Anisotropy in MHD turbulence with mean field: zero parallel cascade?

Roland Grappin (*1*), Andrea Verdini (*2*), Wolf-Christian Müller (*3*)
(1) LPP, LUTH (2) SIDC (3) MPI Garching

This work deals with anisotropy of MHD (incompressible) turbulence with mean magnetic field
The anisotropy issue, in presence of a mean field is important as it determines the scaling and dissipation rate of turbulence and its scaling with B°.
We use two different numerical tools to explore how the cascade occupies Fourier space during the nonlinear cascade.

PLAN

Introduction
(1) Full MHD, periodic boundaries, forcing in volume
(2) Reduced MHD with Shell model for : energy injection at open boundary

Laboratoire de l'Univers et de ses Théories

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Introduction: why anisotropy?

Incompressible MHD equations with unit Prandtl number:

$\partial_t z^+ - B°\partial_x z^+ + (z^-.\nabla)z^+ + \nabla P = R^{-1}\Delta z + f^+$

$\partial_t z^- + B°\partial_x z^- + (z^+.\nabla)z^- + \nabla P = R^{-1}\Delta z- + f^-$

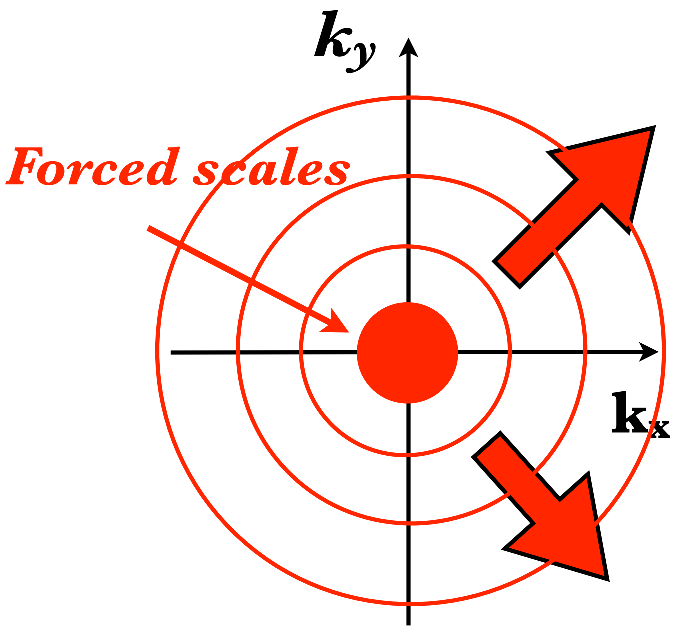
$z+ = u - b$ $z- = u + b$ $(b = (B-B°)/\sqrt{\rho°})$ div $z+ =$ div $z- = 0$

Assume **B° larger than fluctuations** $(b,u,z^\pm)$

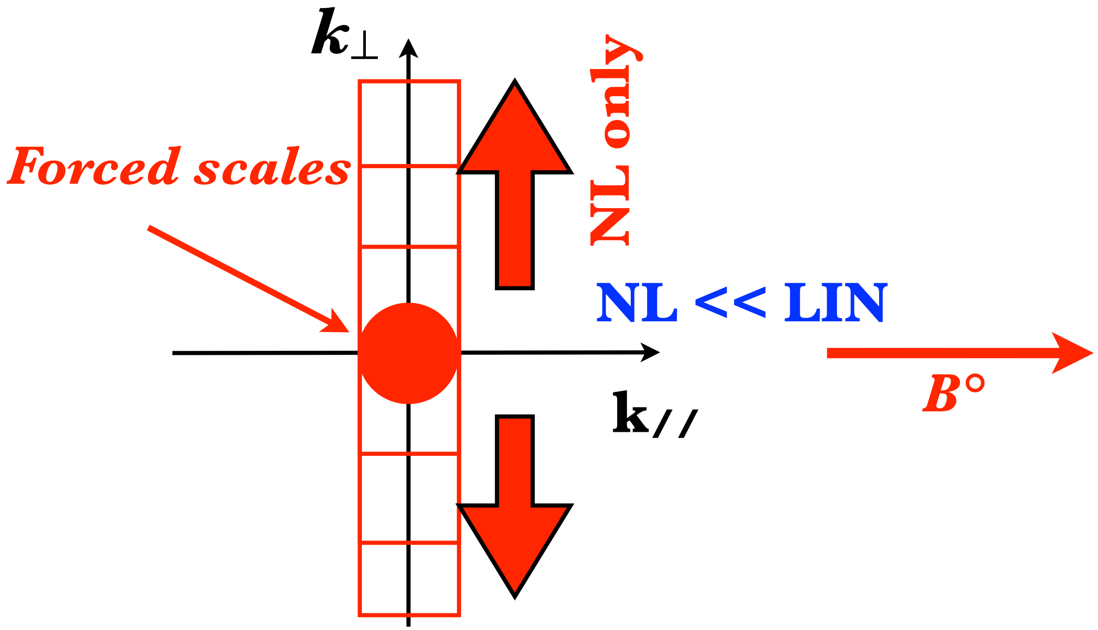
Gradients // to B° have strong $B°\partial_x z^\pm$ => **NL** term **<< LIN** term

Gradients ⊥ to B° have ZERO $B°\partial_x z$ => **NL term alone**

Isotropic case (B°=0)



Anisotropic case (B° large enough)



