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Improved assessment of Deep Thermal Field based on Joint Inversion of Heat Flow, Elevation, Geoid Anomaly data

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Context of the present work

Knowledge of temperature field in deep crustal layers is often of considerable importance in assessment and exploration of Enhanced Geothermal Systems.

Results of regional heat flow studies are usually employed for this purpose. However, there are large uncertainties in downward continuation of gradient and heat flow values measured at shallow depths.

In the present work we propose that elevation and geoid height (which are also indicators of subsurface thermal state) may be used, jointly with heat flow, in obtaining better estimates of the deep thermal field in geothermal areas.

Methodology Employed

Lachenbruch and Morgan (1990) discussed models of crustal and lithospheric structure based on elevation and heat flow data. Fullea et al (2007) extended this approach by incorporating geoid height as an additional constraining parameter.

In this work, we present a refinement of this technique which admits surface heat flow as an input parameter and in addition allow for the effects of vertical variations in thermal conductivity and radiogenic heat production.

The technique employed is based on computationally stable iteration schemes and provide simultaneous checks for compatibility of the inversion results with observational data on surface heat flow, radiogenic heat production, elevation and geoid height.

Basic assumptions in Join Inversion of heat Flow, elevation and Geoid Height

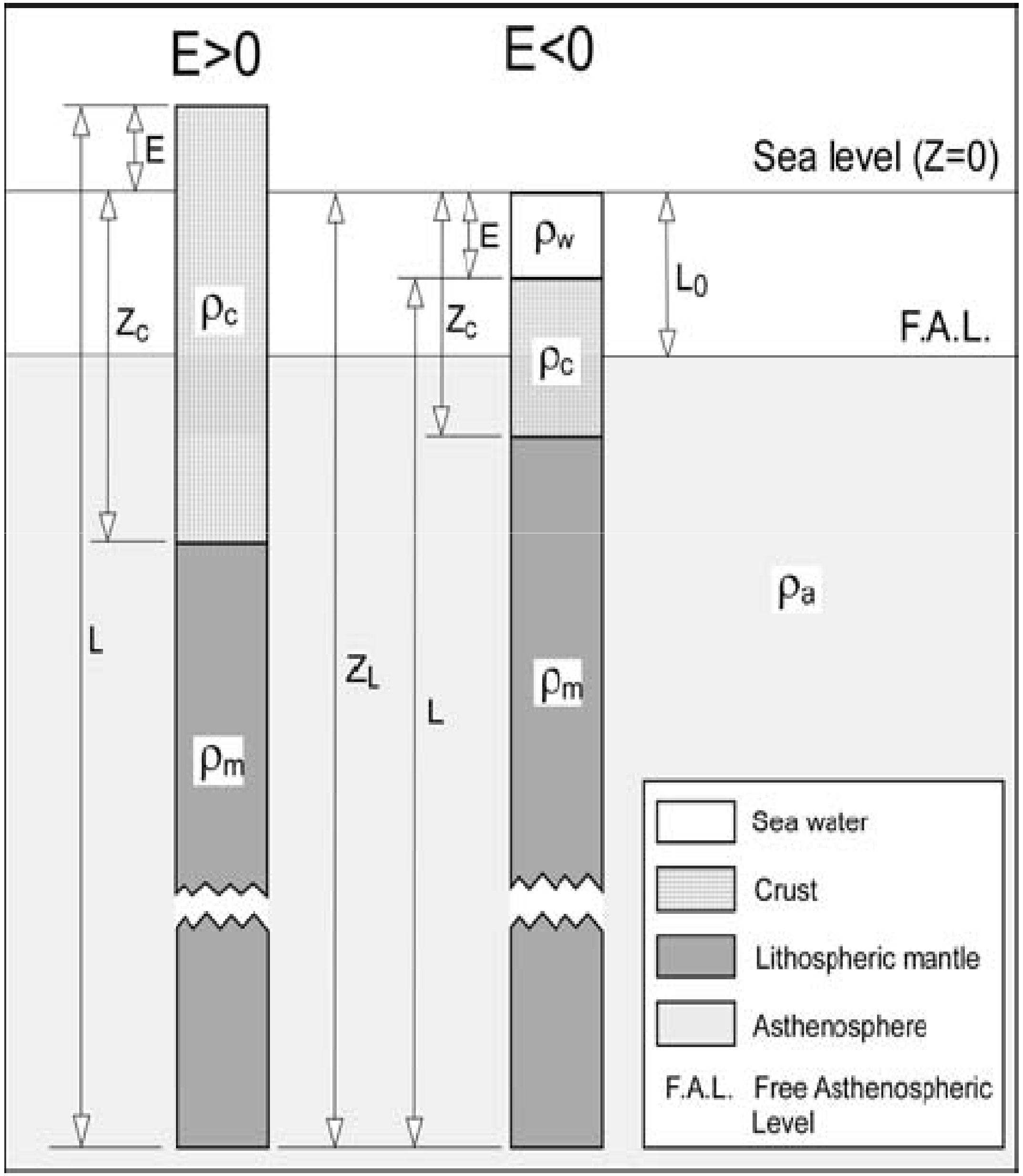
1- Conditions of thermal isostasy prevail;

2- Lateral variations in density are small compared to vertical changes.

Under such conditions the geoid height is proportional to the dipole moment of the vertical distribution of density (Ockendon e Turcotte, 1977; Turcotte e Oxburgh, 1982):

$$N = -\frac{2\pi G}{g} \int_{cL} z \, . \, \rho(z) \, dz + N_0$$

Isostasy of the lithosphere



Above sea level

$$E = \frac{(\rho_a - \rho_l)}{\rho_a} L - L_0 \qquad E > 0$$

Below sea level

$$E = \frac{\rho_a}{\rho_a - \rho_w}\left(\frac{\rho_a - \rho_l}{\rho_a} L - L_0\right) \qquad E < 0$$

Under conditions of local isostasy the relation between elevation and crustal thickness is:

$$z_c = \frac{\rho_a L_0 - E\,\overline{\rho_c} + z_L\left(\overline{\rho_m} - \rho_a\right)}{\left(\overline{\rho_m} - \overline{\rho_c}\right)} \quad \rightarrow \quad E > 0$$

$$z_c = \frac{\rho_a L_0 + E\left(\rho_c - \rho_w\right) + z_L\left(\rho_m - \rho_a\right)}{\left(\rho_m - \rho_c\right)} \quad \rightarrow \quad E < 0$$

The mantle density is assumed to be temperature dependent

$$\rho_m(z) = \rho_a\left(1 + \alpha\left[T_a - T_m(z)\right]\right)$$

This couples isostasy to the thermal field

Coupling isostasy to the thermal field require knowledge of temperatures

Methods of estimating temperatures at the base of the crust

1- Use computed values of depth to base lithosphere (z_l) and estimated value of mantle heat flow (q_m)

Fullea et al, 2007

2- Use computed values of depth to base crust (z_c) and measured values of surface heat flow (q_0)

Alexandrino & Hamza, 2008

The advantages of this latter approach is that crustal temperature field may be derived from experimental heat flow data (rather than estimated moho heat flow) and it is possible to incorporate the effects vertical variations in the thermal properties of the crust.

