When black swans ...

Nick Watkins (British Antarctic Survey)

Session NH9.1/EG8, room 1
Thursday, 07 Apr 2011, 08:30

.... come in bunches: modelling the impact of temporal correlations on the return periods of heavy tailed risk

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With:

- **Sam Rosenberg (now Barclays Capital)**
- **Sandra Chapman (Warwick)**
- **Mark Naylor (Edinburgh)**
- **Mervyn Freeman (BAS)**

Thank:

- KITP 2008 “Physics of Climate Change”
- AGU 2010 Chapman Conference, Hyderabad

Events ...

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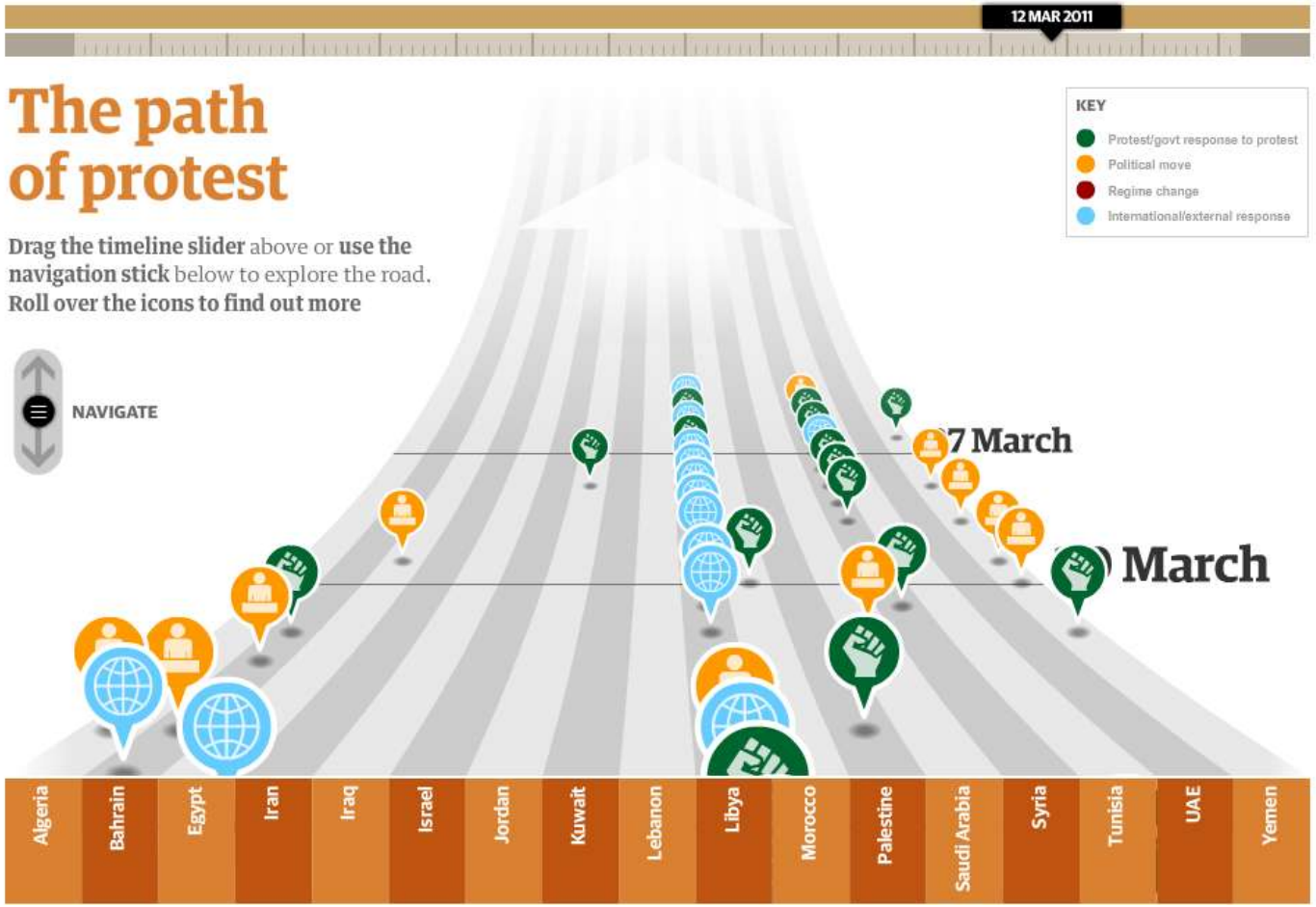
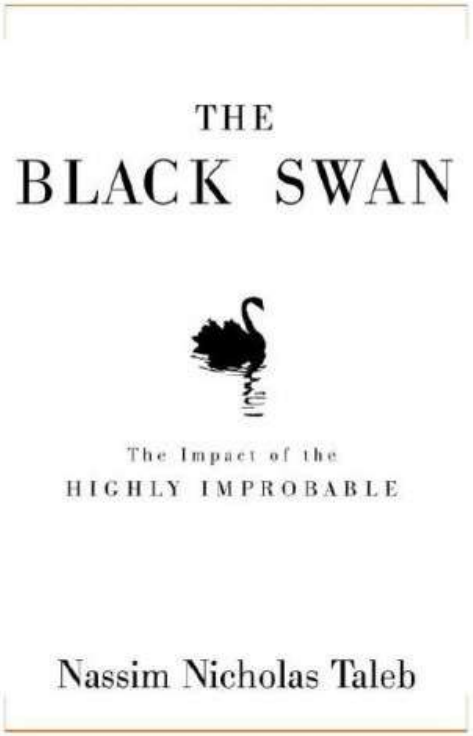
Reactor 4
Fires reported
Reactor 3
Blast, 15 March
Reactor 2
Blast, 14 March
Reactor 5
Reactor 6
Reactor 1
Blast, 12 March
Potential risk
Radiation leak feared