Filling region NL≥ LIN

<u>I Defining NL/LIN dominated regions</u>

•NL time and LIN Alfvén propagation time

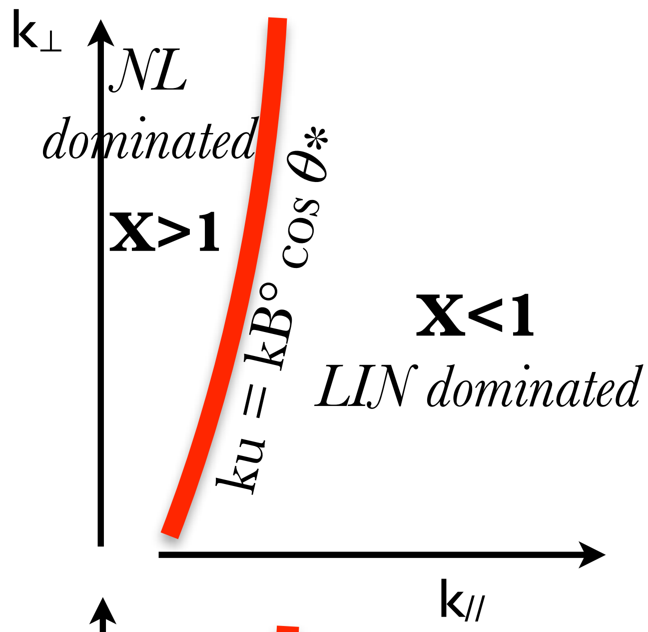
$\tau_{NL} \approx ku$, $\tau_A = kB°\cos\theta = k_{//}B°$

•**Time ratio X = τ_A / τ_{NL}**

• Assuming $u \approx l^{1/3} \approx k^{-1/3}$ (K41)

• NL/LIN boundary X=1

$\cos\theta{*} \approx k^{-1/3}$ or $k_{//} \propto k_{\perp}^{2/3}$



<u>II Critical Balance conjecture</u>

•Assume à la Kolmogorov cascade along $k_{\perp}$

•Assume **zero** cascade along $k_{//}$ (cf. Strauss equ., *reduced MHD*)

Goldreich Sridhar 1995

•**Conjecture 1**:

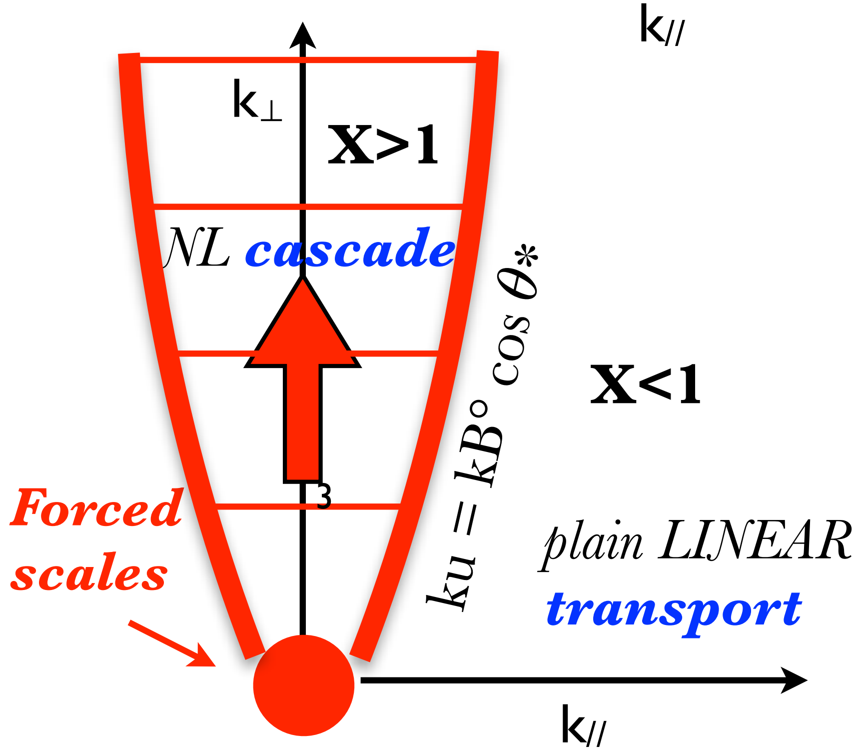
// (linear) and ⊥ (NL) <u>correlation times are equal</u>:

$\tau_{//}^{cor} = \tau_{\perp}^{cor}$ (1)

•**Conjecture 2**:

$k_{\perp}^{-5/3}$ <u>spectrum fills region X≥1</u> (2)

… *with negligible energy outside*



I Full MHD

1024^2 x 256 simulation results, $\varepsilon = b_{rms}/B° = 1/5$

•Continuous: 3D energy contours of 3D energy spectrum $E_3(k,\theta)$

•Dotted: energy contour of *model* 3D spectrum $E_3(k,\theta) = A(\theta) k^{-m-2}$