Thermal model of the crust

Consider the differential equation for steady-state temperature distribution in a medium with heat sources:

$$\frac{d}{dz}\left[\lambda(T)\frac{dT}{dz}\right] = -A_0 \exp(-z/D)$$

where z is the depth, *T* is temperature, *λ(T)* the thermal conductivity, *Ao* is the heat production in near surface layers and *D* is logarithmic decrease of heat production with depth.

The boundary conditions are:

Heat flux at the surface:

$$\lambda(T)\frac{dT}{dz}\bigg|_0 = q_0$$

Temperature at the surface:

$$T(z=0) = T_0$$

Phonon and Radiative processes contribute to overall Thermal Conductivity:

$$\lambda(T) = \frac{\lambda(25)}{1+BT} + C(273.15+T)^3$$

The general equation for isostasy may be expressed as a quadratic relation for the thickness of the lithosphere:

$$z_L^2\left(T_a k_c-\theta\right)+z_L\left(z_c\left(T_a\left(k_m-2k_c\right)+2\theta\right)-\delta+T_a E k_m-\frac{2k_c}{\rho_a\alpha}\left[\left(\rho_a-\rho_c\right)z_c+\eta\right]\right)$$

$$+\left(z_c\left[\delta-T_a\left(z_c\Delta k+Ek_m\right)-z_c\theta\right]-\frac{2}{\rho_a\alpha}\left[\left(z_c\Delta k+Ek_m\right)\left(\eta+\left(\rho_a-\rho_c\right)z_c\right)\right]\right)=0$$

The relation for geoide height becomes:

$$N=-\frac{\pi G}{g}\left[\begin{array}{l}\rho_w E^2+\frac{2\beta}{3}\left(z_c^3-E^3\right)+\left(\beta E+\rho_c^T\right)\left(z_c^2-E^2\right)\\+\left(z_{\max}^2-z_c^2\right)\rho_a+\rho_a\alpha\frac{T_a-T_{mh}}{3}\left[\left(z_L-z_c\right)\left(z_L+2z_c\right)\right]\end{array}\right]+N_0$$

The combined solution of these equations allow analysis of elevation and geoide height under conditions of thermal isostasy

Iterative schemes are necessary because of the non-linearity of the equations

Computational steps of Fullea et al, 2007 and Alexandrino & Hamza, 2008

•1. Estimate the initial values for Z_C and Z_L, assuming constant density for crust and mantle;

•2. Use the initial value of Z_C for calculating the depth to the base of the lithosphere, which couples isostasy to the thermal field;

•3. Calculate temperatures at the base of the crust (T_c) and of the lithosphere (Ta) using values of Z_C and Z_L of step 2 and measured values of surface heat flow q_m;

•4. Calculate the thermal conductivity the crust (λ_c) and lithosphere (λ_m) using values of T_c and Ta of step 3;

•5. Calculate the geoid height using Z_C , Z_L , T_c , T_a , λ_c , and λ_m obtained in steps 3 and 4;

•6. Determine the residual anomaly (calculated – observed);

•7. Change the value of Z_C and repeat the process until the residual anomaly is minimized.

Input data Module

Fullea et al, 2007

Moho Temperature	
Equation 10	
θ	133,79
delta	1,1860E+08
deltaK	0,70

Alexandrino & Hamza, 2008

Moho Temperature	
Equation 10b	
θ	139.32
delta	1.38E+08
deltaK	1.09
Param B	6.82E-04
Param C	6.32E-10
Surface Heat Flow	5.00E-02
Thermal Conductivity	3.00

Modules of Iterative Processes

Fullea et al, 2007

Single iteration - Initial Estimates		
Moho Depth (km)	zc ref	26,95
Lithosphere Thickness (km)	zL ref	91,30

Alexandrino & Hamza, 2008

Multiple Iteration Process		
Moho Depth (km)	zc ref	26.23
Lithosphere Thickness (km)	zL ref	83.80
Feedback of Z_L based on heat flow		83.88

Spreadsheet Layout for Modules of Input Data

Input Parameters		
Density at top	ρc t	2640,00
Density at bottom	ρc b	2920,00
Average density	ρc m	2780,00
Mantle density	ρm	3293,92
Density asthenosphere	ρa	3200,00
Density of water	ρw	1030,00
Compensation Level	z max	300000,00
Coeficient of expansion	α	3,50E-05
Radiogenic heat	Hs	8,20E-07
D parameter	hr	1,05E+04
Crustal conductivity	kc	2,5000
Mantle Conductivity	km	3,2000
Surface temperature	Ts	20,00
Temp. base lithosphere	Ta	1350,00
Elevation	*E*	500,00
Geoide Asthenospheric	Lo	2320,00
Gravitational Constant	G	6,67E-11
PI	pi	3,14
acceleration	g	9,79
Radiogenic heat	f	83,79

Thickness Lithosphere	
Equation 12	
eta	-8,299E+06
	3,241E+03
a	3,241E+03
Term 1	-1,769E+08
Term 2	2,160E+06
Term 3	1,349E+08
	-3,096E+08
b	-3,096E+08
Term 4	2,355E+12
	1,104E+12
c	1,250E+12
	1,250E+12
delta	2,822E+08
r1	4,2252E+03
r2	9,1304E+04
Z_L	9,130E+04
Zc	2,695E+04

Temperature Moho	
Equation 10	
θ	133,79
delta	1,1860E+08
deltaK	0,70
T_{Moho}	511,45

Hydro Geope Anomaly	
Equation 13	
Beta	1,020E-02
a	-2,142E-11
b	2,575E+08
c	1,331E+11
d	1,921E+12
e	2,857E+14
g1	3,131E+01
g2	9,344E+09
g	2,925E+11
Sum	2,880E+14
product	-6,1707E+03
N =	-6,1707E+03