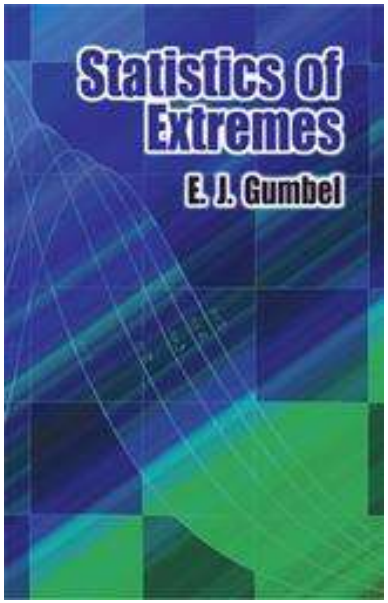
Interacting societal black swans …



Guardian.co.uk, 22nd March 2011

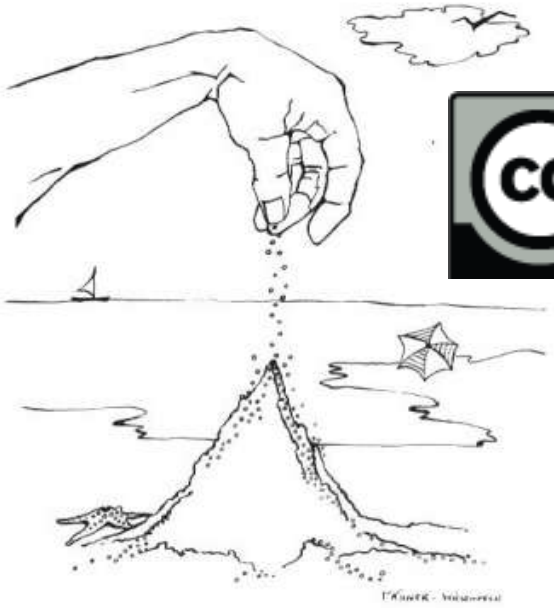
“Black swans”, extremes, bursts …

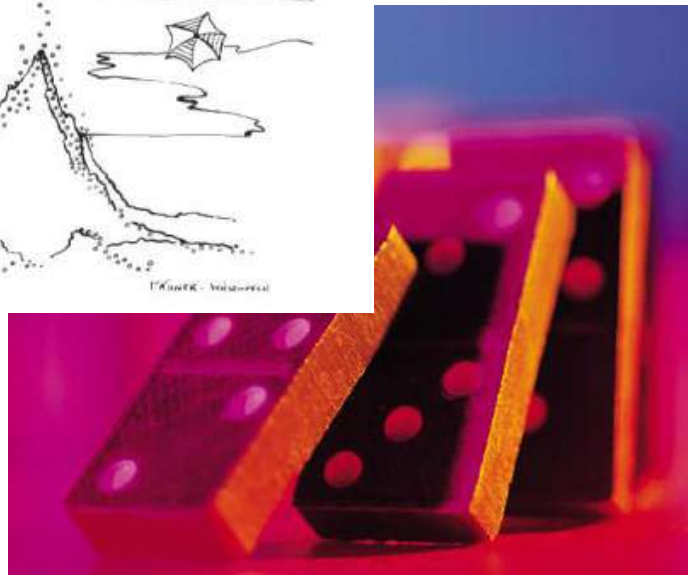