$A(\theta) = (\varepsilon^{-2} \cos^3\theta + \sin^3\theta)^{-(m-2)/2}$ **(1)**

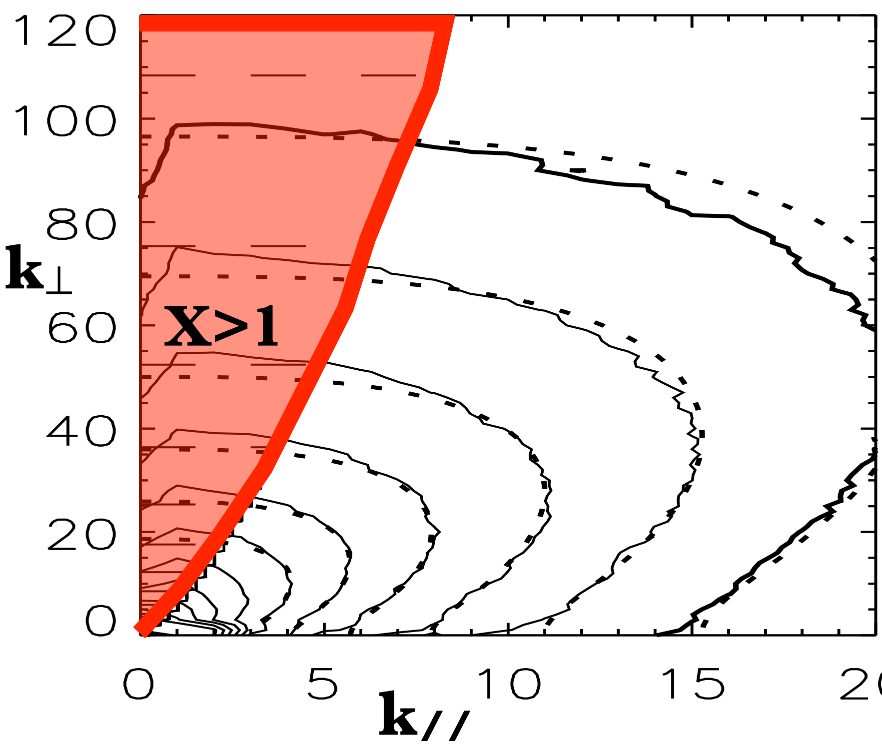
•**Power-law identical in all directions (m=3/2)**

•Inertial range extent varies with θ as $k_d(\theta) = k(\pi/2)(\varepsilon^{-2} \cos^3\theta + \sin^3\theta)^{-1/2}$

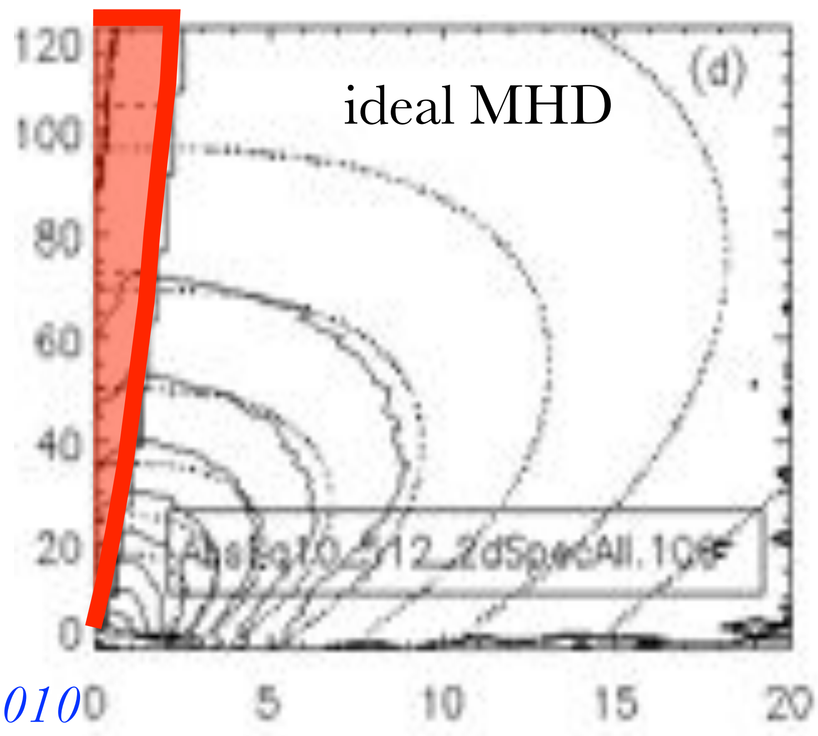
=> $k_d(0)/k_d(\pi/2) = \varepsilon = b_{rms}/B°$

•Excited region largely outside region X>1

Contours of 3D energy spectrum, ***B°=5b$_{rms}$***



Contours of 3D energy spectrum, ***B°=10b$_{rms}$***



Grappin Müller 2010

Anisotropy scaling with B°: quantifying // extent

1. Full MHD properties

Power-law extent given by dissipative wavenumber:

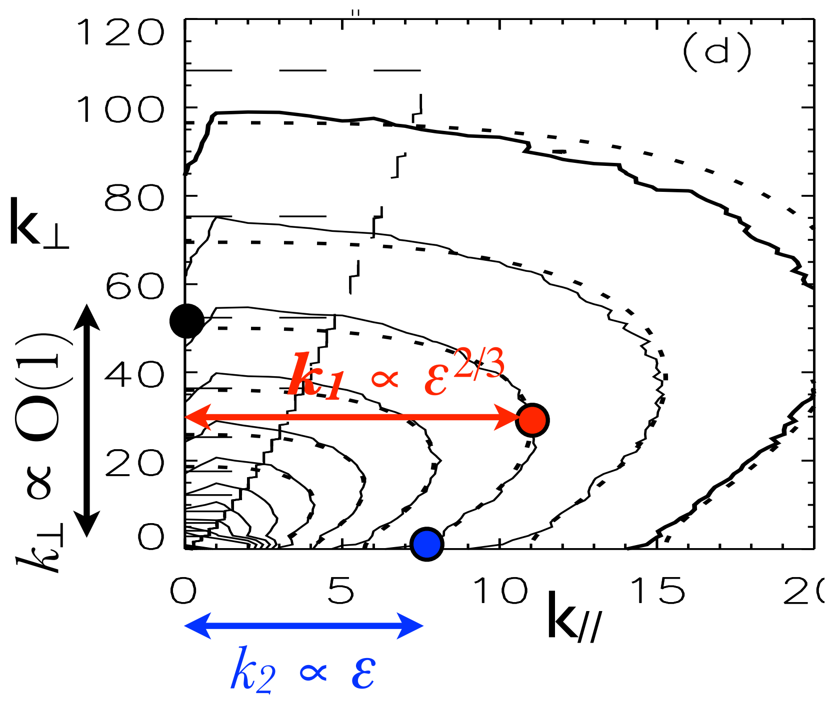
$k_d(\theta) = k(\pi/2)(\varepsilon^{-2} \cos^3\theta + \sin^3\theta)^{-1/2}$

