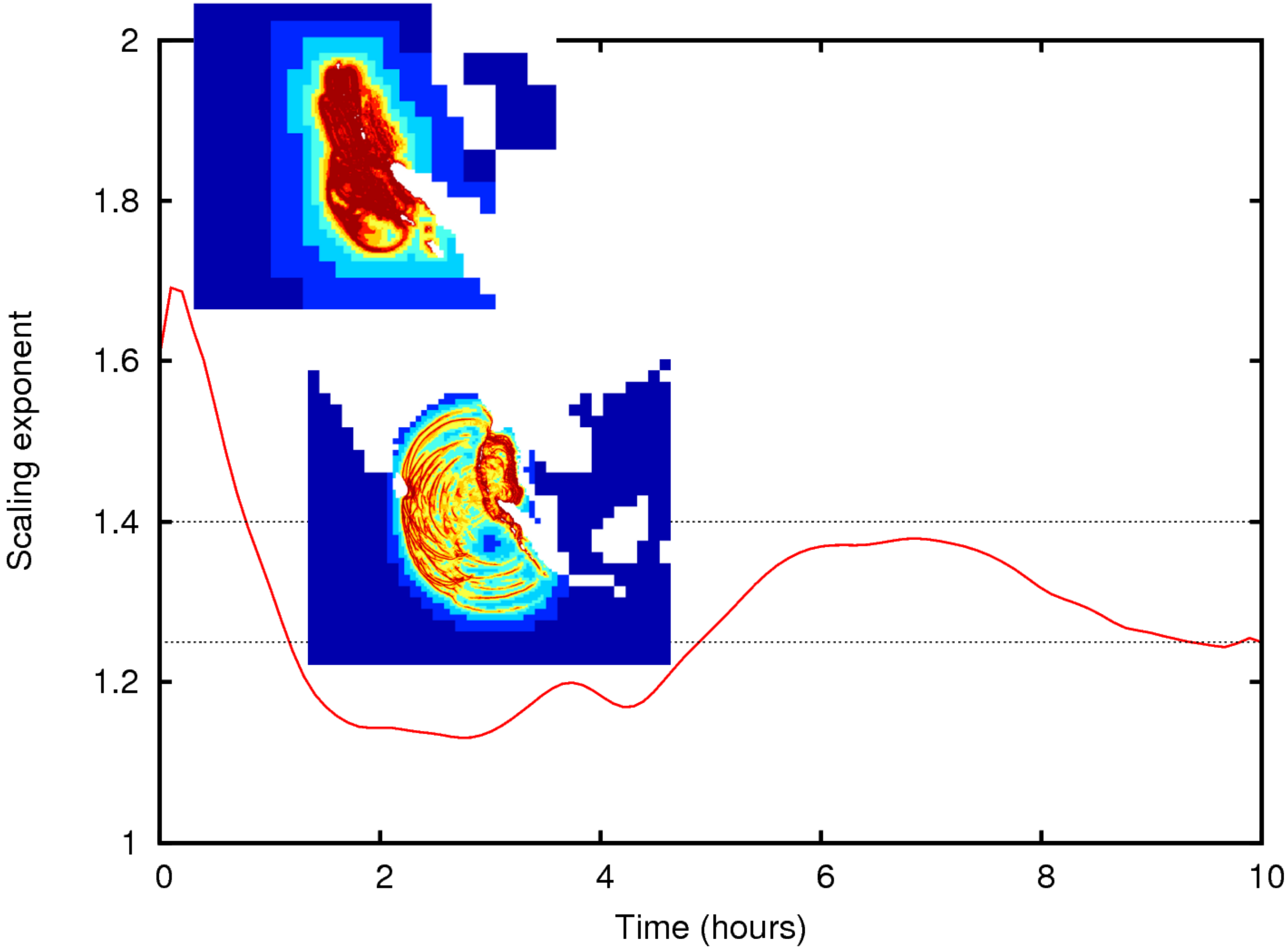


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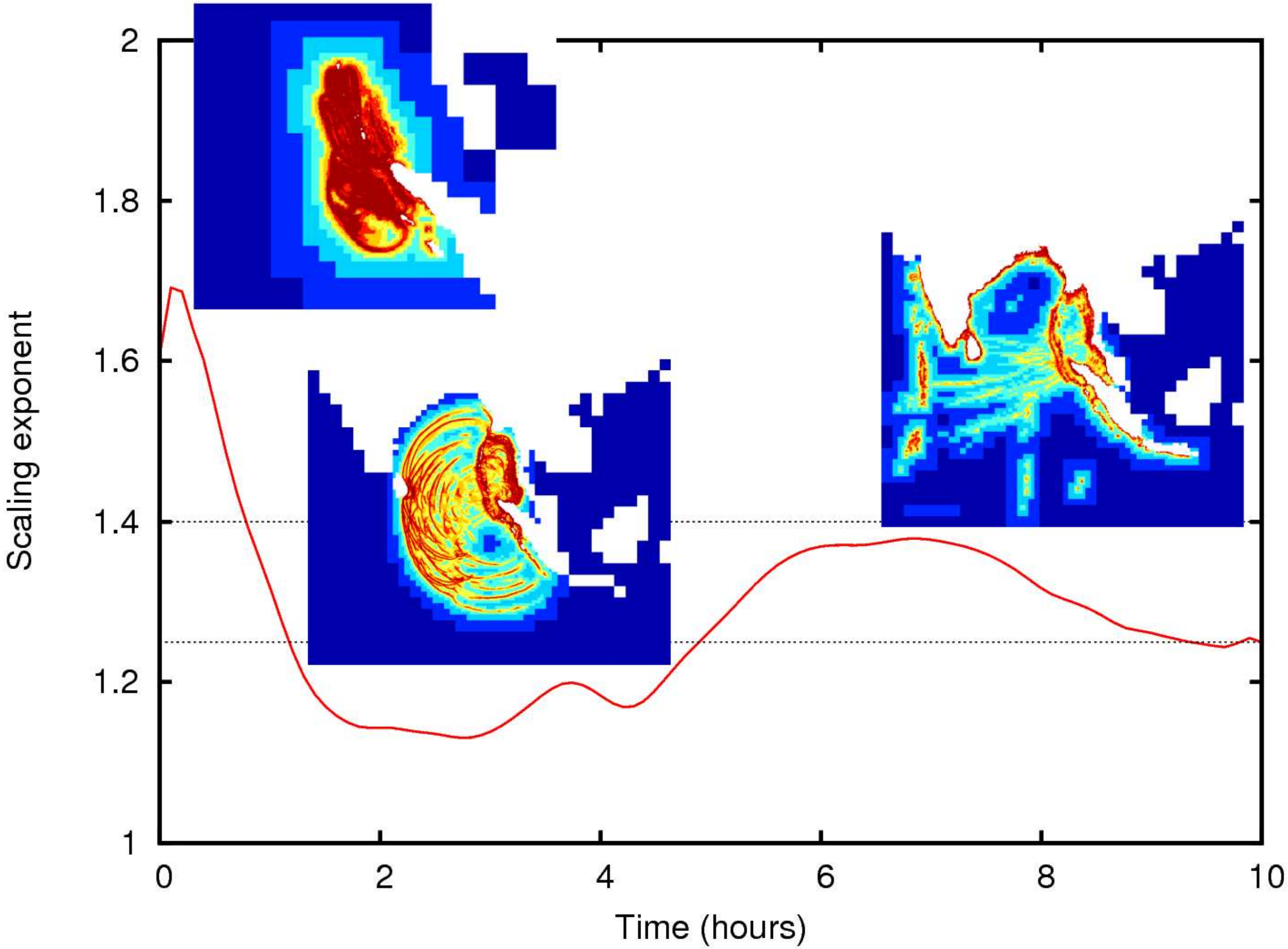


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## Conclusions

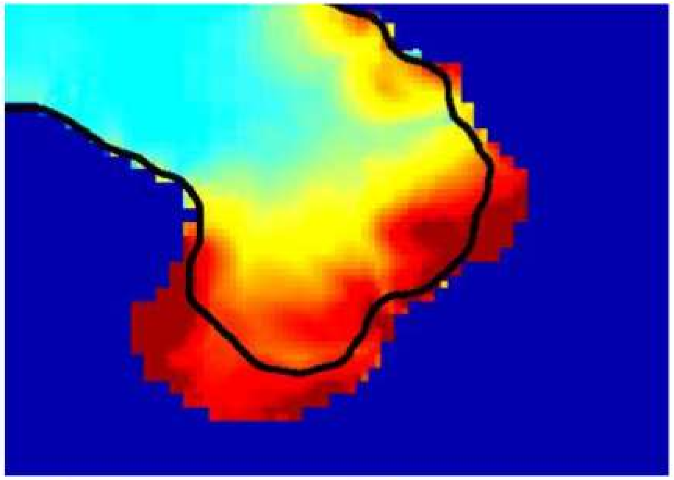
- Accurate and fast solutions for multiscale Saint-Venant problems
- Adaptivity changes the scaling of computing costs: $C\Delta^{-d}$, $d$ is now smaller than the number of dimensions
- This conclusion extends to a range of problems (not just Saint-Venant)
- See also poster for the Tōhoku tsunami

Work in progress

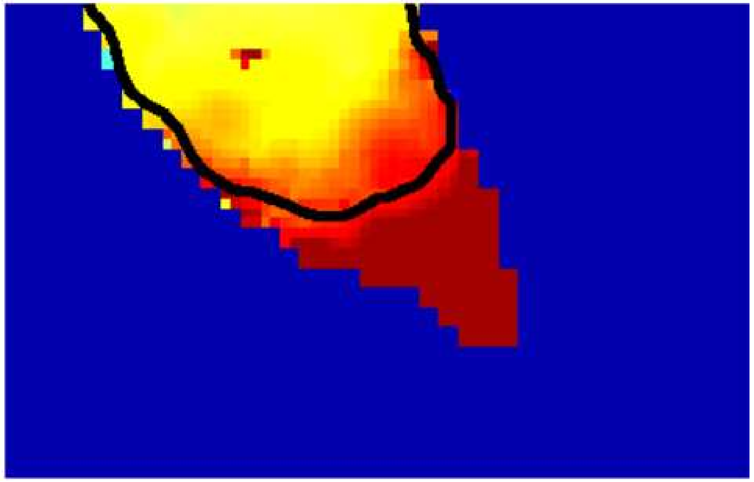
- There is a close link between the physical scale-distribution of (fluid dynamics) problems and the scaling of computing costs: this needs to be explored to make the most of adaptive methods

# Inundation at Tutuila, American Samoa, 2009

Fagasa

Maloata

10 m $\leq$ Spatial resolution $\leq$ 82 km

Simulated domain $\approx$ (3000 km)$^2$

## Maximum runups on shoreline

| Locations | Model | Field surveys |
|---|---|---|
| Aceh (N coast), Indonesia | 8.25 | 10–16 |
| Aceh (W coast), Indonesia | 17.60 | 24–35 |
| Galle, Sri Lanka | 3.16 | 2–3 |
| SE coast, Sri Lanka | 5.60 | 5–10 |
| Chennai, India | 3.01 | 2–3 |
| Nagappaattinam, India | 3.20 | 2–3.5 |
| Kamala Bch., Phuket, Thailand | 5.95 | 4.5–5.3 |