Development of a New Wake Model using Large-Eddy Simulations and Wind Tunnel Data

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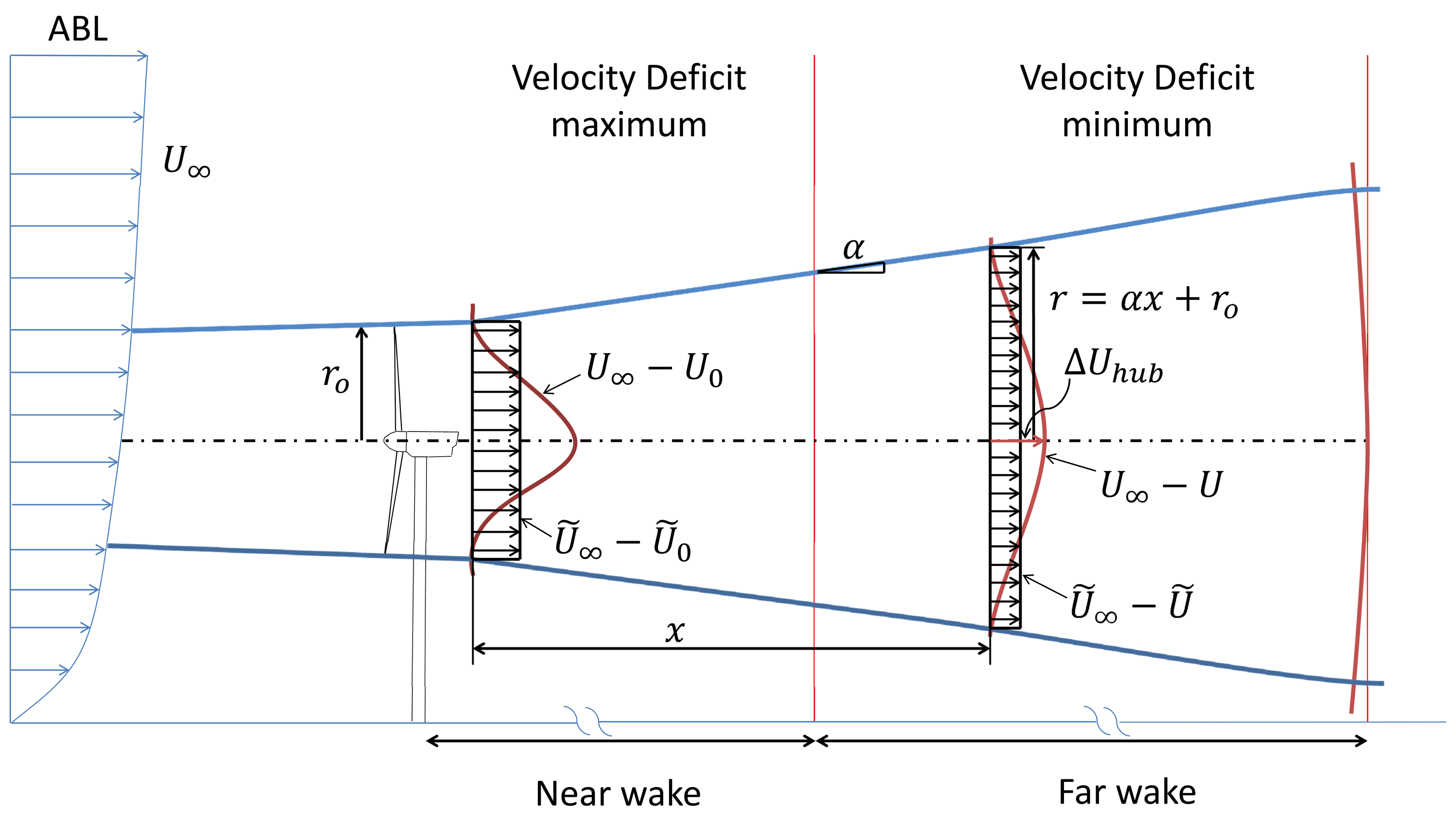
INTRODUCTION

- Aerodynamic research for wind turbines has contributed significantly to the success of modern wind energy.
- Wake models are categorised as Empirical/Analytical models and Computational Fluid Dynamic (CFD) based models.
- Models describing wind turbine wakes were developed mainly in the 1980s, such as the N.O. Jensen model which is a simple analytical model.
- Wind resource software like WAsP utilizes such models to estimate the power loss in wind farms due to the wind speed reduction in wakes from upstream wind turbines. This software is widely used by many wind turbine manufacturers to setup large wind farms.
- The analytical models have advantages from the viewpoint of designing a wind farm due to its simplicity and computational speed in comparison to Computational Fluid Dynamic (CFD) based models.