•When B° grows (or ε decreases):

Max ⊥ extent : $k_\perp \propto O(1)$

MAX // extent $\mathbf{k_1} \propto \varepsilon^{\mathbf{2/3}}$

MIN // extent $\mathbf{k_2} \propto \varepsilon$



2. Critical balance prediction

•Spectrum confined to X>1 region

Energy spectrum as $E_3(k,\theta)=f(X)k_\perp^{-m-2}$ with f(X) negligible when X<1.

•As B° grows ($\varepsilon \rightarrow 0$)

Max ⊥ extent : $k_\perp \propto O(1)$

MAX // extent $\mathbf{k_1} \propto \varepsilon$

=> CB spectrum becomes increasingly different from full MHD spectrum as B° grows

3. Conjecture for large B°

•Full MHD has

- **Reduced decrease of // spectral extent** in oblique directions ($\Delta \mathbf{k}_{//} \approx \varepsilon^{2/3}$)
- **Reduced dissipation** compared to Kolmogorov perpendicular cascade scaling as

$\mathbf{t_{NL}^{-1}} \varepsilon^{2/3}$

Radial cascade?

•In contrast with the **isotropic** forced scales, the scales which play the role of the **large scale input** ($k=k^*\approx 8$) are **NOT isotropic**: they follow a definite $A(\theta)$ dependence which depends on ε.
•Hence, one can define an **effective Reynolds** at each **angle** θ, which controls a (possibly) radial cascade starting with $k=k^*$.

Power-law range extent

1. Hyp. of **pure radial cascade** + standard argument on large scale input (k^*), small-scale sink (k_d), and **Reynolds** $Re \approx A(\theta)^{1/2}$:
$k^* u^{\circ 3} \approx \nu\, k_d^2 u_d^2$, $\qquad u^2_k \approx kE(k,\theta) \approx A(\theta) k^{-1/2}$

$$\mathbf{k_d \approx Re^{2/3}} \qquad \textbf{(1)}$$

2. **Simulation gives** $E_3(k_d,\theta) = \text{const} = A(\theta)\, k_d^{-7/2}$

$$\mathbf{k_d \approx Re^{4/7}} \qquad \textbf{(2)}$$

Conclusion

•power-law range extent **less anisotropic** than would predict **purely radial en. flux**
•radial range NOT inertial, **$\perp$ scales directly feed // scales**

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Interplay of waves and NL coupling

Residual energy $E_R(k)=E_M(k)-E_V(k)$ related to **total energy** $E_T(k)= E_M(k)+E_V(k)$

$\partial_t E_R(k) = -E_R(k)/t_A + E_T(k)/t^*$

t_A = isotropized Alfvén time = $(kB°)^{-1}$, $t^* = t_{NL}\,(b/B°)$ (IK long interaction time)

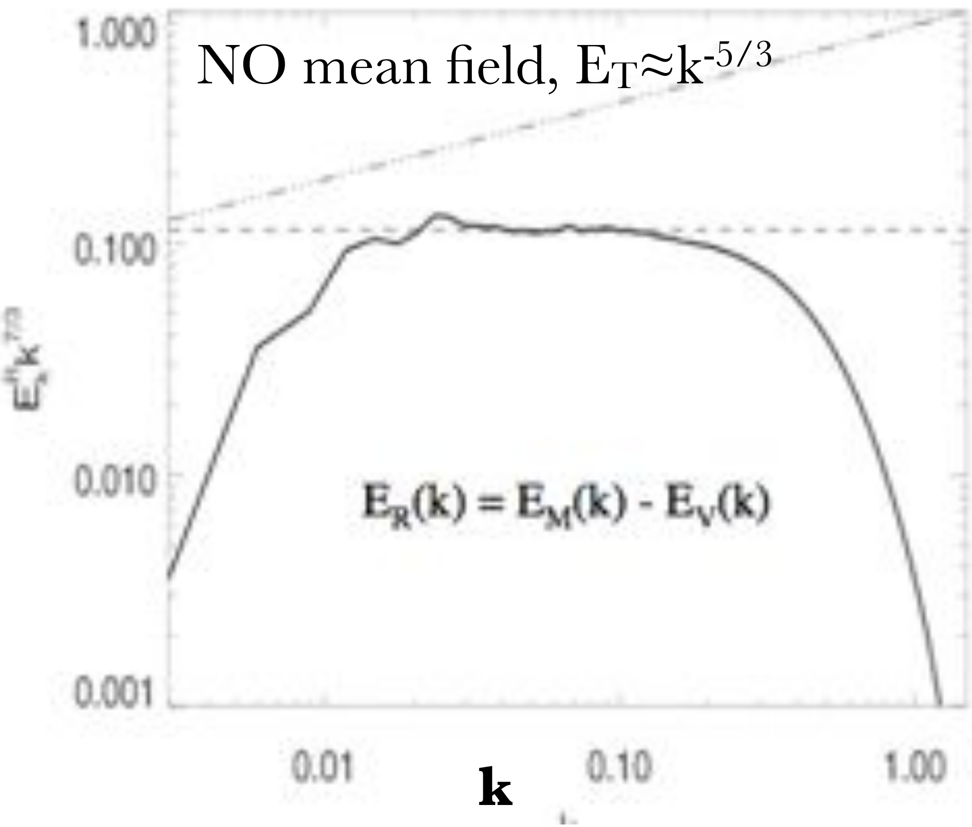
Equilibrium state:

$E_R = (t_A/t_{NL})^2\, E_T \quad => \quad E_R \approx k\, E_T^2$

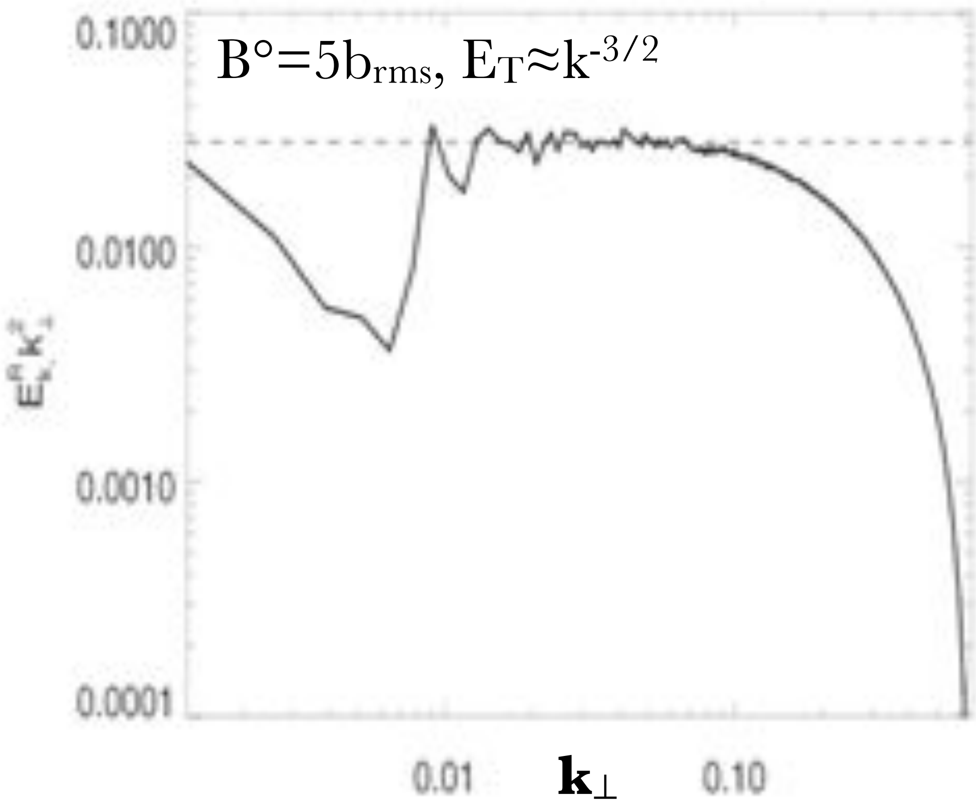
Müller Grappin 2005

- 2 cases: $E_T \approx$ $\mathbf{k^{-5/3}}$ => $E_R \approx$ $\mathbf{k^{-7/3}}$; $E_T \approx$ $\mathbf{k^{-3/2}}$ => $E_R \approx$ $\mathbf{k^{-2}}$
- Indicates quasi-// Alfvén waves play a more active role than just propagating ⊥ fluctuations along mean field

Residual energy spectrum norm. by $\boldsymbol{k^{-7/3}}$



Residual ⊥ energy spectrum norm. by $\boldsymbol{k^{-2}}$



II Reduced MHD - Shell model

•Reduced MHD

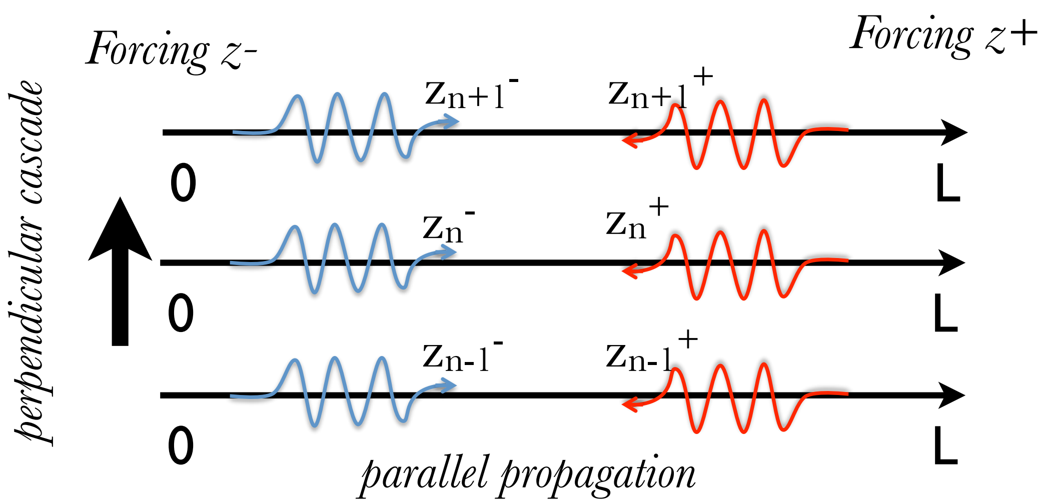
div $z^{\pm}=0$

$\partial_t z^{\pm}$ -+ $B^{\circ}\partial_x z^{\pm} + NL^{\pm} = R^{-1}\,\Delta z^{\pm} + F^{\pm}$ **but** with **$\nabla=(0,\partial_y,\partial_z)$ in $NL^{\pm}$**

•SHELL model:

Replace NL terms by shell model (as many shell models as grid points along x-axis)

NB Shell model increase Reynolds with Log discretization of wave numbers
here, 20 shells <=> $2^{20}\approx 10^{6}$ mesh points in the two directions of the $\perp$ plane...



