

# Consistency of Earth orientation parameters, second degree gravity field coefficients and geophysical excitation functions

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## 1. Introduction

Mass redistributions of the Earth subsystems (atmosphere, oceans, hydrology) cause changes of the relative angular momentum and the tensor of inertia. Consequently the Earth's rotation varies. Geophysical excitation functions (EF) of the subsystems model the variations of the relative angular momentum (motion term) and the tensor of inertia (mass term). As the second degree gravity field coefficients (GFC2) are related to the tensor of inertia, the Earth orientation parameters polar motion and length of day (EOP), the GFC2 and the EF are validated by the combination of the parameters in a least squares adjustment model.

## 2. Methodology

### Relations between GFC2, EOP and EF:

The EF are defined by the linear approximation of the Euler-Liouville equations [Gross, 2009]. The five GFC2 depend linearly on the six elements of the tensor of inertia. Hence a time invariant trace of the tensor of inertia is assumed to solve the under-determined tensor of inertia: The elements of the unknown tensor of inertia can be redundantly obtained from the linear equations:

$$\begin{aligned} \bullet \text{ EOP and motion terms: } \mathbf{A}_{\text{EOP}} (\mathbf{I}_{\text{EOP}} + \boldsymbol{\varepsilon}_{\text{EOP}}) - \mathbf{A}_{\text{motion}} (\mathbf{I}_{\text{motion}} + \boldsymbol{\varepsilon}_{\text{motion}}) &= \boldsymbol{\xi} & (1) \\ \bullet \text{ Mass terms: } \mathbf{A}_{\text{mass}} (\mathbf{I}_{\text{mass}} + \boldsymbol{\varepsilon}_{\text{mass}}) &= \boldsymbol{\xi} & (2) \\ \bullet \text{ GFC2: } \mathbf{A}_{\text{GFC2}} (\mathbf{I}_{\text{GFC2}} + \boldsymbol{\varepsilon}_{\text{GFC2}}) &= \boldsymbol{\xi} & (3) \end{aligned}$$

$\mathbf{A}$  – design-matrices       $\mathbf{I}$  – modeled / observed data  
 $\boldsymbol{\varepsilon}$  – residuals               $\boldsymbol{\xi}$  – unknown tensor of inertia

### Least squares adjustment

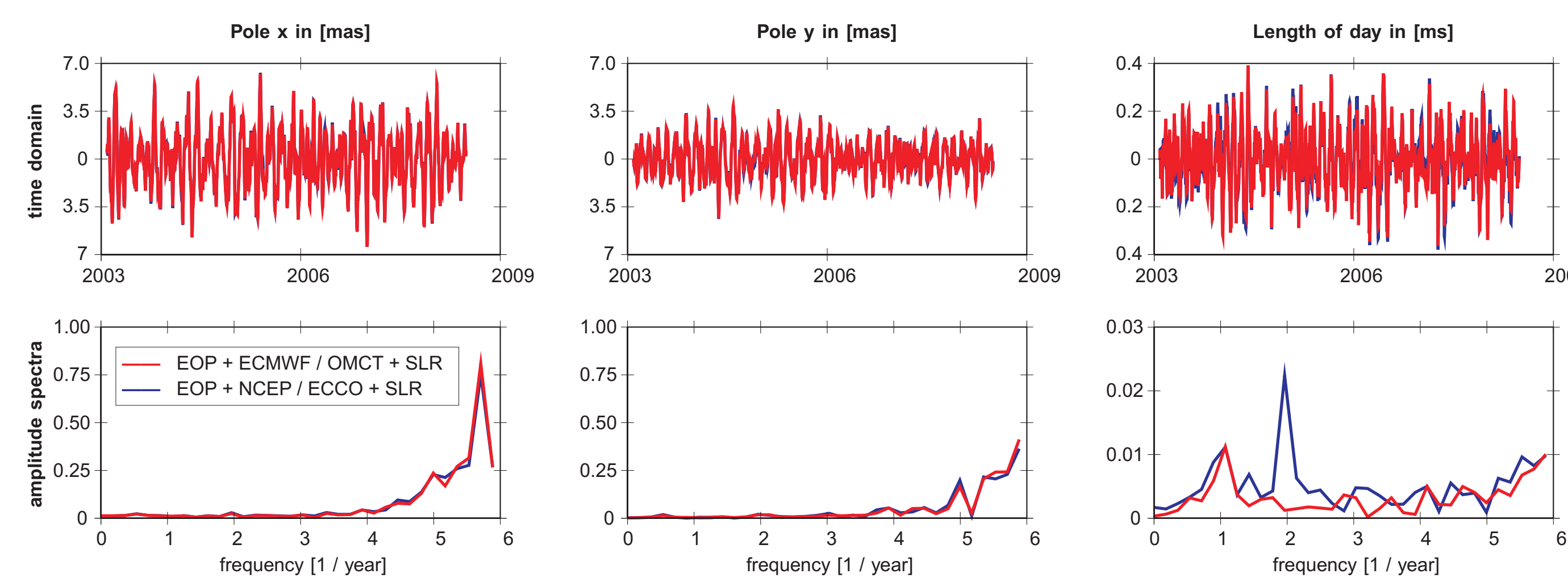
The unknown tensor of inertia and the residuals are determined by a least squares adjustment, which is a weighted average of the Eqs. (1) - (3). The EOP, GFC2 and EF are regarded as (pseudo) observations. Empirical auto- and cross correlation functions approximate missing stochastic information. The weights of the time series are determined by a variance component estimation. For further details see [Heiker et al., 2010]. The results of the adjustment are consistent EOP, GFC2 and EF time series. The residuals (observed / modeled minus adjusted) measure the inconsistencies of the data.