AIM OF THIS STUDY

- A simple wake model is proposed here which is based on conservation of momentum similar to the Jensen model.
- The new model assumes a Gaussian axi-symmetric shape of the velocity deficit in the wake which is clearly shown by several studies and field measurements.
- To create a new wake model which will be cost effective to be used in wind farm modelling software such as WAsP and provide more accurate predictions of the velocity deficit behind a single wind turbine generator.



Approximations
- Axi-symmetric
- Self-similar
- Gaussian

Nomenclature

U_∞ : *ambient wind velocity*
U_0 : *velocity behind rotor*
U : *velocity of wake at distance x*
r_0 : *rotor radius*
r : *wake radius at distance x*
α : *growth rate of wake exapansion*
ΔU_{hub} : *centerline velocity deficit*

Figure 1. Schematic diagram of the wake model with definition of symbols

METHODOLOGY

Using Conservation of momentum gives:

$$\pi {r_0}^2(\tilde{U}_\infty - \tilde{U}_0) = \pi r^2(\tilde{U}_\infty - \tilde{U})$$

where the tilde denotes velocity averaged over the wake cross-section. Assuming Linear expansion of the wake, $r = \alpha x + r_0$

The velocity right behind the rotor can be written as $\tilde{U}_0 = \tilde{U}_\infty(1-2a)$, where $\boldsymbol{a}$ is the induction factor, which is a function of the thrust coeff. (C_T).

Solving for the wake velocity gives Jensen's model (Jensen, 1983):

$$\tilde{U}_\infty - \tilde{U} = \Delta\tilde{U} = 2a\frac{{r_0}^2}{(r_0 + \alpha x)^2}$$

Instead of assuming a constant velocity distribution in the wake, here we **assume that the velocity deficit is axi-symmetric and follows a 2D Gaussian distribution** centered at the hub height, and with amplitude ΔU_{hub} and standard deviation $S_x = S_y = \boldsymbol{S}$

The parameters of the Gaussian distribution (ΔU_{hub} and S) depend on downwind distance (x) and are obtained assuming:

- Linear expansion of the wake ($r = \alpha x + r_0$)
- Self-similar Gaussian wake: r/S is constant with downwind distance
- Conservation of momentum: The integral of the 2D Gaussian velocity deficit should match the momentum deficit at the rotor obtained using 1-D momentum theory:

$$\pi {r_0}^2(\tilde{U}_\infty - \tilde{U}_0) = Vol(2D\ Gaussian)$$

Note that this model requires the specification of two parameters: the ratio ***r/S*** and the growth rate $\boldsymbol{\alpha}$. Using the curve fits for LES data, we obtain an average value of *r/S*=2.75, with little variation with downwind distance. Based on previous studies (Jensen, 1983), we take α=0.1.

(See Figure 2 for the velocity deficit prediction of the new model in comparison to the Jensen & Risoe models)