Buchlin & Velli. ApJ (2007), Nigro et al PRL (2004)

Spectra $E^{\pm}(x,k_{\perp})$

$b_{rms}/B^{\circ} = 0.05$; **$L_{\perp}=2\pi/k^{\circ}=0.01\ L$** (aspect ratio 100)

⊥ modes k_n (n=2,3,4) injected at x=0 and L

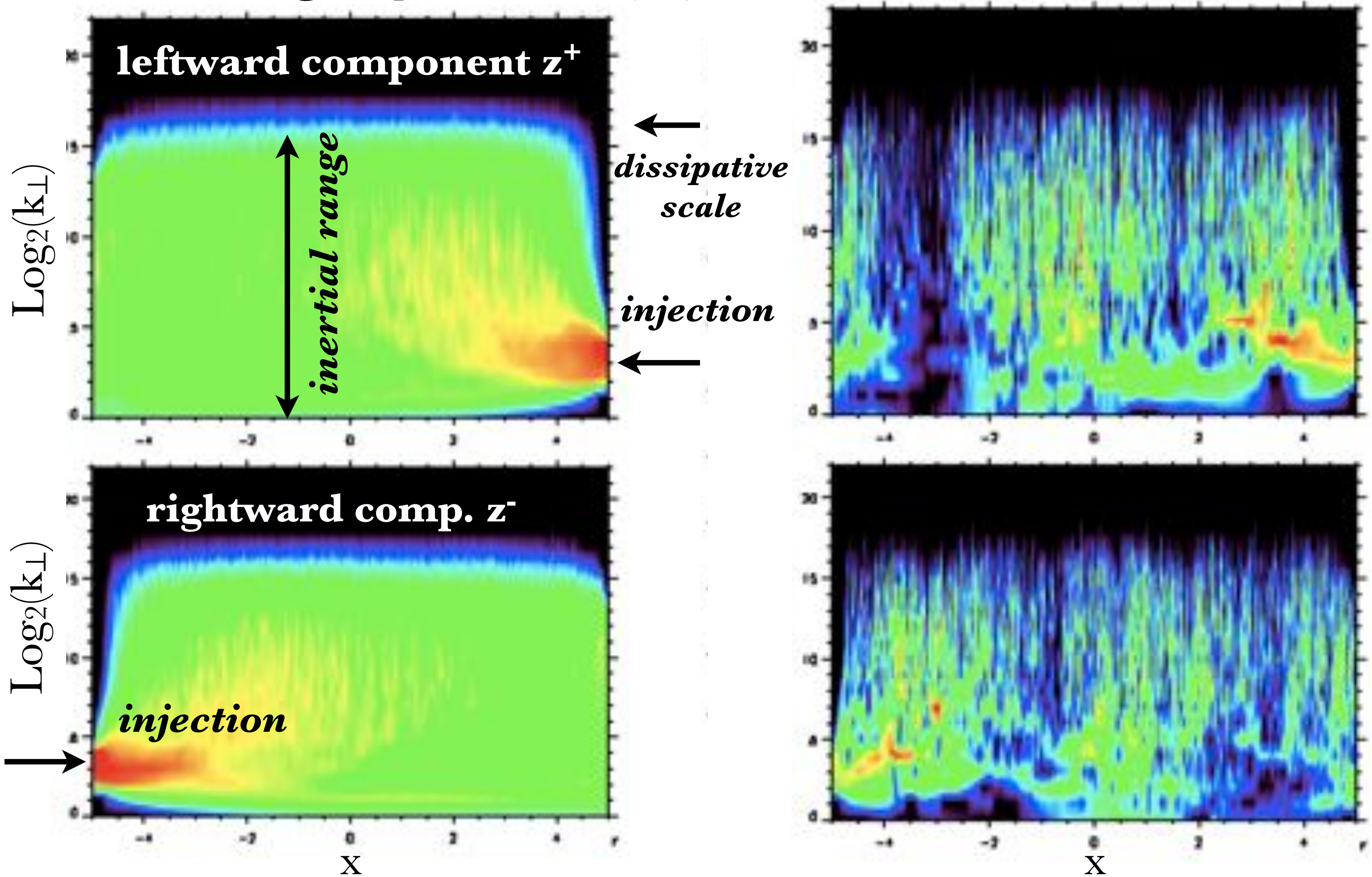
When z^+ and z^- modes meet, they start a **cascade** to larger $k_{\perp}$ (larger n).

