A GENERALIZATION OF THE LOG-LOGISTIC DISTRIBUTION FOR THE PROBABILITY OF DAILY PRECIPITATION

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Methodology

The proposed model is a modification of log-logistic distribution (Fig. 1) according to:

\[ \pi(p > P) = \frac{1}{1 + \lambda \exp(-\lambda s^*)} \]

where: \( \lambda = \frac{p - P}{P} \), and \( s^* \) is the cumulative probability, i.e., the probability that a station registers a precipitation equal to or higher than \( P \). The parameters \( \lambda, P \), and \( k \) are positive parameters. To compare the results, a modified version of Gumbel Distribution were used:

\[ \pi(p > P) = \exp \left[ - \exp \left( \lambda + k \right) \right] \]

The proposed model and commonly used models were adjusted for daily precipitation of every month of year, for 108 observatories from Spain. To adjust all the functions we have used the maximum likelihood inference. Profile Log-Likelihood. In addition, some test of goodness-of-fit were applied: NMAE, Akaike Index, Anderson-Darling and Kolmogorov-Smirnov tests.

Introduction

The daily precipitation in climate models can be used for various environmental studies, but it has a systematic error that is difficult to correct. Probability of distribution is very high around the lower rainfall (most likely zero) and decreases sharply to high rainfall. For that reason, the commonly used models have problems to fit all daily precipitation.

The objective of this work is to present and evaluate an alternative model of probability, based on a generalization of the Log-Logistic Distribution. Some test of goodness-of-fit was done by comparing the Akaike Information Index, Anderson-Darling, Kolmogorov-Smirnov and the Normalized Mean Absolute Error (NMAE). The proposed model and the commonly used probability models (GEV, GPD, Gamma, Gumbel, Weibull, Exponential and Log-normal) have been applied to daily rainfall for 108 observatories from Spain.

Discussion

- The normalized absolute error is similar for all scales of precipitation (Fig. 2), i.e., the generalized Log-Logistic can accurately represent the entire range of probability of daily precipitation.
- In most cases, this alternative model shows better results than the modified version of Gumbel Distribution (Fig. 3, 4 and 5).
- The fact of using four parameters can lead to difficulties in adjustment by computer algorithms, as it requires initial values very close to the best estimators of parameters.
- This model can be used to characterize and correct the probability distribution of any set of daily precipitation data.
- The parameters fitted for a probability distribution of precipitation should adequately reflect climate characteristics (see p[i]).
- The fit of the alternative model of probability for the different months of the year and different zones of Spain suggests a good performance for different climates.

Conclusions

- Most commonly used models were not able to adjust precisely enough the set of daily precipitation, included every frequency. It was therefore necessary to use models with three or four parameters.
- The generalized Log-Logistic model with four parameters presented the best results than other models for the most observatories.
- To be precise, it obtained a mean absolute error of less than 10% in most of the stations analyzed, and it is similar for all scales of precipitation.
- The fit is worse for the summer months, due to the few days of precipitation. Therefore, probably the proposed model works best for rainy climates.
- This model can be used to characterize the climate patterns of precipitation and to correct numerical model outputs.

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References


Photo of León Forado (2005)