# Non-equilibrium statistical mechanics of geophysical flows

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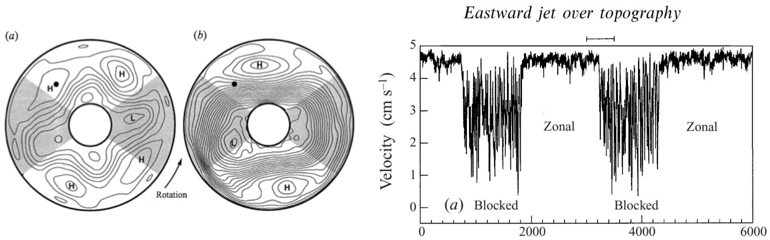
# Non-equilibrium phase transitions in real flows

## Rotating-tank experiments (quasi-geostrophic dynamics)

Transitions between blocked and zonal states:

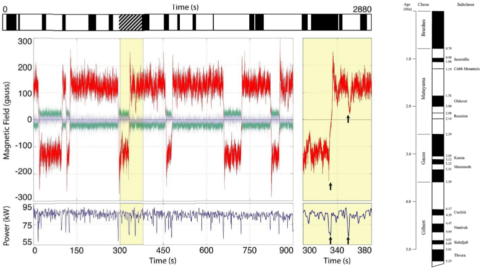



Y. Tian *et al.*, J. Fluid Mech. (2001) (groups of H. Swinney and M. Ghil)

Barotropic quasi-geostrophic dynamics

# Random transitions in other turbulence problems

## Magnetic field reversal (turbulent dynamo, MHD)




VKS experiment, Earth

Other examples:

- Turbulent convection, Von Kármán and Couette turbulence
- Random changes of paths for the Kuroshio current, weather regimes, . . .

# 2D Stochastic Navier–Stokes equations

## Limit of weak forcing and dissipation

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \omega = \nu \Delta \omega - \alpha \omega + \sqrt{2\alpha} f_s,$$

- Time scale separation: turnover time $= 1 \ll 1/\alpha =$ forcing or dissipation time
- At lowest order, we are left with the conservative dynamics. The flow self-organizes and converges towards steady solutions of the 2D Euler equations, i.e.,

  $$\mathbf{u} \cdot \boldsymbol{\nabla} \omega = 0 \quad \Leftrightarrow \quad \omega = f(\psi),$$

  where the stream-function $\psi$ is given by $\mathbf{u} = \mathbf{e}_z \times \boldsymbol{\nabla} \psi$.
- Degeneracy: how is $f$ selected?

# Equilibrium: 2D Euler equations

- Hamiltonian, time-reversible
- Nonlinear transport of the vorticity $\omega = (\boldsymbol{\nabla} \times \mathbf{u}) \cdot \mathbf{e}_z = \Delta\psi$
- Invariants:

$$\text{Energy:} \qquad \mathcal{E}[\omega] = \frac{1}{2}\int_{\mathcal{D}} \mathbf{u}^2 = -\frac{1}{2}\int_{\mathcal{D}} \psi\omega = E_0$$

$$\text{Vorticity distribution:} \qquad d(\sigma) = \frac{\mathrm{d}A}{\mathrm{d}\sigma} \quad \text{with} \quad A(\sigma) = \int_{\mathcal{D}} \chi_{\{\omega(\mathbf{r}) \leq \sigma\}}$$

# Robert–Sommeria—Miller (RSM) theory

## Equilibrium statistical mechanics (most probable vorticity field)

- Probabilistic description of the vorticity field $\omega(\mathbf{r})$: local probability $\rho(\sigma,\mathbf{r})$
- Measure of the phase space volume (here, number of microscopic fields $\omega$ corresponding to probability $\rho$):

$$\text{Boltzmann–Gibbs entropy:}\quad \mathcal{S}[\rho] = -\int_{\mathcal{D}} \mathrm{d}\mathbf{r} \int_{-\infty}^{+\infty} \mathrm{d}\sigma\, \rho(\sigma,\mathbf{r}) \log \rho(\sigma,\mathbf{r})$$

- Microcanonical RSM variational problem (MVP):

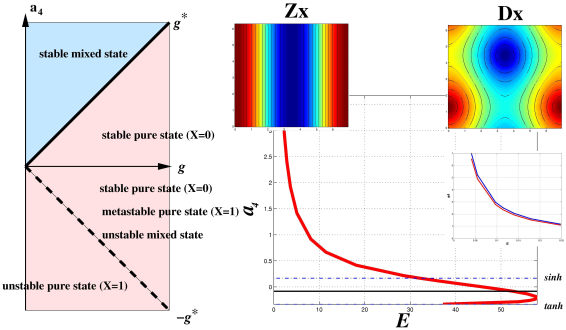
$$S(E_0,d) = \sup_{\{\rho \mid N[\rho]=1\}} \left\{ \mathcal{S}[\rho] \mid E\left[\overline{\omega}\right] = E_0, D\left[\rho\right] = d \right\}$$

- Critical points are stationary flows of the Euler equations:

$$\omega = f(\psi).$$

- Statistical equilibria can be proven to be invariant measures of the 2D Euler equations. F. Bouchet and M. Corvellec, J. Stat. Mech. (2010) P08021

# Equilibrium States for 2D Euler (doubly periodic BC)



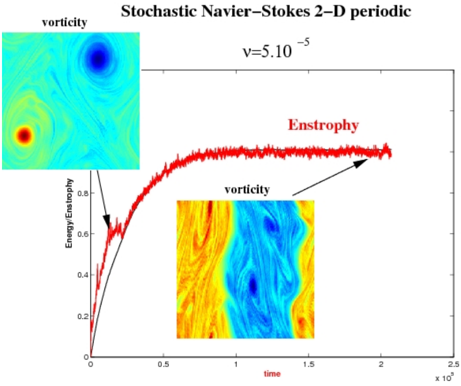

Normal form analysis (Liapunov–Schmidt reduction)