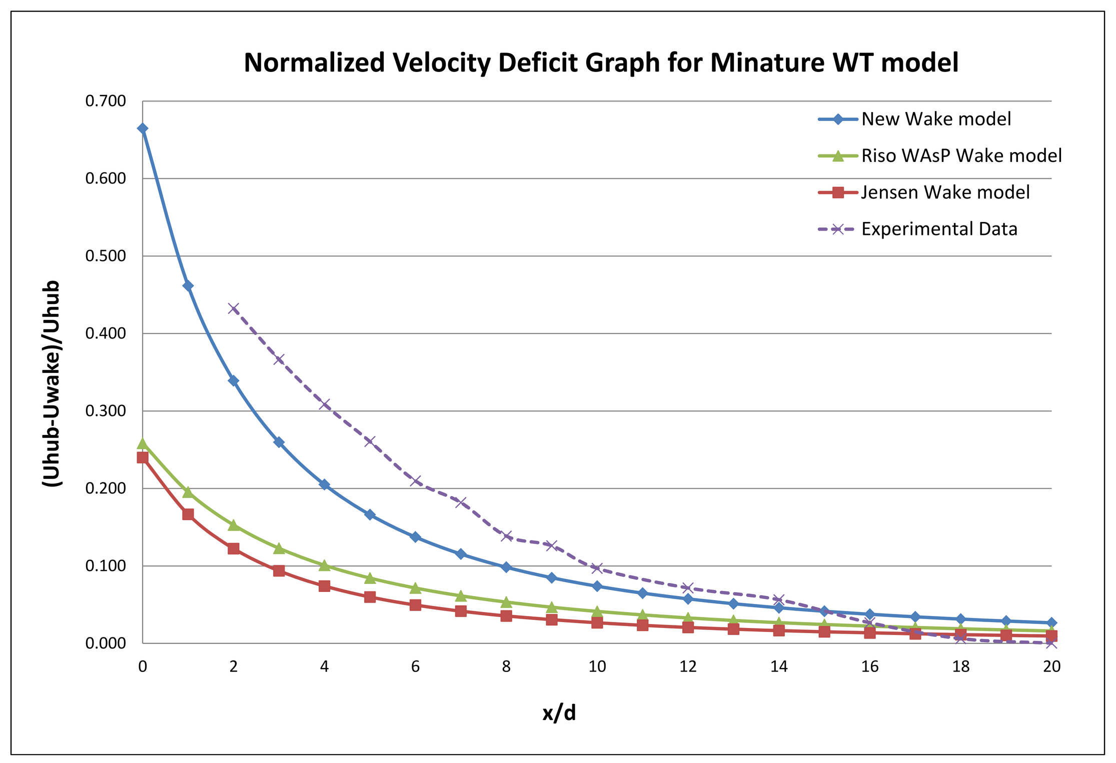
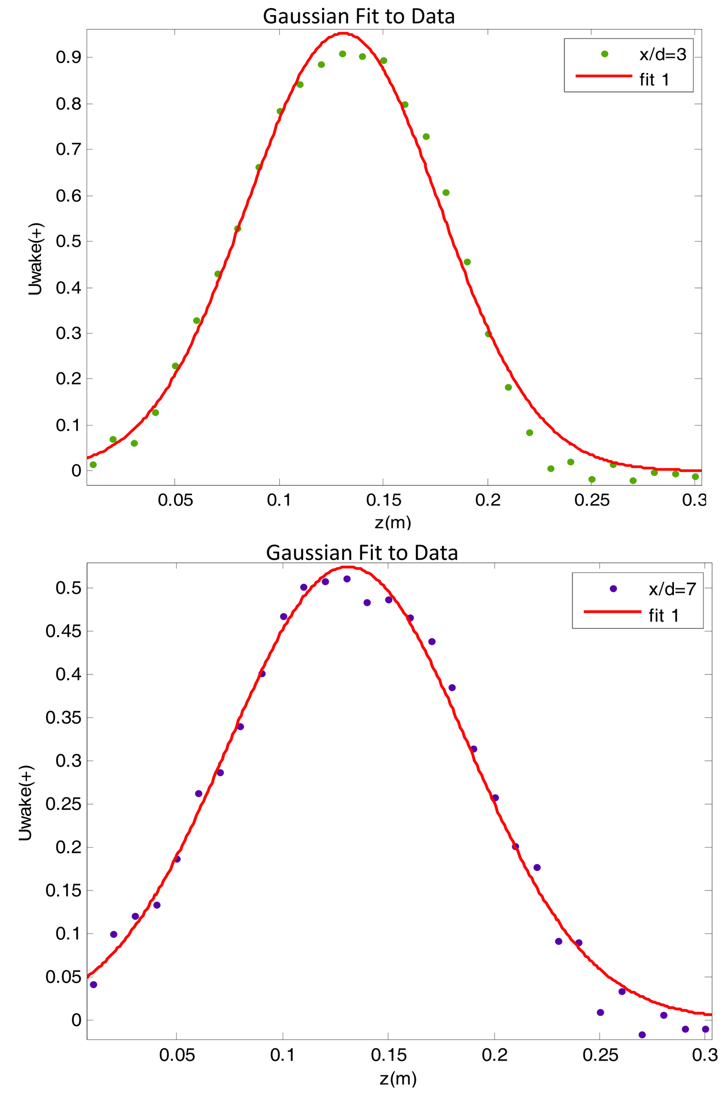
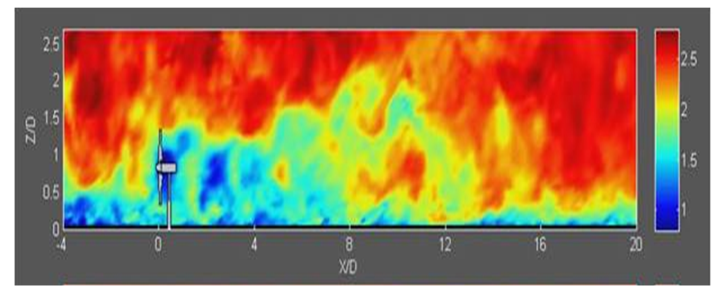