Reference Hydro Geope	
Equation A4 (case b)	
π G / g	2,142E-11
((ρm-ρw)/(ρm-ρa)) E	1,205E+04
2 ρa L_0	1,485E+07
(ρa-ρw) E	2,115E+06
2 ρa L_0 + (ρa-ρw) E	1,696E+07
Product 1	2,044E+11
Z_0^2 ρa	2,880E+14
$(ρa\ L_0)^2$ / (ρm-ρa)	5,869E+11
Sum	2,888E+14
Noc =	6,1872E+03

Equation A1	
Termo 1	257500000,00
Termo 2	1,8351E+12
Termo 3	2,5284E+13
Termo 4	2,6132E+14
Soma	2,8844E+14

Equation A2	
a	7,424E+06
b	8,050E+05
c	8,5751E+06
soma	1,680E+07
d	6,539E+02
divisão	2,570E+04
Zc	2,570E+04

Equation A3	
kapa	8,299E+06
Termo 1	2,381E-03
Termo 2	5,472E+00
Termo 3	6,8873E+13
Termo 4	-1,863E+14
Termo 5	2,5517E+14
Z_L	1,0873E+05

Equation A4 (case a)	
a	2,142E-11
b	-4,375E+08
c	2,880E+14
d	1,640E+11
Soma	2,878E+14
No - N =	6166,73

2 – Modules for Iterative Steps

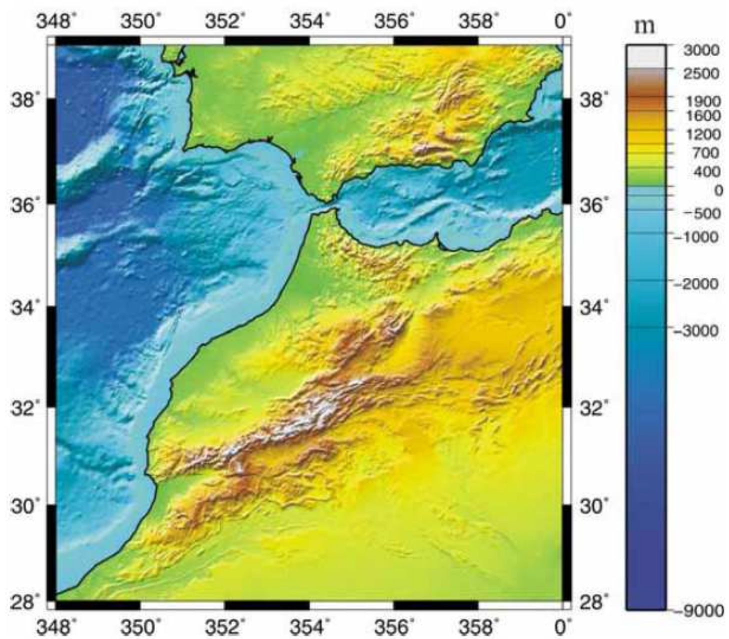
Estimates of Iterative Process		
Moho depth (km)	zc ref	26,95
Base of lithosphere (km)	zL ref	91,30

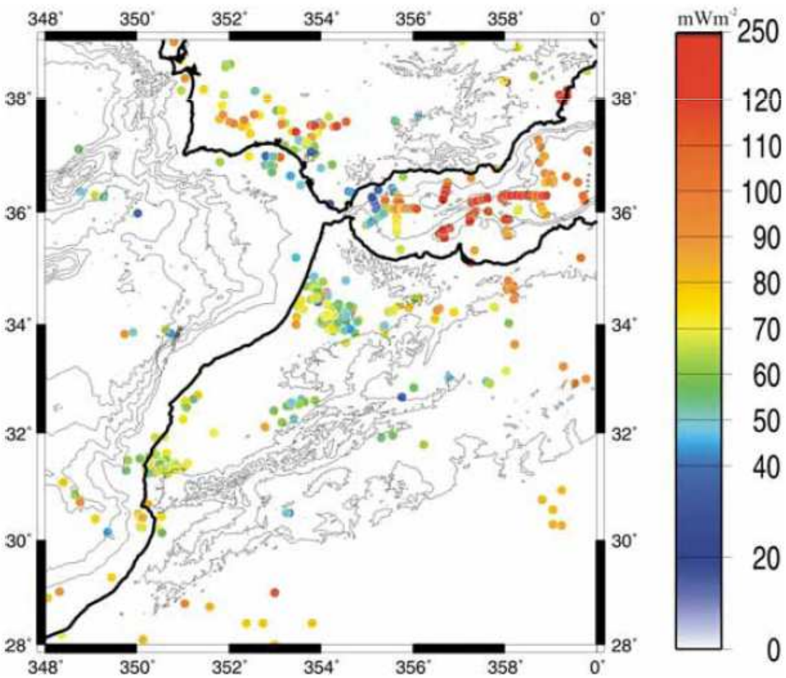
Hydro Geope Residual	
Geoide Height - calculated	-4,00
Geoide Height - observed	-4,00
Residual (observed - Calculated)	0,00

Mantle density	
Equation 11	
ρm m	3246,96

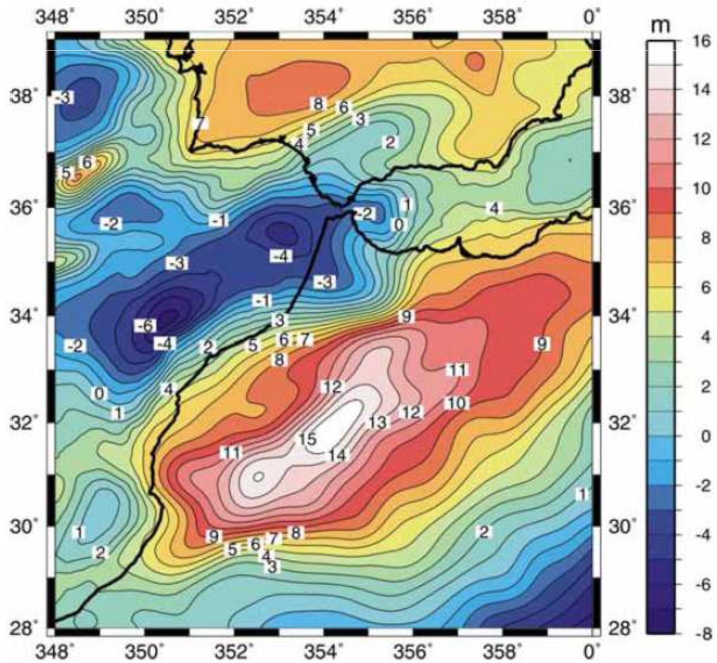
Study area:

Gibralter Arc System



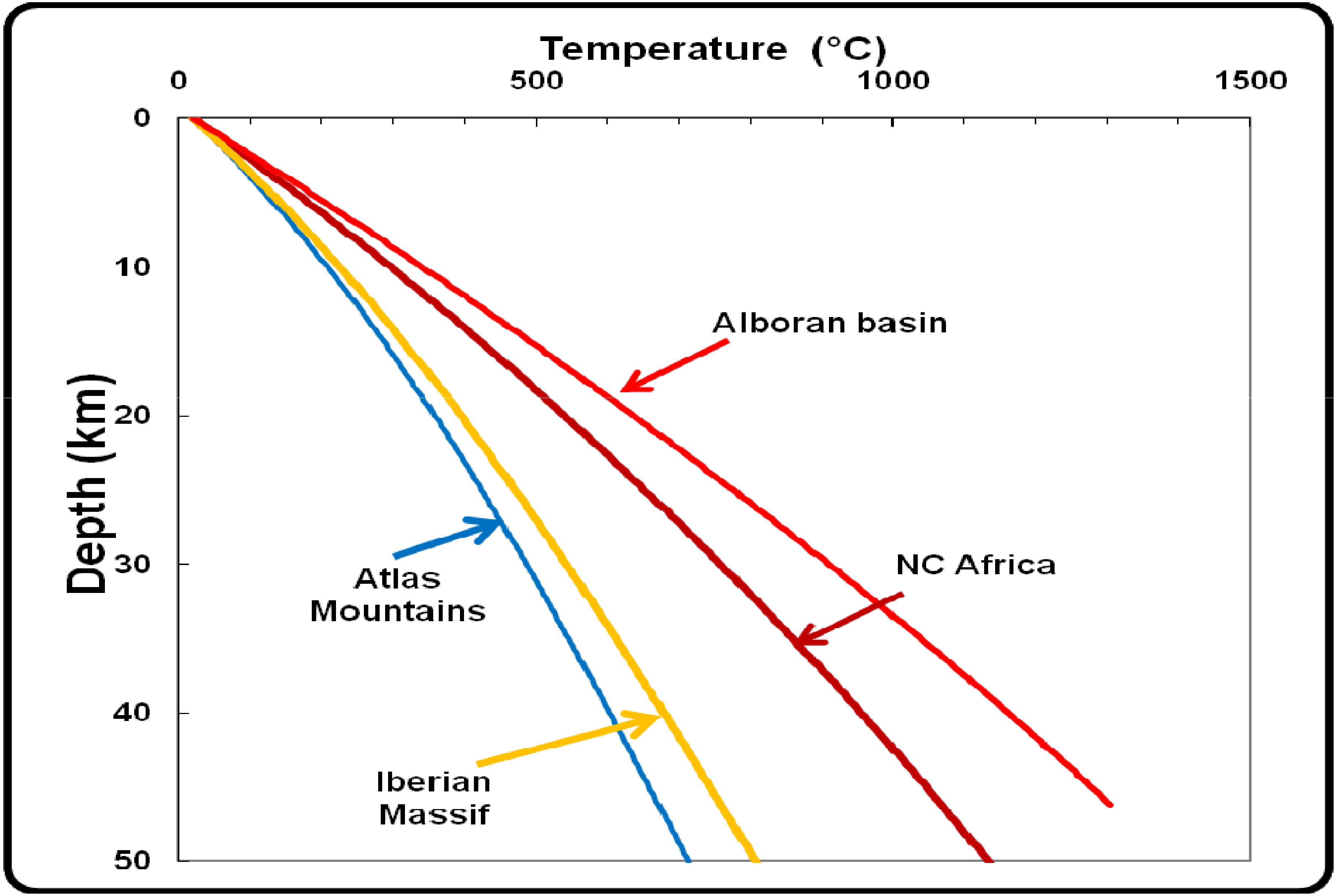


Heat Flow



Geoid Height

Temperature Distributions



Comparisons illustrating the differences between the results of Fullea et al (2007) and this work

Eastern Alboran Basin, Mediterranean

Parameter	Fullea et al	This work	Difference	Error (%)
Moho Depth (km)	21.5	18.4	3.1	14.5%
Depth to base of Lithosphere (km)	86.2	64.2	22.0	25.5%
Moho Temperature	425.6	541.9	-116.3	-27.3%

Atlas Mountains, Northwest Africa

Parameter	Fullea et al	This work	Difference	Error (%)
Moho Depth (km)	35.3	31.3	4.0	11.1%
Depth to base of Lithosphere (km)	160.3	138.5	21.7	13.6%
Moho Temperature	399.4	500.1	-100.6	-25.2%

Effect of Hydrothermal circulation

This is a major source of error in estimation of deep temperatures in enhanced geothermal systems.

The conventional methods based on results of gradient and heat flow measurements in shallow boreholes is incapable of prviding satsfactory solution.

The temperatures are usually overestimated.

In the method based on joint inversion of Elevation – Geoid Height- Heat Flow this problem can be solved by introducing in the input and interactive modules a geologically reasonable estimate of the fluid circulation depth.

