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On the Statistical Properties of Record-Breaking Temperatures

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A systematic study of the statistics of record-breaking temperatures is presented. We first consider temperature time series to be Gaussian white noises and give the classic record-breaking theory results for any independent and identically distributed (i.i.d.) process. We then carry out Monte Carlo simulations to determine the influence of long-range correlations and linear temperature trends. For the range of fractional Gaussian noises that are observed to be applicable to temperature time series, the influence on the record-breaking statistics is small. We next superimpose a linear trend on a Gaussian white noise and extend the theory to include the effect of an additive trend. We determine the ratios of the number of maximum to the number of minimum record-breaking temperatures. We find the single governing parameter to be the ratio of the temperature change per year to the standard deviation of the underlying white noise. To test our approach, we consider the 30-year record of temperatures at the Mauna Loa Observatory. We determine the temperature trends by direct measurements and use our simulations to infer trends from the number of record-breaking temperatures. The two approaches give values that are in good agreement. We find that the warming trend is primarily due to an increase in the (overnight) minimum temperatures while the maximum (daytime) temperatures are approximately constant. F

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Influence of correlations and trends on record-breaking statistics

C1

Results for a white-noise time series

Mean number of record breaking maximum or minimum values $\langle n_{rb} \rangle$ as a function of the length n of the time series [1]

$$\langle n_{rb}(n) \rangle = 1 + \frac{1}{2} \dots \frac{1}{n} \approx \ln(n) + \gamma \quad (1)$$

where $\gamma = 0.5772$ is the Euler-Mascheroni constant.

Results for fractional Gaussian noises and fractional Brownian walks

Self-affine correlated time series have a power-law dependence of the power-spectral density S on the frequency f [2]

$$S(f) \propto f^{-\beta} \quad (2)$$

$\beta=0$ is a white noise, $\beta=2$ is a Brownian walk, $-1< \beta <1$ are stationary fractional noises, $1< \beta< 3$ are nonstationary fractional walks.

Results for a fractional Gaussian noise (Monte Carlo simulations)

C2

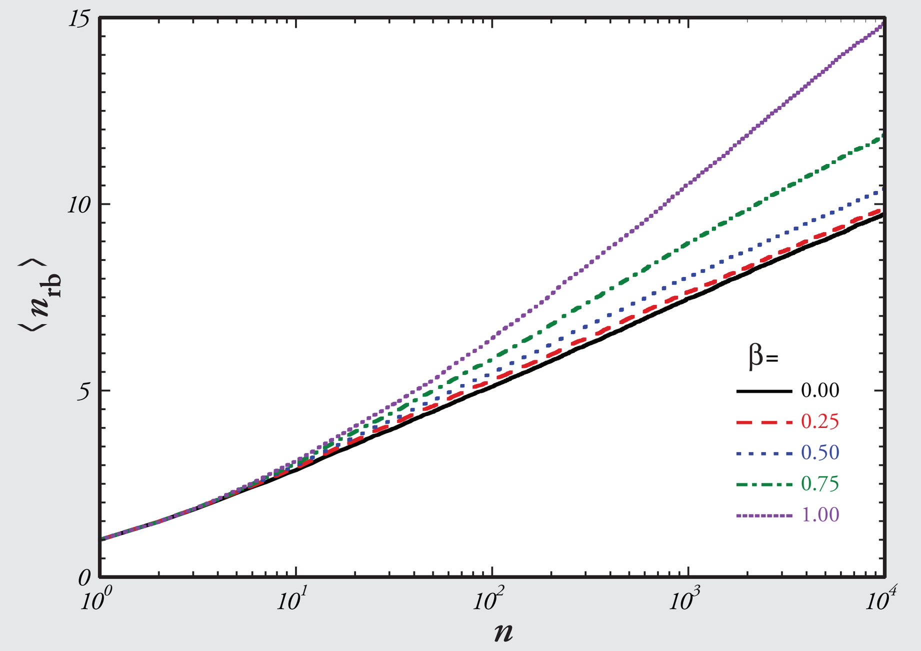

FIG. 1. Dependence of the mean number of record-breaking values $\langle n_{rb} \rangle$ as a function of the number of events n. These results do not depend upon using maximum or minimum values. These results are shown for fractional Gaussian noises with β = 0, 0.25, 0.50, 0.75, and 1.00. The results for the white noise β = 0 are identical to the i.i.d. random variable theory given in *Eq. (1)*.