### Data

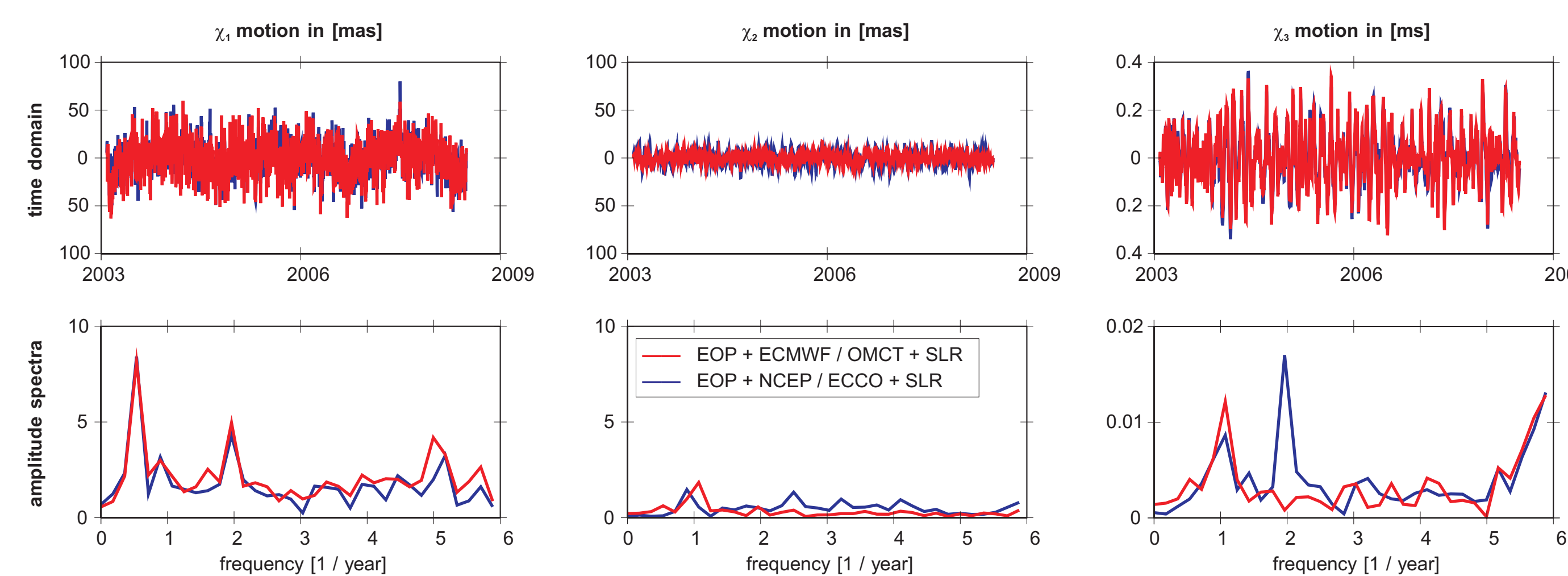
These time series are analyzed in each possible combination.

- EOP: – EOP 05 C04 from the International Earth Rotation Service
- EF: – Atmosphere: NCEP (Reanalysis) + Oceans: ECCO (Version kf079)  
– Atmosphere: ECMWF(ERAinterim) + Atmosphere: OMCT
- GFC2: – SLR-Coefficients derived by UT/CSR  
– GRACE-Coefficients derived by UT/CSR

