Towards oscillation-free implementation of the immersed boundary method with spectral methods for direct and lardy-eddy simulation

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1. Introduction

- ☞ *Immersed boundary method (IBM)* offers a simple strategy to use a regular computational grid while solving flow problems with complex geometry. Due to this advantage, IBM has gained popularity in many applications such as direct and large-eddy simulation. The original idea of the IB methods is to model the no-slip condition at the solid body surface by introducing a source term in the momentum equations, the so-called immersed boundary force, in such a way that the flow inside the solid domain is frozen. IB methods can be classified into two main approaches. In the first approach [1], termed as the continuous forcing approach, the immersed boundary force is introduced into the partial differential equations for momentum before discretization. In the second approach [2], named the discrete forcing approach (also known as the direct forcing approach), the immersed boundary force is applied after the equations are discretized. For flows without moving boundaries, the discrete forcing approach is more attractive because it enables a sharp representation of the immersed boundary and does not introduce additional numerical stability restriction.
- ☞ *Spectral-like methods* such as the pseudo-spectral method and high-order compact schemes have been widely used in the field of computational fluid dynamics due to their very favorable accuracy properties. However, the spectral-like methods usually adopts a regular computational grid and can not be applied to problems with complex geometries. To remove this limitation, some attempts have been made to incorporate IBM to the spectral-like methods. When doing so, spurious oscillations occur in the calculated velocity derivatives near the immersed boundary. This is associated with the *Gibbs phenomenon* of spectral methods. Different methods [2, 3, 4, 5, 6, 7] have been proposed to reduce the oscillations, but none of them has accuracy, efficiency, and generality in one.
- ☞ The purpose of this research work is to develop a general smoothing technique for the implementation of IBM with the spectral-like methods aimed at eliminating or reducing efficiently the Gibbs oscillation without affecting the flow field outside the body. We will focus on the combination of the discrete forcing approach with the pseudo-spectral method.

2. Radial basis function based smoothing

A signed distance function φ is used to identify the points inside the body ($\varphi \leq 0$) and those in the flow domain ($\varphi > 0$). In general, we would like to approximate a variable, the velocity component $u(\boldsymbol{x})$ for example, inside the body in a smooth way without introducing a discontinuity. The idea proposed here is to interpolate the variable value at any point inside the body by using the known variable values at the points near the body surface and satisfying the boundary conditions. The interpolation formula is given as

$$u(\boldsymbol{x}) = \sum_{i=1}^{N} c_i \phi \left(\|\boldsymbol{x} - \boldsymbol{x}_i\| \right), \qquad (1)$$

where c_i is the interpolation coefficient and ϕ is a radial basis function whose value depends only on the distance from the origin. Commonly used types of radial basis functions include Gaussian, Multiquadric, and Polyharmonic spline. The radial basis function adopted here is Gaussian. From the given variable values $u(\boldsymbol{x}_i)$ ($i = 1, 2, \ldots, N$), the unknown coefficients c_i ($i = 1, 2, \ldots, N$) are obtained by solving the corresponding linear algebra equations.
To satisfy the no-slip condition at the body surface, two layers of auxiliary points are introduced. The first layer is on the body surface ($\varphi = 0$) where the velocity is known to be zero. The second layer is inside the body and at a distance δ to the surface ($\varphi = -\delta$). The auxiliary points are arranged in such a way that the average spacing between them is also around δ. First, the velocity at each auxiliary point of the second layer is obtained by a linear interpolation. Let $\boldsymbol{x}$ be the position of an auxiliary point and $\boldsymbol{n}$ denote the surface normal passing through $\boldsymbol{x}$. A corresponding point at a distance δ' from the immersed boundary is defined as $\boldsymbol{x}' = \boldsymbol{x} + (\delta + \delta')\boldsymbol{n}$. Let $\boldsymbol{v}$ be the velocity at the auxiliary point and $\boldsymbol{v}'$ denote the velocity at the corresponding point. The linear interpolation gives $\boldsymbol{v} = -\boldsymbol{v}'\delta/\delta'$ in which zero velocity on the immersed boundary is applied. Then, the auxiliary points of the two layers are used to construct the RBF interpolator given by Eq. (1). Finally, the obtained RBF interpolator is applied repeatedly for all the regular grid points inside the body ($\varphi < 0$). Usually, the velocity at the corresponding point is not known and needs to be interpolated. In this paper, the trilinear finite element interpolation is used. The RBF-based smoothing procedure described above, named as *RBFS* hereafter, is illustrated in Fig. 1.