Input data Module for minimizing the effects of hydrothermal circulation

Standard Input

Moho Temperature	
Equation 10b	
θ	139.32
delta	1.38E+08
deltaK	1.09
Param B	6.82E-04
Param C	6.32E-10
Surface Heat Flux	5.00E-02
Thermal Conductivity	3.00

Modified Input

Moho Temperature	
Equation 10b	
θ	139.32
delta	1.38E+08
deltaK	1.09
Param B	6.82E-04
Param C	6.32E-10
Perturbed Surface Heat Flux	12.00E-02
Conductivity	3.00
Hydrothermal Circulation Depth	3.00

Iterative module for hydrothermal Circulation

Standard

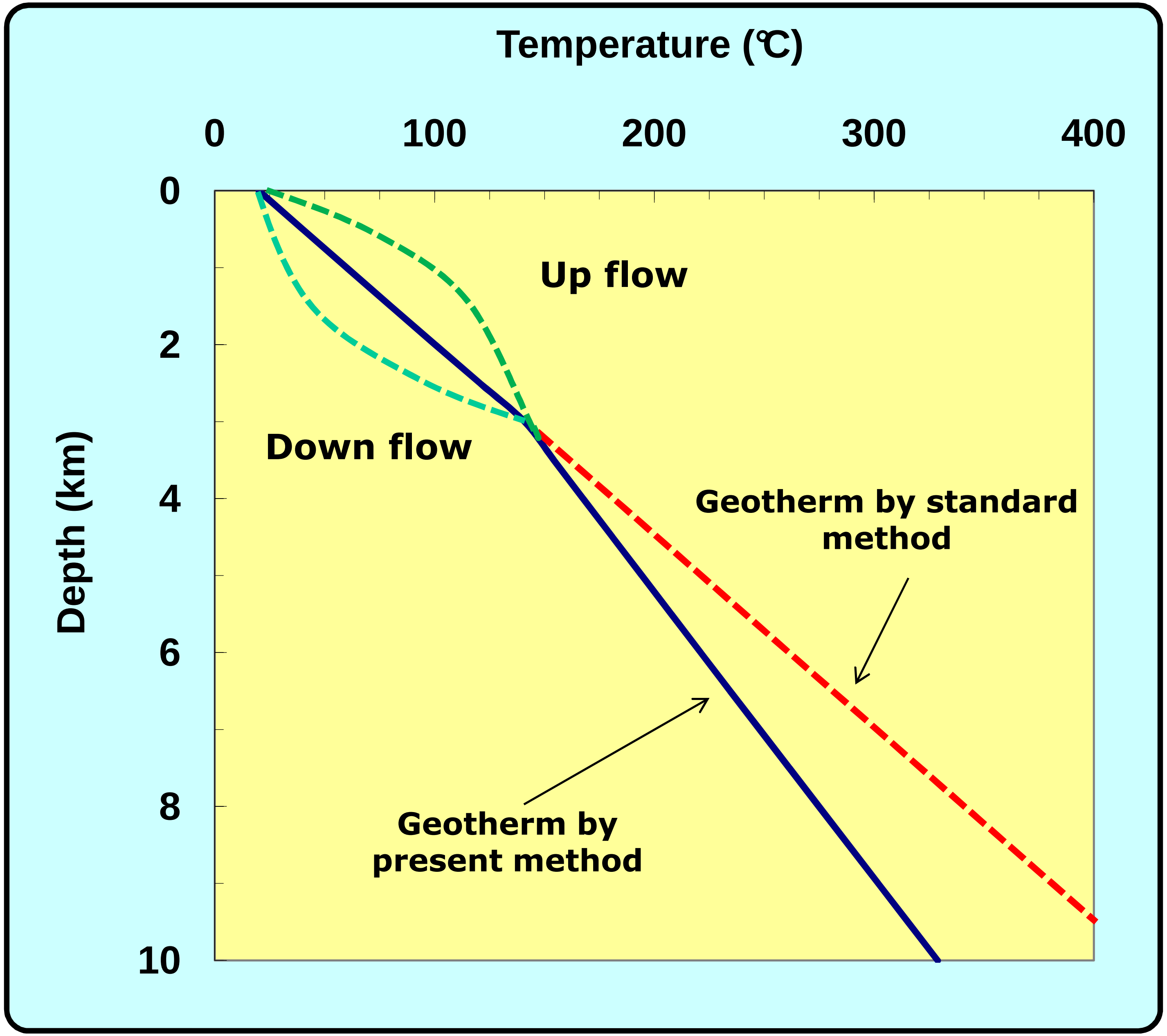
Multiple Iteration Process		
Moho Depth (km)	zc ref	26.23
Lithosphere Thickness (km)	zL ref	83.80
Feedback of Z_L based on heat flow		83.88

Modified

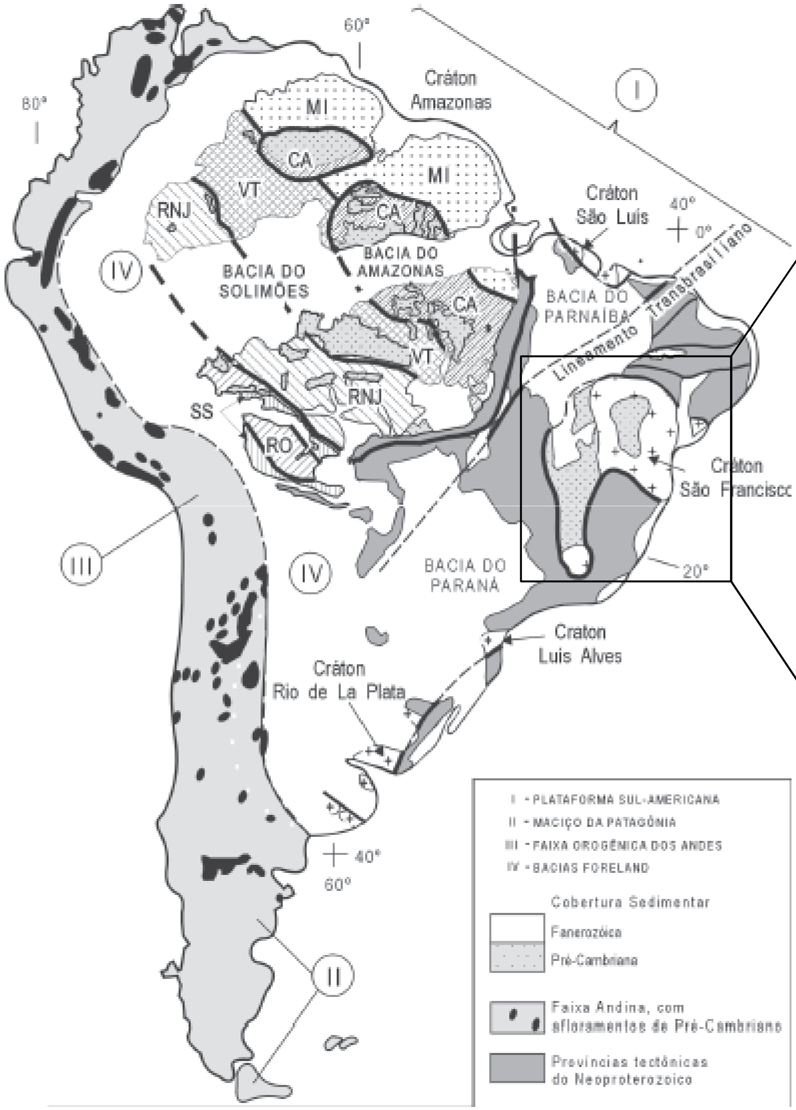
Multiple Iteration Process		
Estimated Depth of hydrothermal circulation	Z_h	3.00*
Moho Depth (km)	zc ref	26.23
Lithosphere Thickness (km)	zL ref	83.80
Feedback of Z_L based on heat flow		83.88