Figure 2. Normalized velocity deficit graph of the new wake model in comparison with the Jensen and Risoe/Park analytical models

WIND TUNNEL DATA





LES DATA

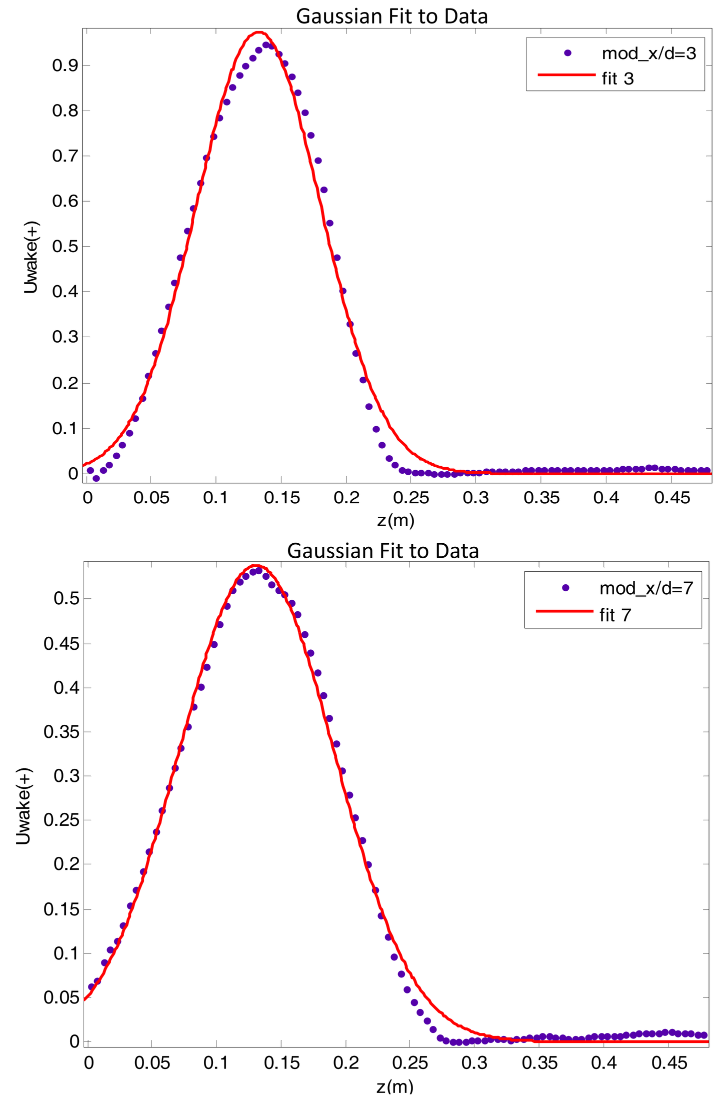


Figure 3. Gaussian curve fits of Wind tunnel Data (Chamorro L and Porté-Agel, 2010) and Large-Eddy Simulation(LES) Data (Wu Yu-Ting and Porté-Agel, 2011) with a 95% confidence Interval. Wake data of x/d = 3 & 7 distances are shown here.

CONCLUSION

- The new wake model shows better agreement with experimental data in comparison to the Jensen and Risoe wake models.
- Further comparisons of the new model with field measurements and Large-Eddy Simulations (LES) will be carried out in the future.
- Emphasis will be placed on understanding the dependence of the wake growth rate on effects such as turbulence intensity, thrust coefficient of the rotor and and mean shear of the ambient flow.

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