“ Is it a bird, an extreme event, or a cliche … ?”
– The Economist, March 2011

My personal motivation …





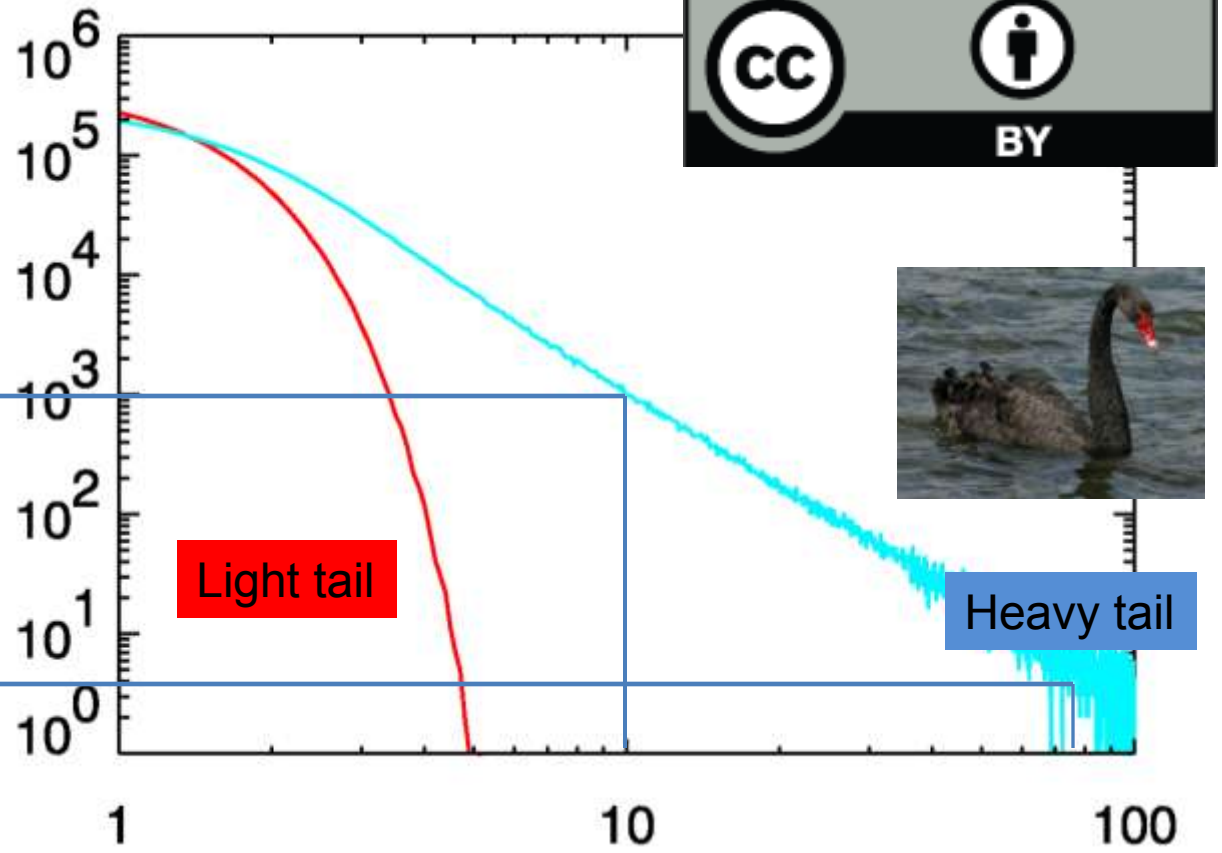
Common threads: intermittent energy release & toppling events, in complex systems.