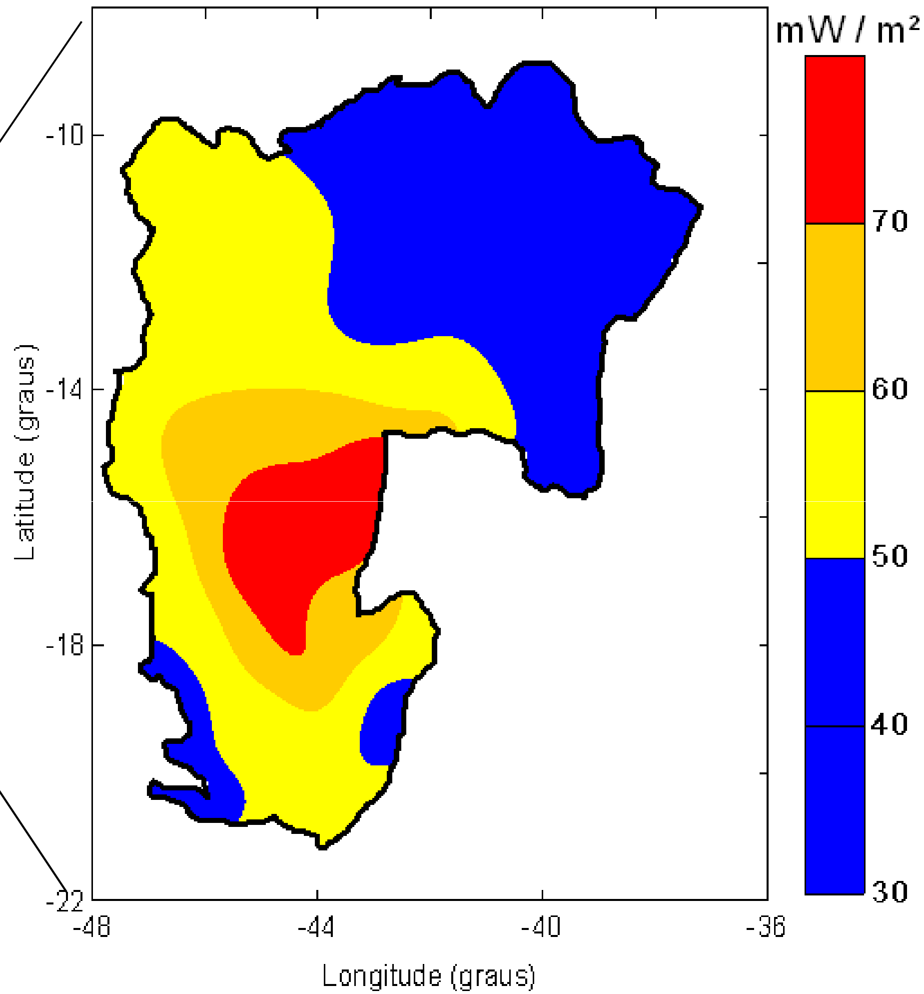
*** Estimate based on related geophysical (for example seismic) and geological data**

Results of Numerical simulation for q_0 = 120 mW/m²



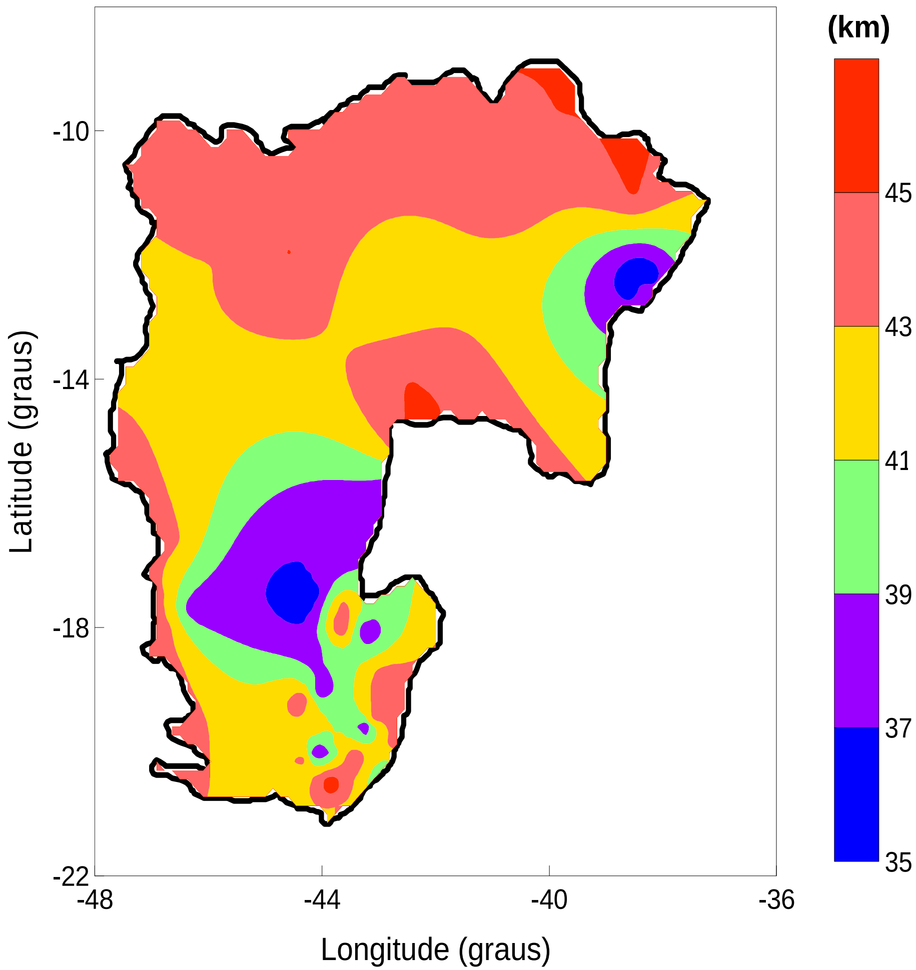
Application in area of tectonic stability: São Francisco structural province