Parameters

- $g$: domain geometry
- $E$: energy
- $a_4$: nonlinearity of the vorticity–stream-function relation $(f(\psi))$

# Numerical simulation of 2D SNS

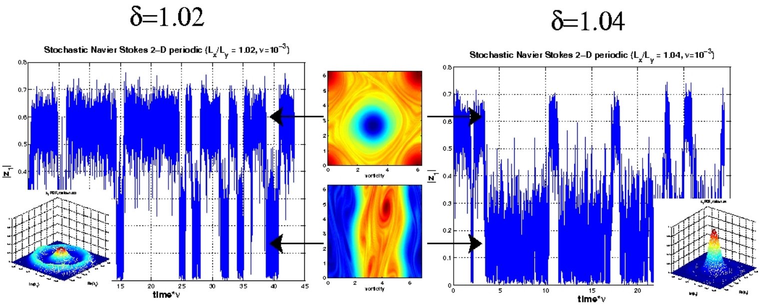
(Play movie)




Very long relaxation times ($\sim 10^5$ turnover times)

# Out-of-equilibrium phase transitions

## Time series and PDF of the order parameter




Order parameter: $z_1 = \int \mathrm{d}x\mathrm{d}y\;\; \exp(\mathrm{i}y)\omega(x,y)$

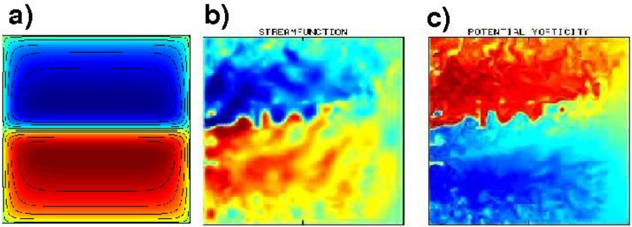
For unidirectional flows, $|z_1| \simeq 0$; for dipoles, $|z_1| \simeq 0.6$–$0.7$.

F. Bouchet and E. Simonnet, Phys. Rev. Lett. **102**, 094504 (2009)

# Strong eastward jets are statistical equilibria.

(Play movie)

## Statistical equilibria of the QG $1^{1/2}$-layer model in a closed basin:

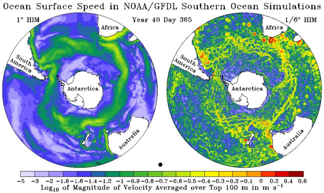


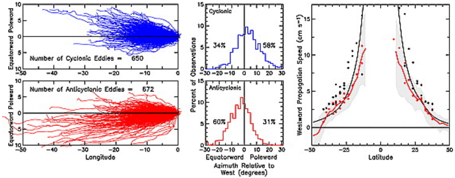

- States with positive PV to the north (eastward jet) and states with positive PV to the south (westward jet) are equivalent.
- Beta effect $h(y) = \tilde{\beta} y$ breaks the symmetry between westward and eastward jets.

A. Venaille and F. Bouchet, submitted to J. Phys. Oceanogr.

# Ocean rings (mesoscale ocean vortices)

## Gulf Stream rings, Agulhas rings, Meddies, etc.

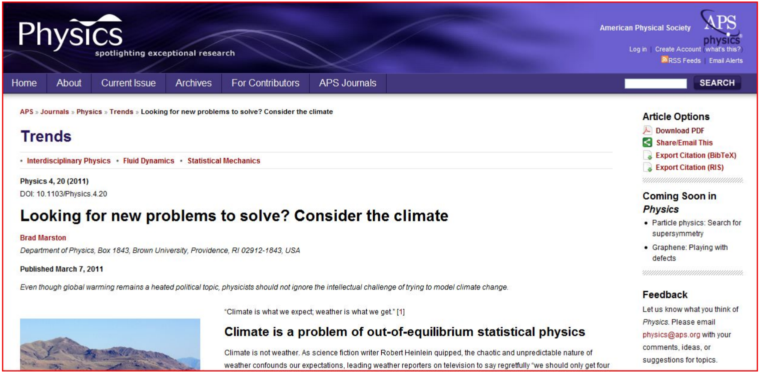



Hallberg and Gnanadesikan, JPO (2006) Chelton *et al.*, GRL (2007)

- A large part of mesoscale variability is explained by rings of size $100$–$200\ km$ (several Rossby deformation radii $R_d$).
- Both cyclonic and anticyclonic rings drift westward with velocity $\tilde{\beta} R_d^2$.

# *Physics* Trends



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## Trends

• Interdisciplinary Physics • Fluid Dynamics • Statistical Mechanics

Physics 4, 20 (2011)
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### Looking for new problems to solve? Consider the climate

Brad Marston
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Published March 7, 2011

*Even though global warming remains a heated political topic, physicists should not ignore the intellectual challenge of trying to model climate change.*

"Climate is what we expect; weather is what we get." [1]

#### Climate is a problem of out-of-equilibrium statistical physics

Climate is not weather. As science fiction writer Robert Heinlein quipped, the chaotic and unpredictable nature of weather confounds our expectations, leading weather reporters on television to say regretfully "we should only get four

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B. Marston, Physics **4**, 20 (2011)
http://physics.aps.org/articles/v4/20

# Summary

## Situations of bistability are common in geophysical turbulent flows.

Take-home messages:

1. Using statistical mechanics, phase transitions can be predicted for the 2D Navier–Stokes equations and Quasi-Geostrophic model.

2. We can observe the expected out-of-equilibrium phase transitions in the 2D SNS.

3. Ocean rings and jets are statistical equilibria.