Frequency-magnitude relationships

200 000 events of size “1”

1000 events of size “10”

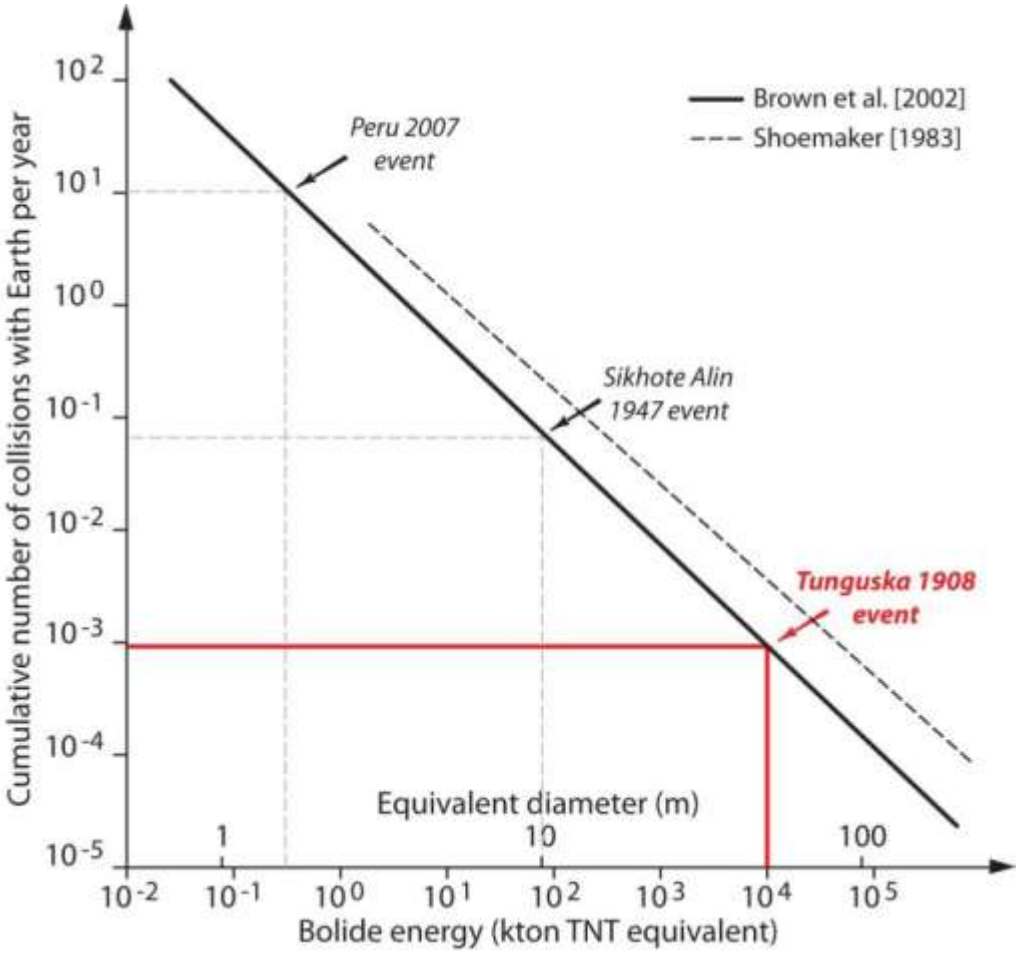
~5 events of size 100



Magnitude “1” ~ 200 times more frequent than magnitude “10” which in turn is about 200 times more frequent than magnitude 100. Histogram tail ~ power law.