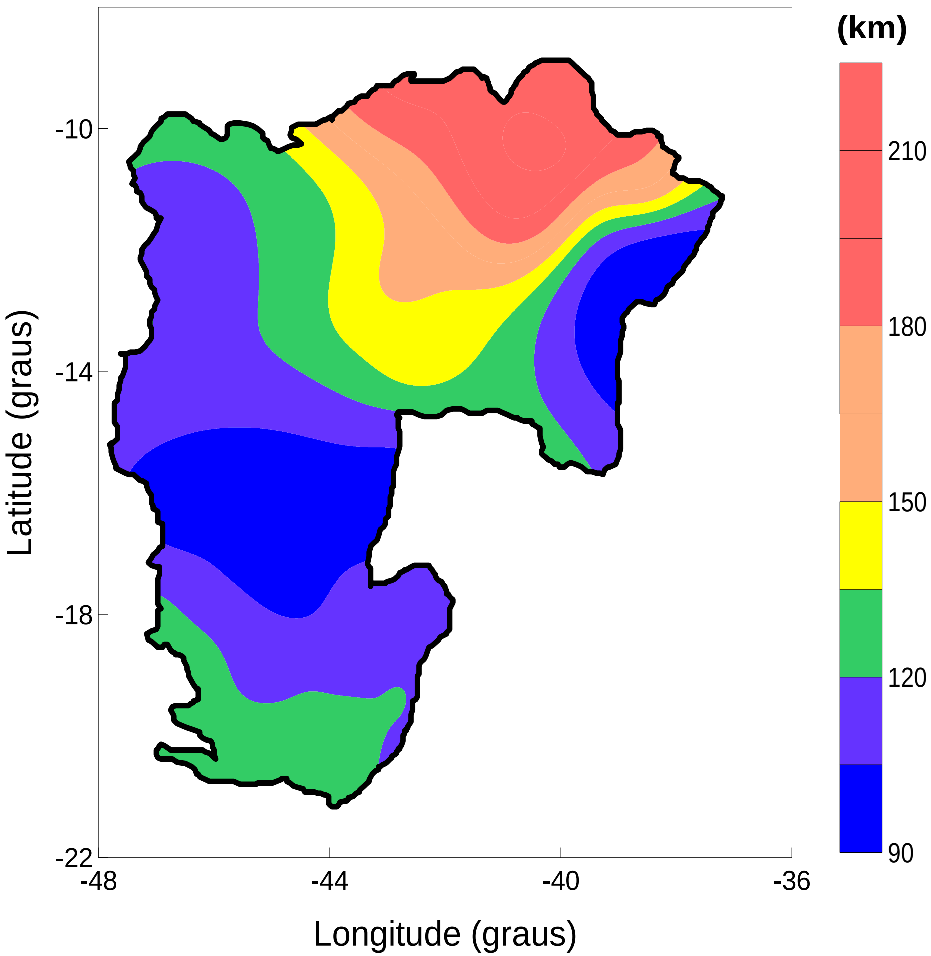


Heat Flow Map

Isostasy in São Francisco structural province

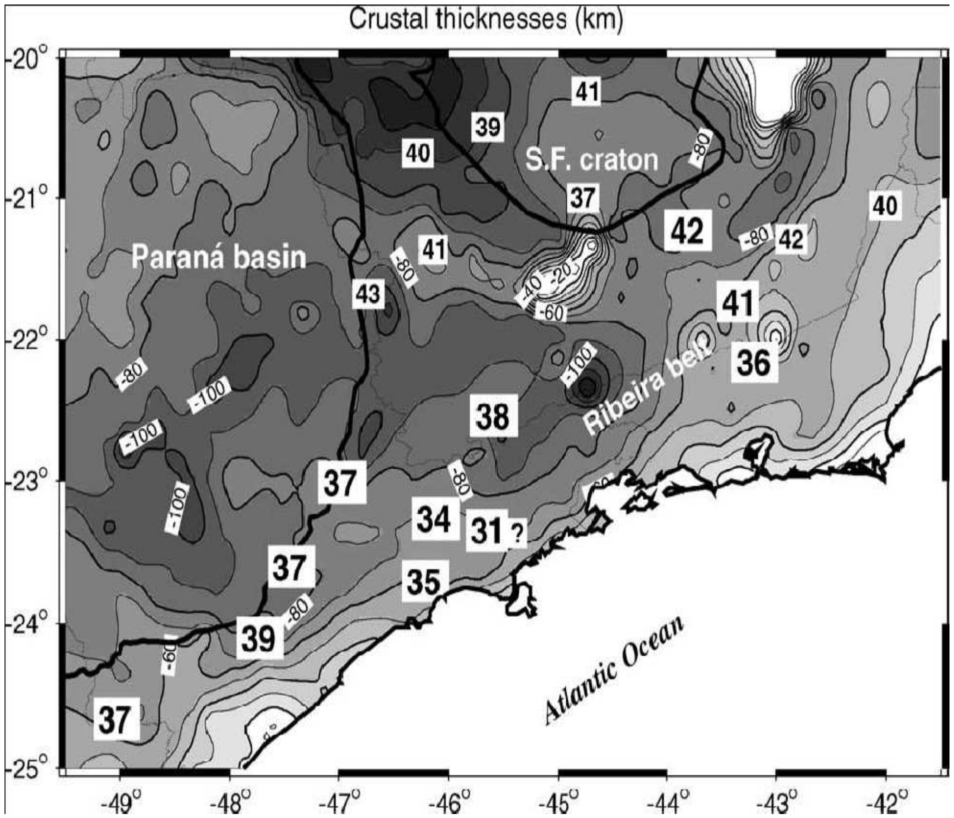


Moho Depths

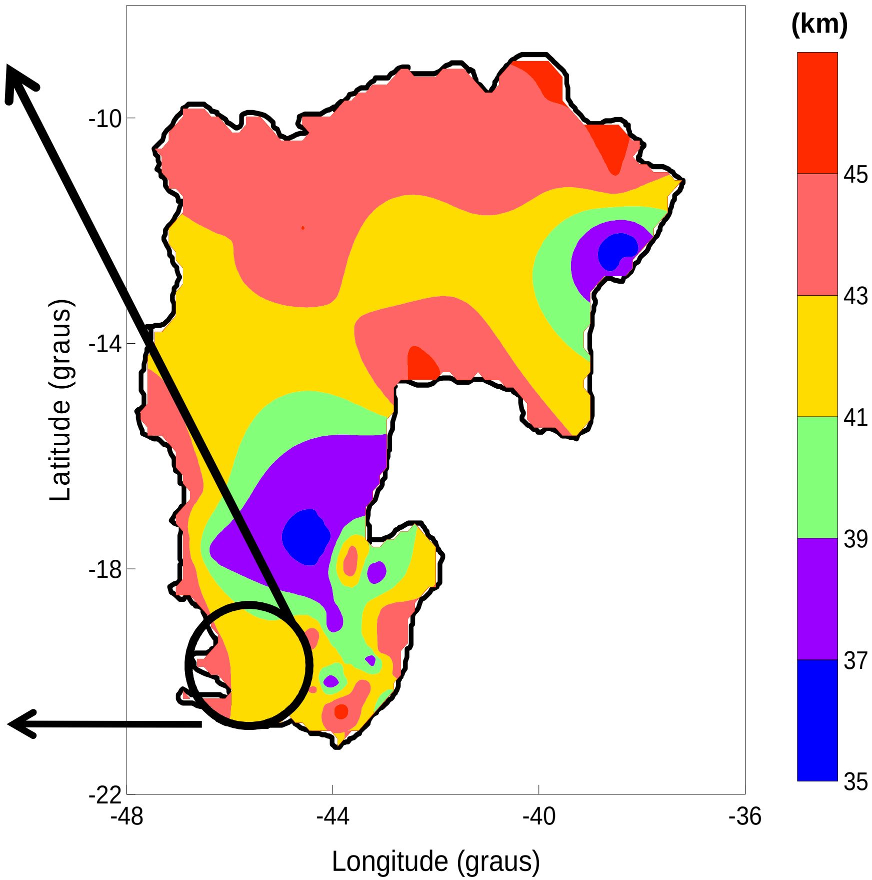


Lithospheric Thickness

Comparison with Seismic Tomography

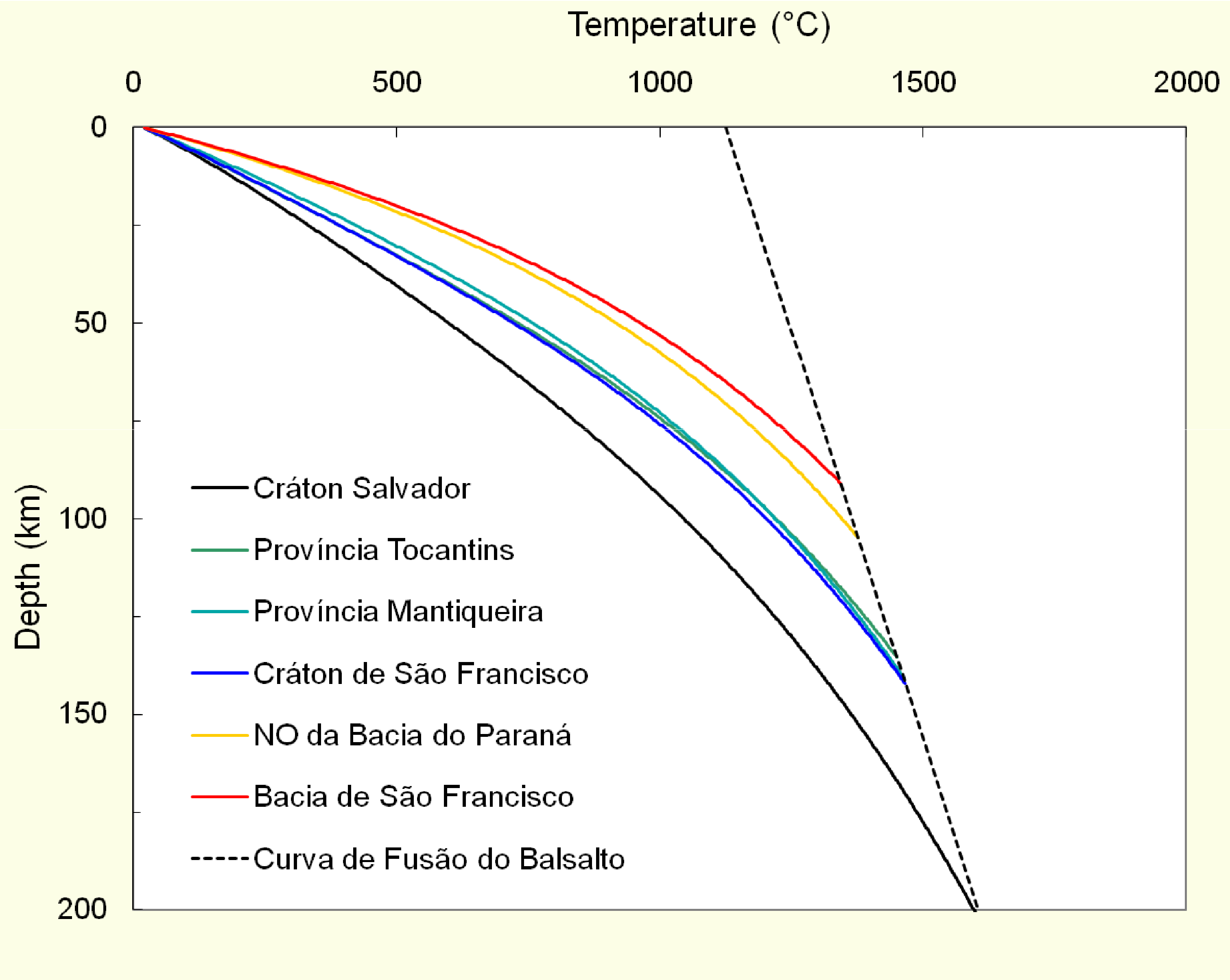


França and Assumpção (2004)



This Work.

Temperature distributions in the main tectonic units



Conclusions

Methods based on simultaneous inversion of heat flow, elevation and geoid height, provide better estimates of deep thermal field in enhanced geothermal systems.

In addition, it takes into consideration effects of vertical variations in thermal properties.

The method also provides reliable results in the presence of perturbations induced by hydrothermal circulation.

Thanks for your attention