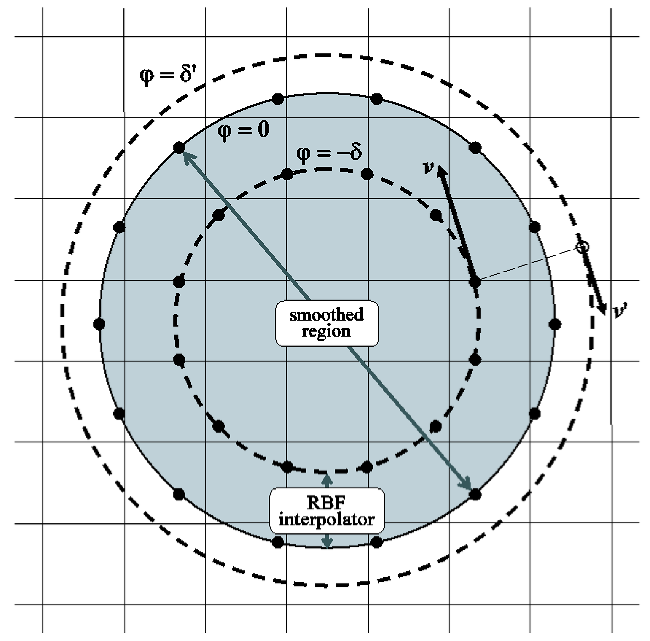

Figure 1: *Schematic showing the smoothing procedure of RBFS. Here, all the auxiliary points used for constructing the RBF interpolation are represented by solid circles.*

To test the proposed smoothing techniques, we here consider a typical velocity profile as shown in Fig. 2. The smoothed velocity profiles are also shown in Fig. 2, while the corresponding derivatives calculated by the pseudo-spectral method are presented in Fig. 3. It is observed that RBFS is more effective than the Laplacian smoothing in removing the Gibbs oscillations presented in the calculated derivatives.

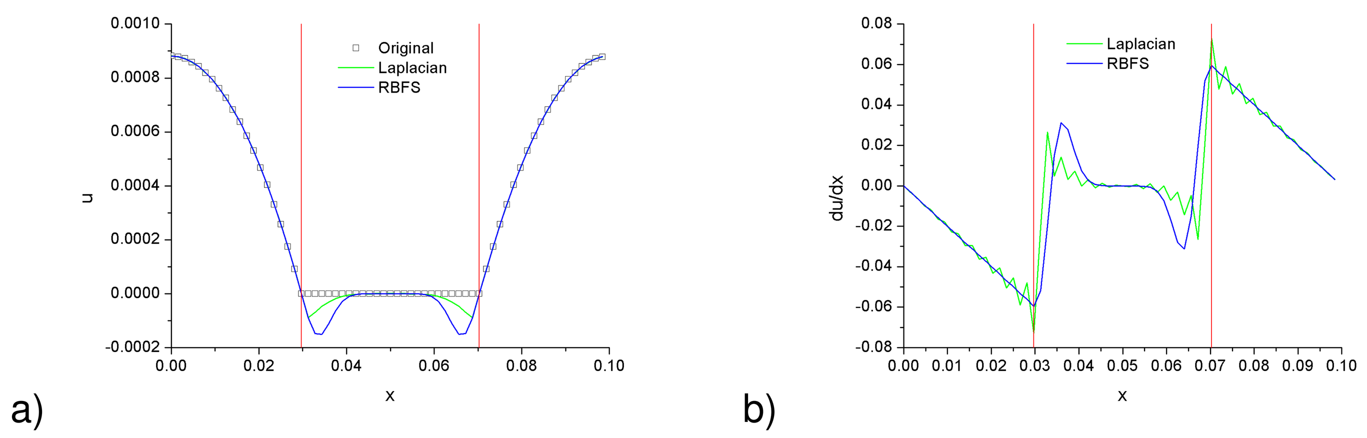

Figure 2: *Flow pass a bluff body represented by the two vertical lines: a) The velocity profile and the smoothed ones; b) The velocity derivatives.*