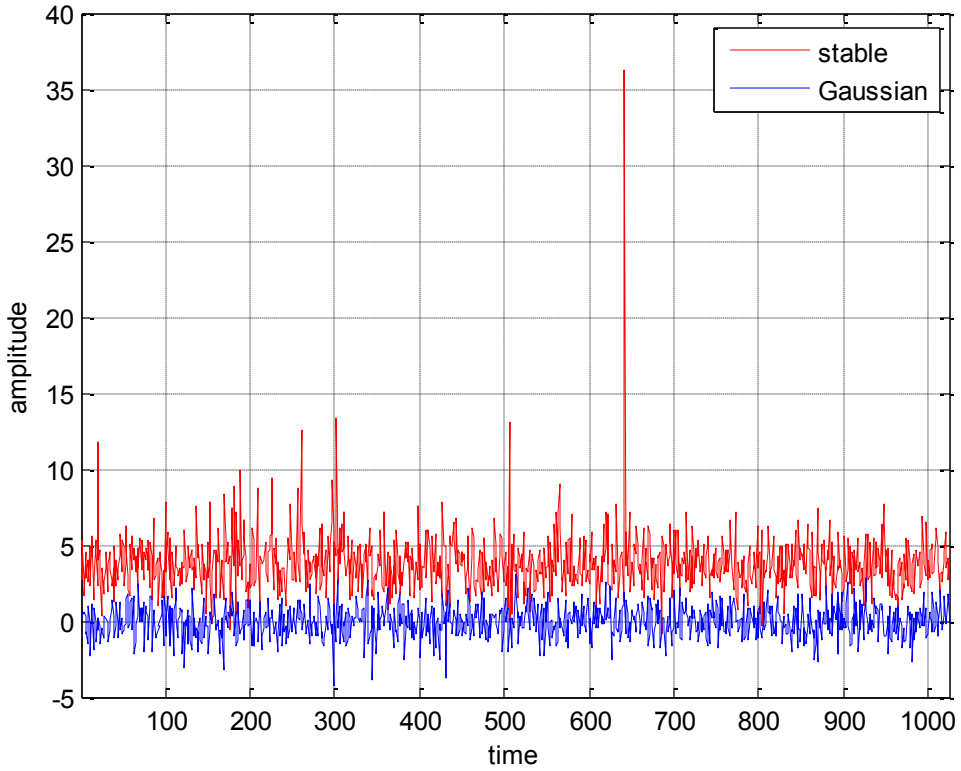
For a power law the PDF P(x) then has tail decaying as power –(1+ α) .

Example: Near Earth Objects



RMS, 2009

Heavy versus light tails



- Top trace shows skewed α-stable noise (α=1.8). Lower trace shows a Gaussian white noise (essentially no events outside +/- 3σ)

But does knowing how often you see events always indicate how long you'll wait for next one?

- **No ! Not necessarily –** after all, even white Gaussian or α -stable noises can show apparent clusters
- **But more profoundly**--magnitudes need not be independent … may be autocorrelated (e.g. AR(1))

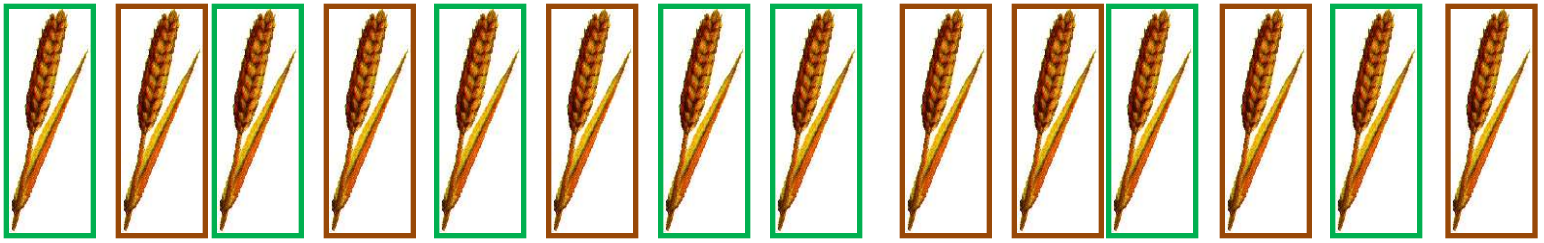
  or even long range dependent ("1/f") …