Typically, for a temperature true series we have $\beta \approx 0.5$ [3] so that the influence of long range correlations is small.

Results for a fractional Brownian walk (Monte Carlo simulations)

C3

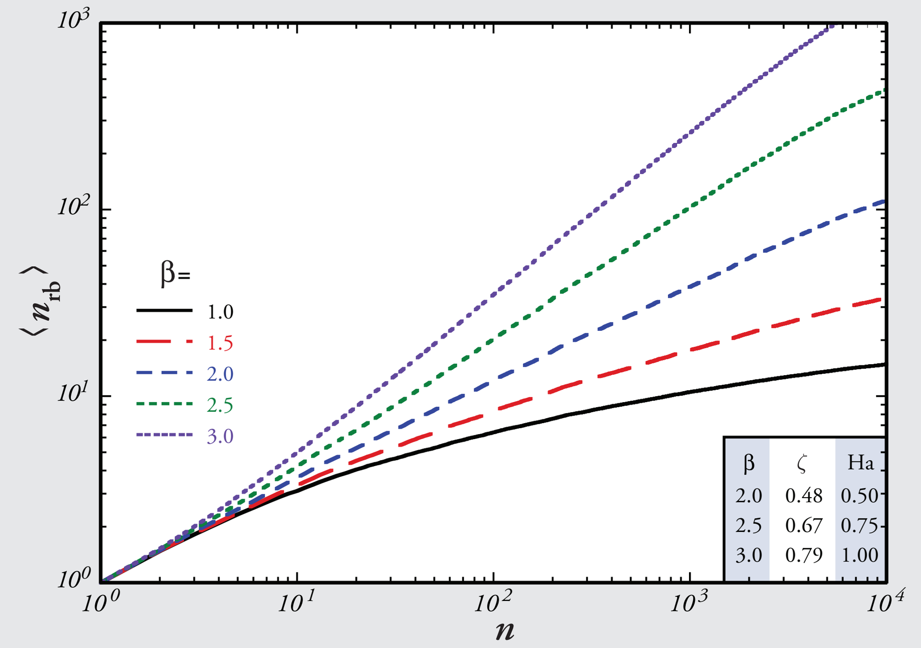

FIG. 2. Dependence of the mean number of record-breaking values $\langle n_{rb} \rangle$ as a function of the number of events. Results are shown for fractional Brownian walks β = 1.0, 1.5, 2.0, 2.5, and 3.0. The results for β = 2.0, 2.5, and 3.0 are compared with the power-law correlation given in *Eq. (3)* for ζ and *Eq. (4)* for Ha.

For β=2, 2.5, and 3 the results correlate with

$$\langle n_{rb}(n) \rangle \propto n^{\zeta} \quad (3)$$

A property of fractional Brownian walks is [2]

$$\sigma \propto n^{Ha} \qquad Ha = \frac{1}{2}(\beta - 1) \quad (4)$$

where Ha is the Hausdorff measure.

Results with a linear trend

T1

We assume that the mean of a Gaussian white noise with unity standard deviation is given by