=> spectrum $E^{\pm}(k_n,x)$, <u>different at each point x</u>

•Forcing such as $t_{NL} << t_A = L/V_A$: spectra have time to develop before leaving domain

***Time-averaged** spectra $k^{5/3}\ E^{\pm}(x,k)$*

***Snapshot** of spectra*



1. <u>Time-averaged spectra</u> (7 Alfvén times): normalizing by $k^{-5/3}$ reveals **5/3 slope** except close to boundaries (x=0, L)

2. <u>Snapshots</u> reveal // structures which decrease at small ⊥ scales

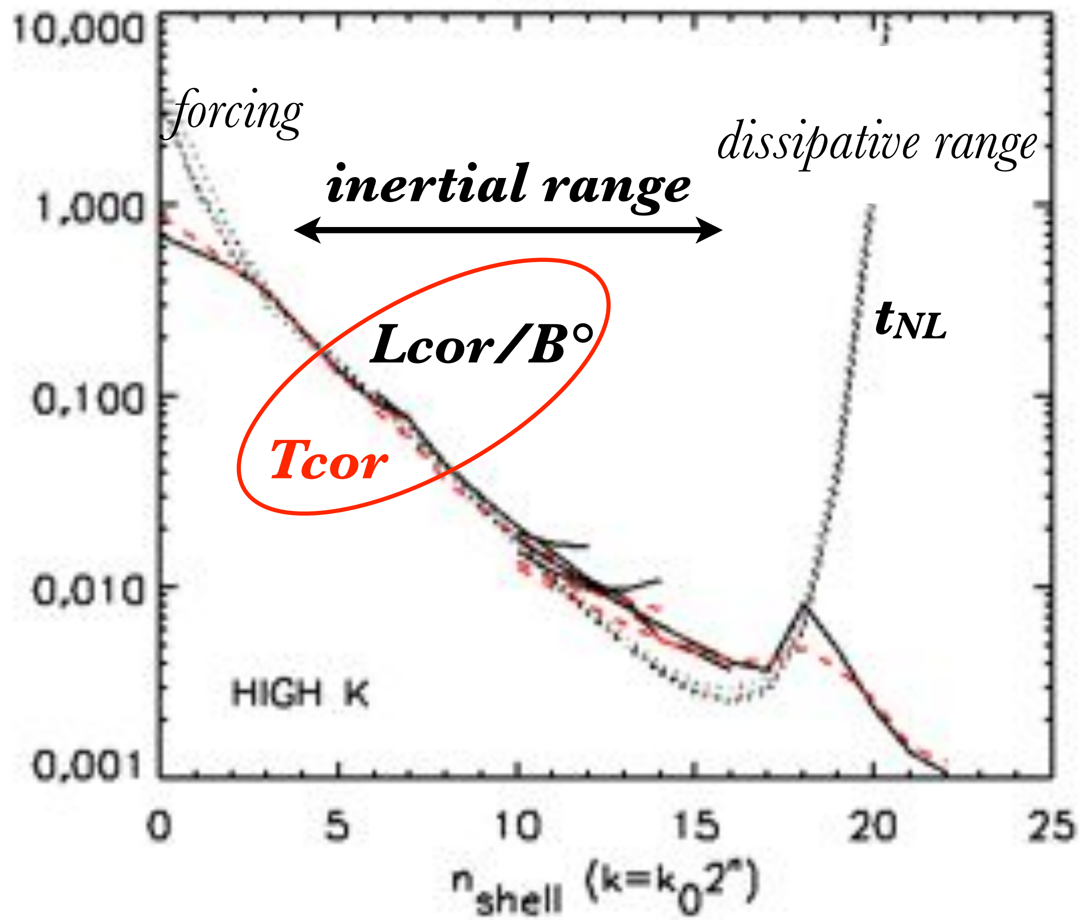
=> Test for critical balance and anisotropy ?

Critical balance between perp and // times?

Each perpendicular mode in the domain [0,L] x [1,N] is a random signal
One may define for each mode at each point x of the domain:

- a correlation time => and average on x
- a nonlinear time based on amplitude of u_n and wavenumber k_n=> average on x
- a parallel correlation length

Tcor, Lcor/B°, t_{NL}



For each n, compute characteristic times ($n=\log_2(k)$):

1. ⊥ correlation time $T_{cor}(n)$
2. // correlation length $L_{cor}(n)$ and // correlation time $L_{cor}/B°$
3. NL time $t_{NL}(n) = (k_n|u_n|)^{-1}$

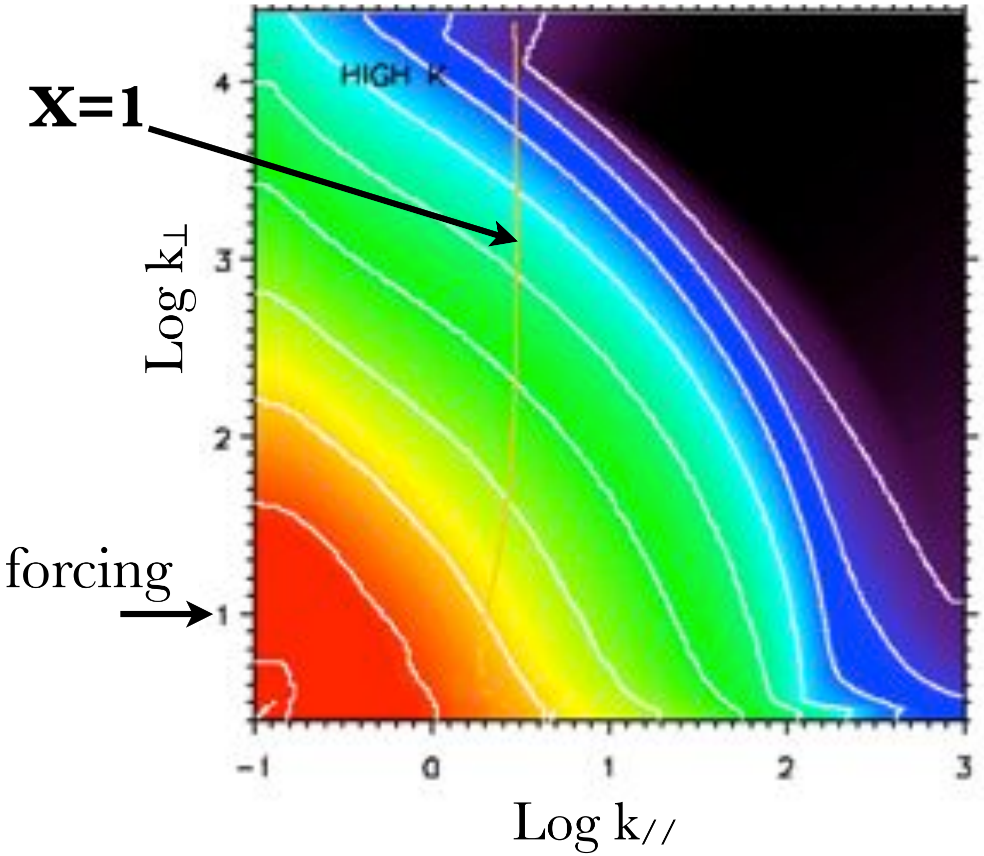
In inertial range, <u>all three time scales coincide</u>:
=> Shell model show critical balance of NL perp and linear // times