Black swans may be bunched

Mandelbrot:

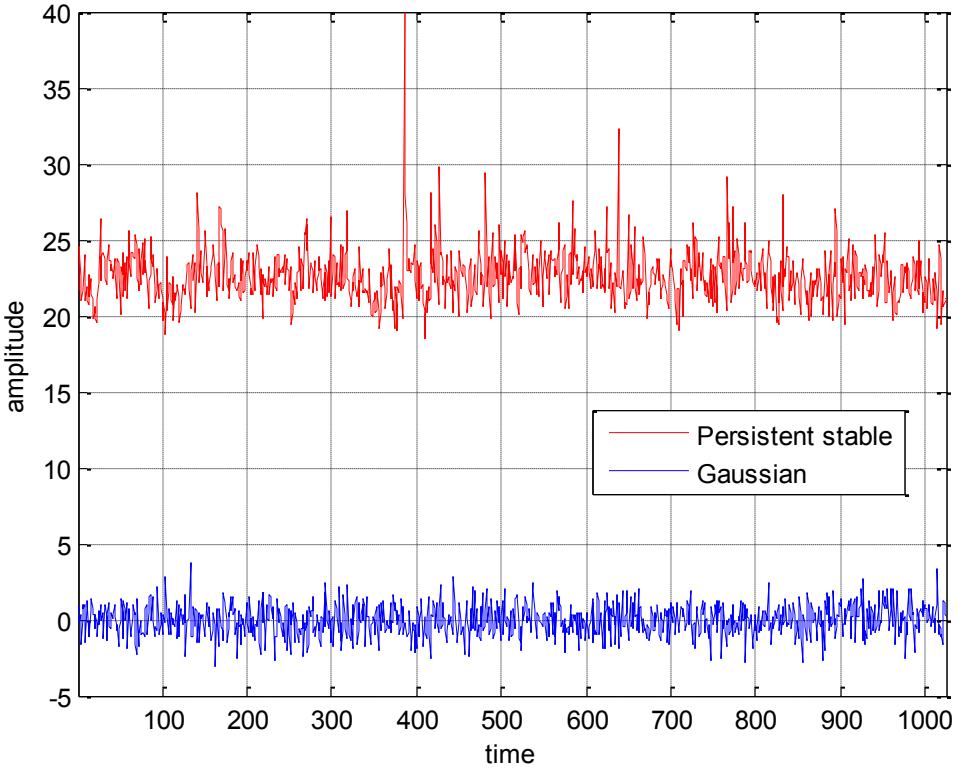
Long range dependence, or "the Joseph effect"

My "physicist's wheat"---illustrates Pharoah's dream of 7 years of plenty (green boxes) and 7 years of drought (brown boxes). Now shuffle series ...



Point is that frequency distribution (of this sample at least) unaffected by shuffling, but that the two series represent very different worlds.