## 3. Results

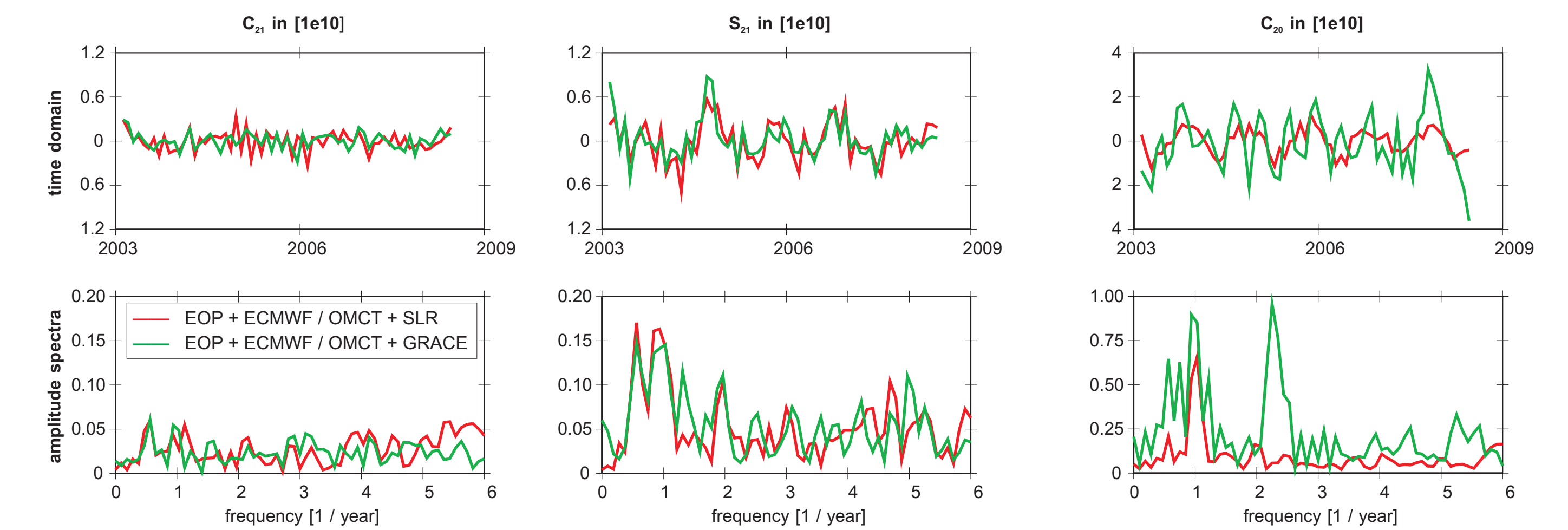


**Fig. 1: Residuals of the EOP** in the time domain (upper row) and the frequency domain (lower row). The residuals result from the combination of the EOP and SLR-GFC2 with different EF (red: ECMWF / OMCT and blue: NCEP / ECCO). The small polar motion residuals do not contain periods longer than three months. The comparably large amplitudes of the periods of approximately two months are artefacts as the EOP and EF with their daily temporal resolution are compared with monthly GFC2 (see the Nyquist–Shannon sampling theorem). A clear semiannual signal is present in the NCEP / ECCO length of day residuals resulting from the motion term of the EF (see explanation in Fig. 2).



**Fig. 2: Residuals of the motion term of the EF** in the time domain (upper row) and the frequency domain (lower row). The residuals result from the combination of the EOP and SLR-GFC2 with different EF (red: ECMWF / OMCT and blue: NCEP / ECCO). The  $\chi_1$  residuals show larger variations than the  $\chi_2$  residuals. The  $\chi_1$  residuals contain periods of two years and six months whereas the  $\chi_2$  residuals do not reveal any systematic effects. Periods longer than two years might be present. However, they can not be identified as the time series are too short (65 months). The  $\chi_3$  residuals present a similar amplitude spectrum as the length of day residuals. According to Eq. (1) the difference between the EOP and the motion terms of the EF can solely be validated. Hence the semiannual peak in the length of day residuals is caused by the NCEP / ECCO model.

The **residuals of the mass term of the EF** are not shown. The  $\chi_1$  /  $\chi_2$  mass residuals of the NCEP / ECCO model show larger variations than the ECMWF / OMCT residuals. The amplitude spectra reveal a dominant yearly period in the residuals of both EF. The  $\chi_3$  mass residuals do not indicate any significant difference between the two EF.



**Fig. 3: Residuals of the GFC2** in the time domain (upper row) and the frequency domain (lower row). The residuals result from the combination of the EOP and ECMWF / OMCT with different GFC2 solution (red: SLR and green: GRACE). The  $C_{21}$  residuals do not contain any systematic effects and vary less than the  $S_{21}$  residuals. The GRACE  $C_{20}$  residuals show a large semiannual period which is not present in the SLR residuals. This period might be caused by an erroneous GRACE background model.

## 4. Summary

- Best fitting combination: EOP 05 C04 + ECMWF / OMCT + SLR
- Residuals in EOP can be caused by inconsistent motion terms of the EF.
- Longer time series are needed to analyse long periods (greater than two years). Please provide SLR gravity field coefficients (before 2003) and updated EF (after 2008).
- The parameter  $\chi_2$  which is connected with  $C_{21}$  and the polar motion show less inconsistencies than the connected parameter  $\chi_1$ ,  $S_{21}$  and the polar motion (see Euler-Liouville equations [Gross, 2009]). The reason is probably that the EF are modeled with different quality over land and over sea. Model errors over the Eurasian continent might affect  $\chi_1$  more than  $\chi_2$ .

### References

- Gross, R. (2008): *Earth Rotation Variations - Long Period*, in Schuber G. (ed.), *Treatise on geophysics: Volume 3 - Geodesy*, Elsevier
- Heiker, A., Kutterer, H., Müller, J: *Stabilization of Satellite Derived Gravity Field Coefficients by Earth Rotation Parameters and Excitation Functions*, *Geodesy for Planet Earth*, Proceeding of the IAG Scientific Assembly 2009 in Buenos Aires, Argentina, accepted March 2010

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