(preliminary) Full angular spectra $E(k_{//},k_\perp)$: anisotropy

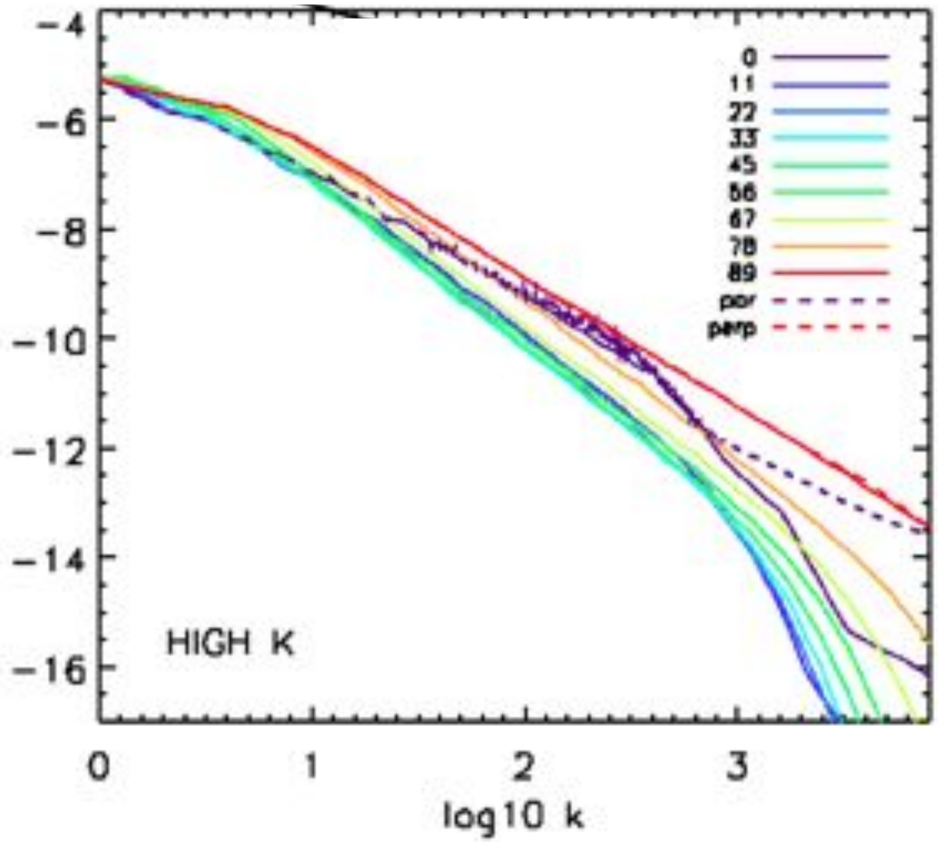
- Spectra $E^{\pm}(x,k)$ **transformed into *full spectra*** $E\pm(k_{//}, k_\perp)$ via FFT along x axis
- NL forcing time << L/V_A

NB Using cartesian-Log coordinates spoils anisotropy in Fourier space
$E^{\pm}(\log|k|)$ in each direction θ should be used instead, as below
NB2 Needs more checking: normalization AND interpolation (*very large aspect ratio*)

(Polar) angular energy spectrum $E(\log|k|,\theta)$



1D energy spectra along various directions



Summary

1. Full MHD, periodic boundaries, volume forcing

Properties distinct from critical balance predictions are found when $B° > b_{rms}$

A uniform 1D spectral slope is found in all directions; anisotropy has a weak scaling with $\varepsilon = b_{rms}/B°$, dissipation is reduced, varying as $\varepsilon^{2/3}$. This is valid for:

- power-law range as well as dissipative range
- resistive and ideal runs (1D slope resp. m=3/2 and m=5/3)
- Reynolds numbers from 100 to 1000
- $B°/b_{rms}$ = 5 and 10

2. Reduced MHD, injection through open boundaries

This model adopts from start zero nonlinear parallel interactions

It thus allows to explore in detail the basic predictions of the critical balance (CB) paradigm

•Each scale of the perpendicular **inertial range** (5/3 power-law) have the 3 time scales equal: (i) the perpendicular correlation time (ii) the nonlinear time (defined as $k|u|^{-1}$) (iii) the linear propagation time (defined as L_{cor}/V_A)

•However, anisotropy of energy spectrum $E(k_{//}, k_{\perp})$ **might be** at odd with CB predictions