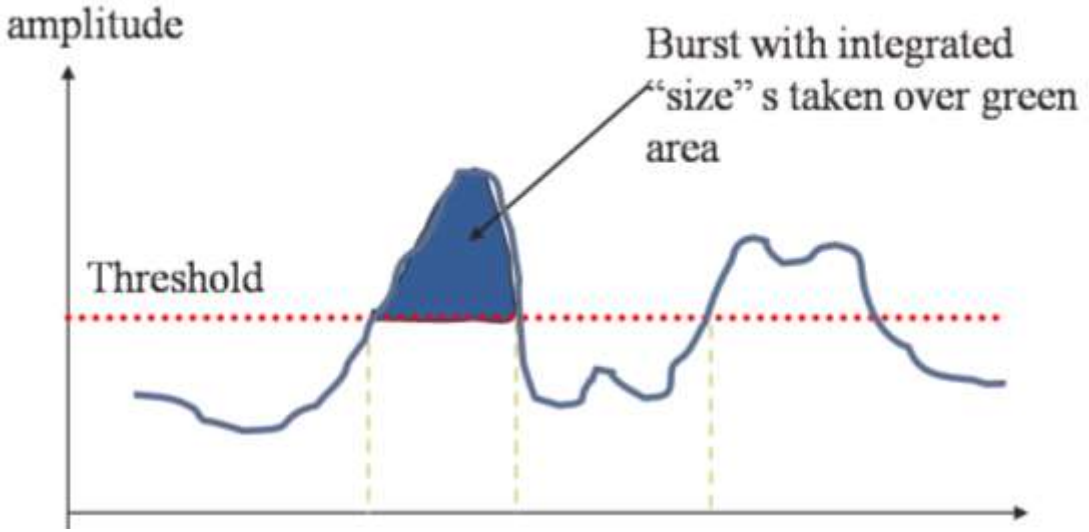
Heavy tails plus Joseph effect



- Top trace adds lrd to α-stable noise (d=0.2)
- Lower trace compares a Gaussian white noise

Bursts

Less obviously, strong correlation may integrate several "mediocre" events into a longer-lived "extreme" burst … activity burst concept from Bak et al's Self Organise Criticality naturally interpolates between this & individual spikes.



Linear Fractional Stable Motion, a random walk model of how heavy tails & Joseph effect conspire to produce bursts

$$X_{H,\alpha}(t) = C_{H,\alpha}^{-1} \int_R \left((t-s)_+^{H-\frac{1}{\alpha}} - (-s)_+^{H-\frac{1}{\alpha}} \right) dL_\alpha(s)$$

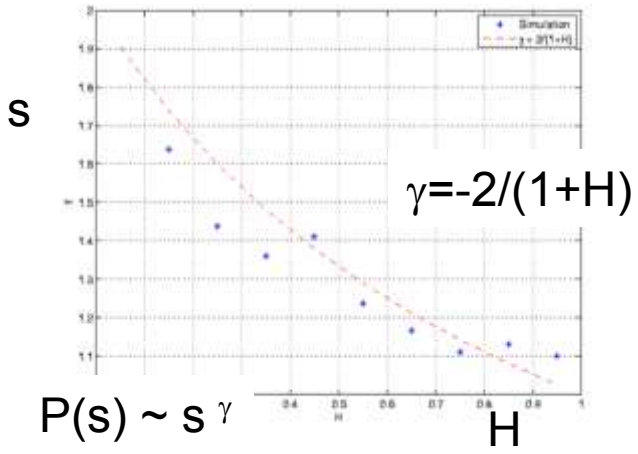
Memory kernel: Joseph effect

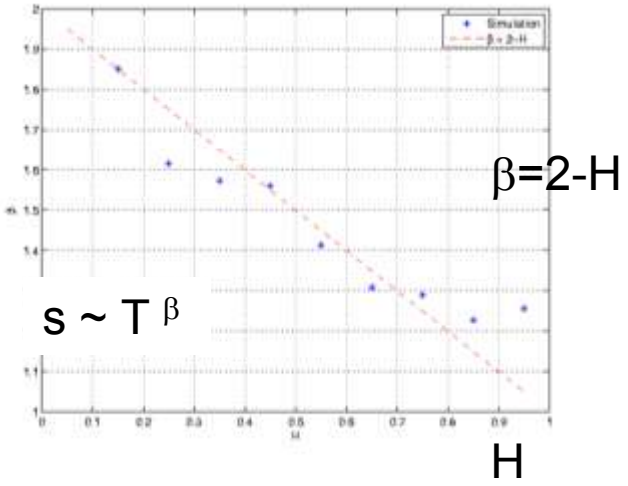
α-stable jump: heavy tails

- Mandelbrot's fractional Brownian motion but integrates α-stable rather than Gaussian noise (e.g. **Samarodnitsky & Taqqu book**).

Simulations of light-tailed bursts

- Tail pdf of burst size s, and dependence of s on duration T predicted to have exponents γ=-2/(1+H) & β=2-H & respectively **Watkins *et al*, PRE, 2009** .