$$\langle y_n \rangle = \langle y_1 \rangle + \alpha\,(n-1') \quad (5)$$

Results of Monte Carlo simulations are

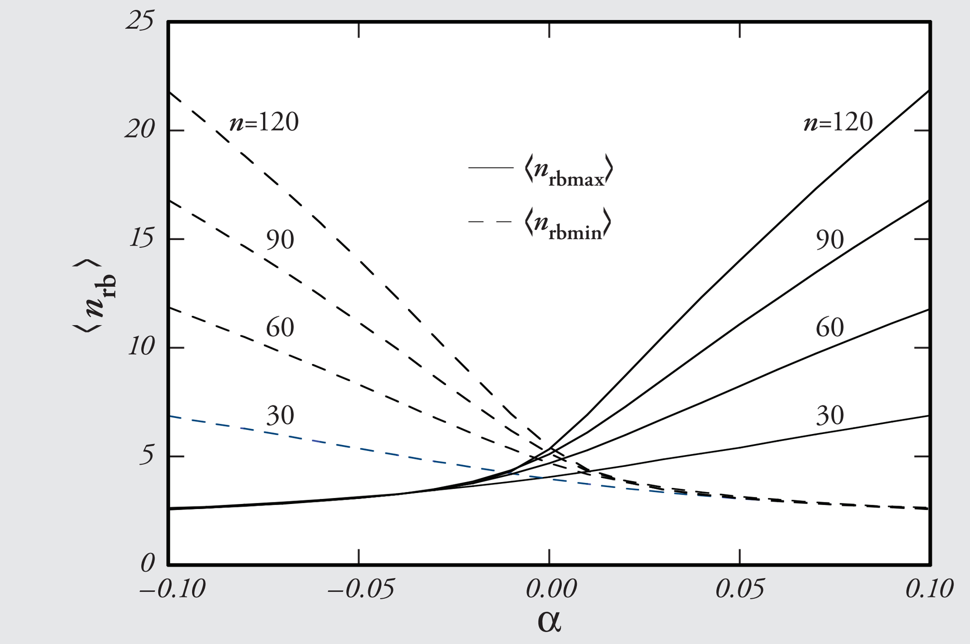

FIG. 3. Mean numbers of record-breaking maximum values $\langle n_{rbmax} \rangle$ and record-breaking minimum values $\langle n_{rbmin} \rangle$ are given as a function of the slope α for η = 30, 60, 90, and 120 values.

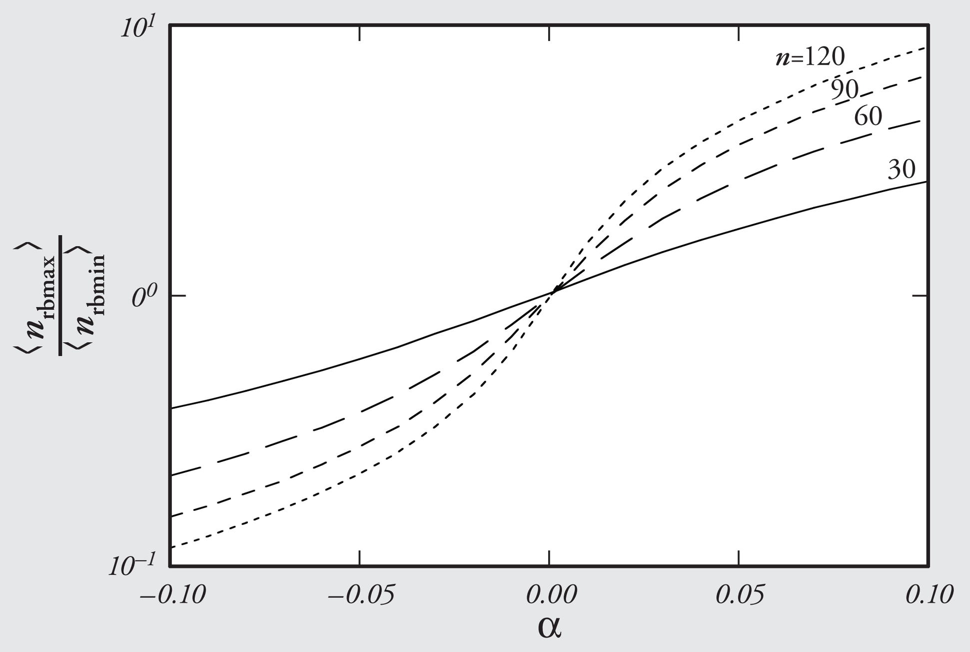

FIG. 4. Ratios of the mean number of record-breaking maximum values to the mean number of record-breaking minimum values $\langle n_{rbmax} \rangle / \langle n_{rbmin} \rangle$ are given as a function of the slope α for n = 30, 60, 90, and 120 values.