3. Combined IBM/Spectral method for direct numerical simulation

For direct numerical simulation (DNS), the governing equations are the Navier-Stokes equations. The projection method is adopted to satisfy the incompressibility. The spatial derivatives are computed by the pseudo-spectral method in the horizontal directions, while in the vertical direction they are computed with the second-order centered finite difference scheme in a staggered grid formulation. The nonlinear convection terms are de-aliased in Fourier space by the 3/2 rule. The Poisson equation for pressure with the Neumann boundary condition on the top and bottom boundaries is solved by using the spectral transform in the horizontal and second-order finite differences in the vertical with a tridiagonal solver. The discrete-time momentum forcing is applied to simulate the effects of obstacles in the flow. The scheme of simply setting the velocity inside the body to zero is named as *IBM-Simple*. In addition to the inherent problem of Gibbs oscillation, another flaw of IBM-Simple is that it cannot satisfy the non-slip boundary condition exactly unless the grid points happen to be located on the body surface. The new scheme which applies RBFS is called as *IBM-RBFS* and the scheme of Mohd-Yusof [2] is denoted as *IBM-NS*. In IBM-NS, velocity reversing is performed for the inner grid points close to the body surface and the velocity field further inside the body is developed freely by solving the governing equations as outside. The flow through a periodic lattice of cylinders is taken as a benchmark problem for testing the different schemes.

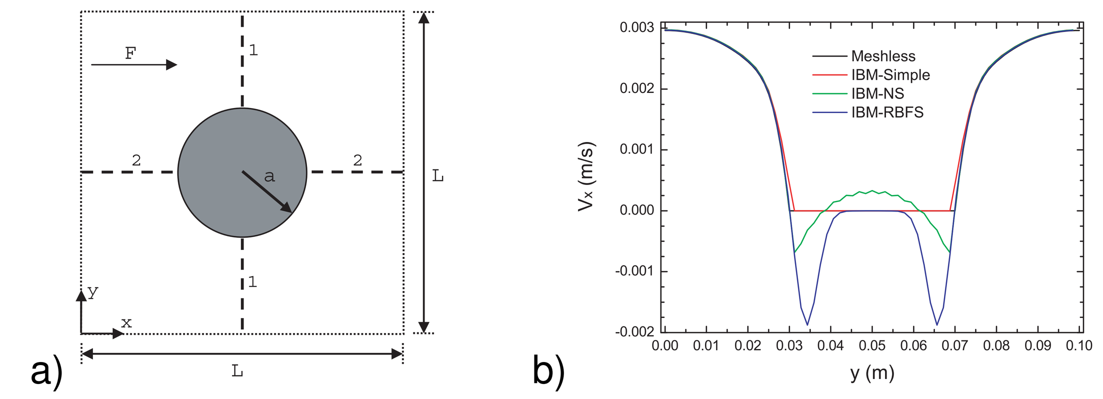

Figure 3: *a) Single cylinder within a periodic lattice and paths for comparison of numerical solutions. b) Velocity profiles along path 1 at $t = 200$ s.*

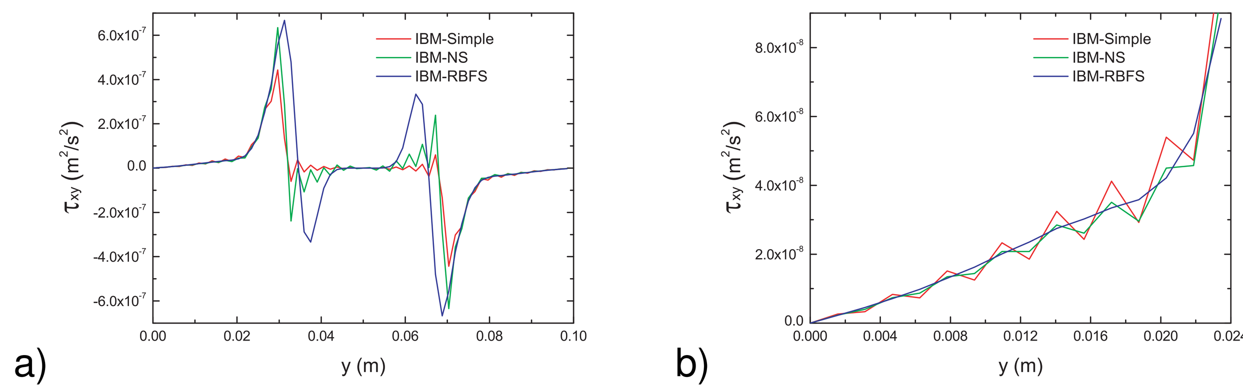

Figure 4: *a) τ_{xy} along path 1 at $t = 200$ s. b) The left part of a) enlarged.*

4. Conclusions and remarks

IBM-RBFS is an accurate and robust numerical method for solving the incompressible Navier-Stokes equations. Compared to the other IBM implementations without the same kind of smoothing, the Gibbs oscillation is significantly alleviated by the proposed method. Although the proposed smoothing technique is only applied to the velocity field in this paper, it can be easily extended for other physical variables such as temperature for heat transfer, concentration for scalar transport, and subgrid-scale stress components for large eddy simulation, after minor modifications in order to meet different boundary conditions on the body surface.

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