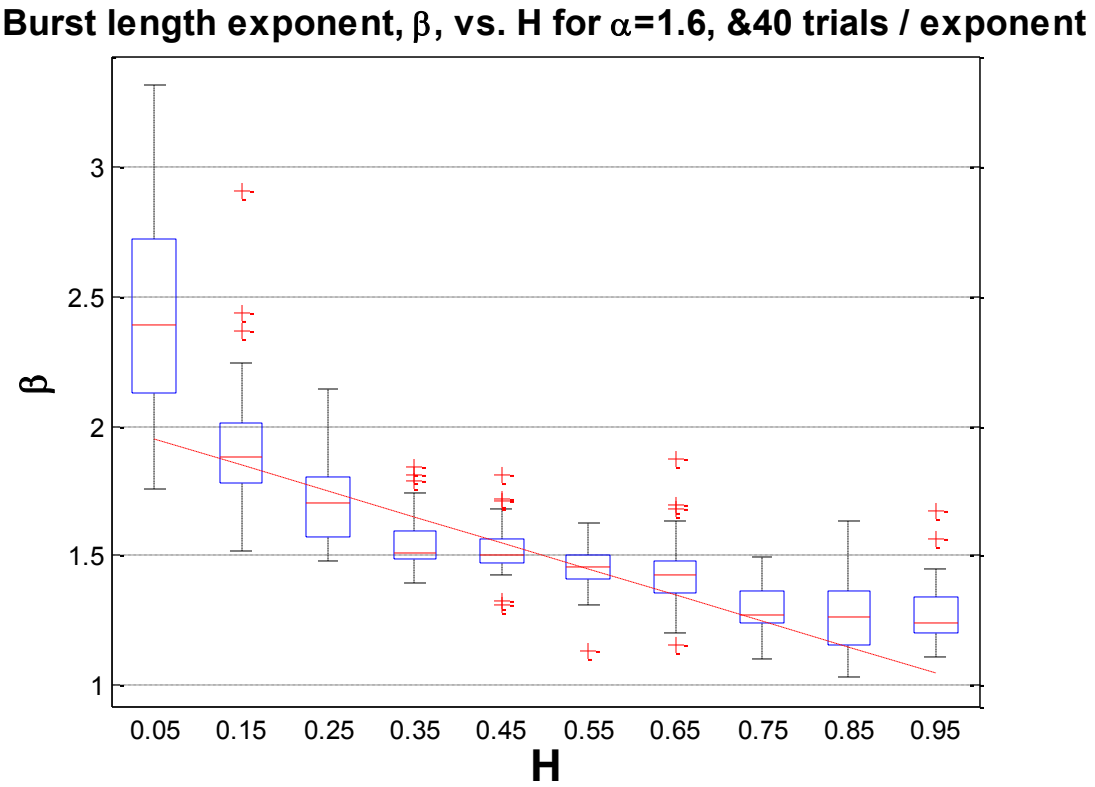


- **Good agreement in Gaussian (fBm) limit**: Confirmed findings of **Carbone *et al* [PRE, 2004] & Rypdal and Rypdal [PRE, 2008].**

Simulations of heavy-tailed bursts

- **Watkins et al, PRE, 2009** found expressions also reasonable down to $\alpha \sim 1.6$, but to fail completely by $\alpha = 1$:

Plot taken from work in progress on detailed uncertainty analysis, larger ensemble etc.



Conclusions I

- Assessing hazard from Black Swans requires consideration not only of their relative frequency but also their bunching.
- Motivated by data and by models like SOC (which sought to unify these effects), we have begun to look theoretically and numerically at bursts in a rich toy model, LFSM **[Watkins et al, PRE, 2009].**
- Initial results promising-but need to flesh out.

Conclusions II

- Now need to: Explore stationary noises like α-stable FARIMA as well as self-similar walks like LFSM
- Link our burst results to Extreme Value Theory and theory of records
- Examine our assumptions about stationarity and about data coming from single distribution (c.f. **Sornette's** "Dragon Kings")
- Build models of coupling between variables as well as model the bunching within one time series ---SuperCats ?

In Memoriam:

- **Benoit Mandelbrot (1924-2010)**
- **Per Bak (1948-2002)**

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