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Application to record-breaking temperatures at the Mauna Loa Observatory, Hawaii

ML1

Measurements at this laboratory established the systematic increase in global CO_2.

It can be argued that temperature trends at this site are representative of global values.

We utilize daily maximum and minimum temperatures for the period January 1, 1977 to December 31, 2006 [4].

We obtain best fit linear trends for each day of the year

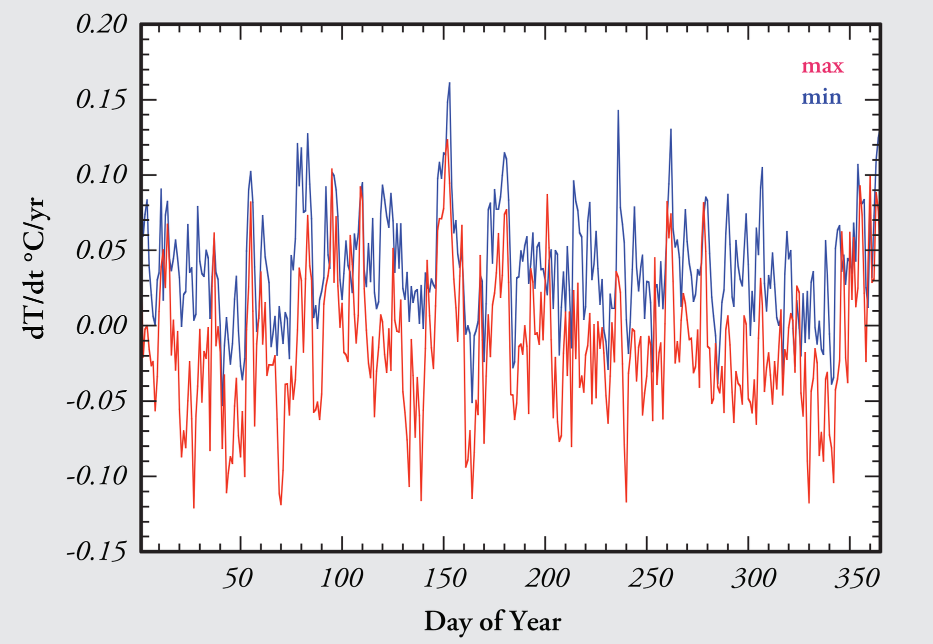

FIG. 5. The best fit linear temperature trends *dT/dt* for the 30 years are given for the 365 days of the year. The trends of both maximum daily temperatures dT_{max}/dt and minimum daily temperatures dT_{min}/dt are given.

ML2

Annual mean maximum and minimum temperatures and best fit linear trends

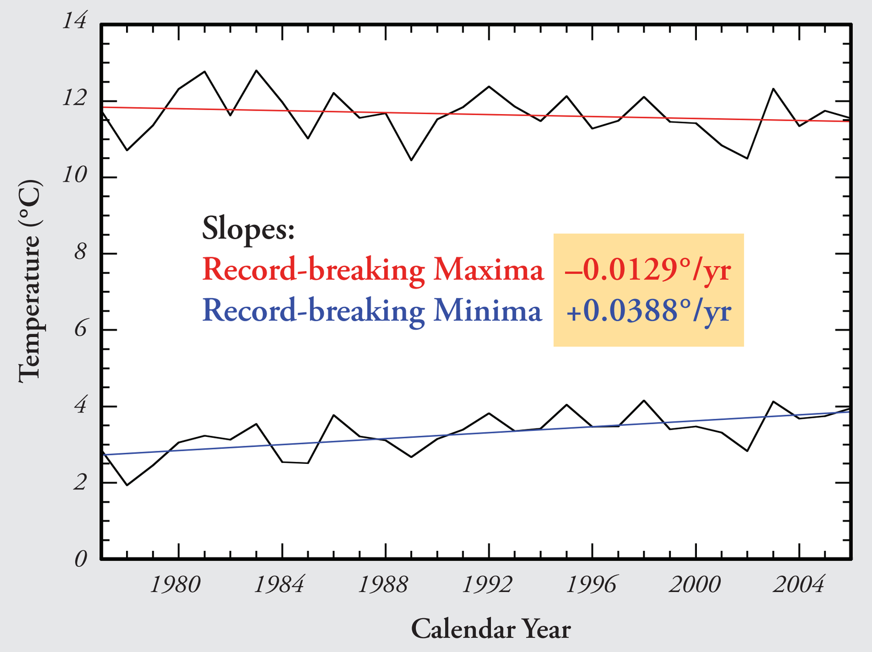

FIG. 6. The mean annual maximum and minimum temperatures are given as a function of time from 1977 to 2006.

Average number of record-breaking maximum and minimum temperatures each year

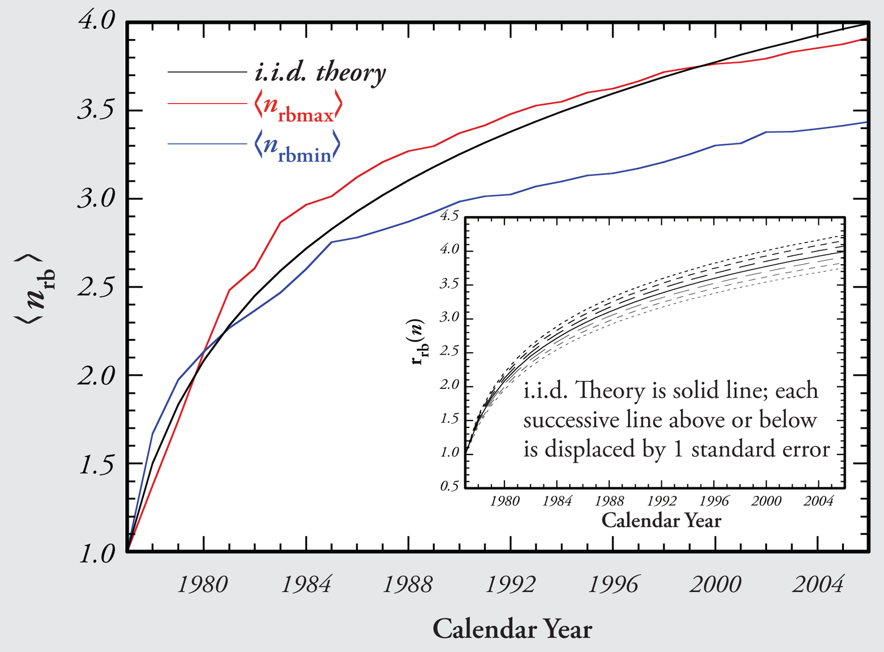

FIG. 7. The average number of record-breaking maximum and minimum temperatures, $\langle n_{rbmax} \rangle$ and $\langle n_{rbmin} \rangle$, as a function of time measured forward from January 1, 1977. The average is over the 365 days of the year. Also included is the number expected for an i.i.d. random process from *Eq. (1)*.

ML3

We next determine temperature trends inferred from 30 year record-breaking data.

	Max. Temps	Min. Temps
From Figure 7 $\langle n_{rbmax} \rangle$	3.90	3.42
From Figure 3 α	−0.0037	0.0213
Standard deviation σ, °C	2.44	1.79
$dT/dt \propto \sigma$ °Cyr^{-1}	−0.0091	0.0381

Summary of Results

	Mauna Loa direct	Mauna Loa record-breaking	Climate Change 2007 [5]
dT_{max}/dt °Cyr^{-1}	−0.0129	−0.0091	0.014
dT_{min}/dt °Cyr^{-1}	0.0388	0.0381	0.020
dT/dt °Cyr^{-1}	0.0130	0.00145	0.017