MULTIFRACTAL PARAMETERS AND EXTREME BEHAVIOUR OF HIGH RESOLUTION RAINFALL TIME SERIES

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INTRODUCTION

Reliability of precipitation statistics drastically depends on the quality of available data. The measuring and recording techniques may introduce statistical biases that have been somewhat discussed in the literature. We show that most of the tipping-bucket time series have a lower recording frequency than that is assumed (Fig. 1). This often leads to spurious scaling breaks that in turn results in spurious rainfall estimates with the consequences for operational hydrology.

It is therefore essential to quantify the quality of the rainfall time series before their use. We propose the SERQUAL procedure [1] extracting sub-series that satisfy the required quality criteria and hence could be used to calibrate or validate hydrological models.

We use high quality data selected by the SERQUAL procedure to investigate the multifractal properties of high-resolution rainfall time series in France. Within the framework of Universal Multifractals (UM) having two relevant parameters (α and *C*1), we discuss the uncertainties of parameter estimates and how to reduce them. We focus on the question of multifractal phase transitions being associated either with a sample size limitation or with fat tailed probability distributions, i.e. extremes of rainfall. We discuss how to improve the estimates of the corresponding exponents and how to use them assesing hydrological impacts of climate change in the Ile-de-France region.

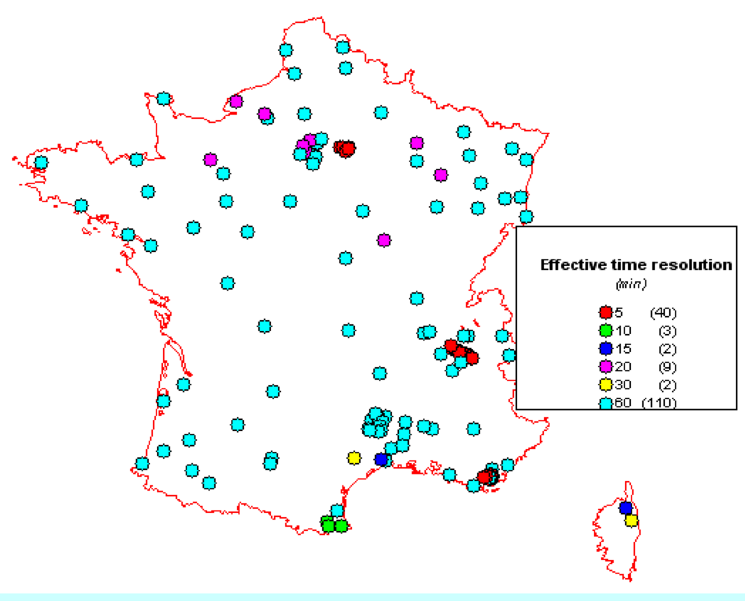

Fig. 1 Geographical maps of the effective resolution of rainfall time series (in minutes), the corresponding number of stations is displayed beside (between parenthesis).

MULTIFRACTAL PARAMETERS

❖ Estimation of the multifractal parameters with the help of the Trace Moment (TM) analysis [2]:

$$\left\langle R_\lambda^q \right\rangle \approx \lambda^{K(q)} \qquad K(q) = \begin{cases} C1(q^\alpha - q)/(\alpha-1) & \alpha \neq 1 \\ C1 q \ln q & \alpha = 1 \end{cases} \quad (1)$$

Rain rates R_λ at various resolution $\lambda<\Lambda$ are obtained by successive aggregations of the original time series R_Λ. In the framework of UM [2], the scaling moment function K(q) of a conservative process is given by Eq. 1, where:

- Index α measures the multifractality of the process;
- Co-dimension C1 of the mean field measures its mean fractality

Larger α and C1 => stronger extremes

α and C1 estimates are based on the estimate of K(q)

+ TM1 Method [3]:

$$\hat{K}(q) = \begin{cases} (q^\alpha - q)/(\alpha-1) & \alpha \neq 1 \\ q \ln q & \alpha = 1 \end{cases}$$

If the multifractal model is valid: $C1(q) = K(q)/\hat{K}(q) = const = C1$

Solution: to find α for C1(q)=const, then C1=<C1(q)>

+ TM2 Method: to estimate first two derivatives of K(q)

C1=K'(1) , α = K''(1)/C1

❖ Estimation of the multifractal parameters using the Double Trace Moment (DTM) method [4]:

$$R_\Lambda \Rightarrow R_\Lambda^\eta \Rightarrow \langle (R_\Lambda^\eta)_\lambda^q \rangle \approx \lambda^{K(q,\eta)} \qquad K(q,\eta) = \eta^\alpha K(q) \quad (2)$$

α and C1 are estimated on the plot of log(K(q,η)) vs log(η)

The bounds [η_{min} η_{max}] (see Eq. 2) can be theoretically estimated:

$\eta_{min}=(c_o/C1)^{1/\alpha}$ max (1.1/q) ; $\eta_{max}=(d/C1)^{1/\alpha}$ min (1.1/q)

We first define an interval [η_{min} η_{max}] using an approximation of the slope (α):

$$K(q,\bar{\eta}) = (K(q,\eta)_{min} K(q,\eta)_{max})^{1/2}$$

then two methods are used to estimate α and C1:

+ DTM-IP Method: (α, C1) are estimated with the help of an inflection point located inside of the interval [η_{min} η_{max}]

+ DTM-RR Method: DTM-IP is an iterative procedure that is first used for a second approximation of (α, C1) to estimate [η_{min} η_{max}], then new parameters (α, C1) are estimated with the help of a linear regression over [η_{min} η_{max}].

Table 1. Nash estimates for all 4 methods tested on UM simulations

Method	TM1		TM2		DTM-IP		DTM-RR	
Parameter	α	C1	α	C1	α	C1	α	C1
Nash	0.53	0.89	0.85	0.87	0.97	0.86	0.95	0.86

UM parameter estimates on the rainfall time series

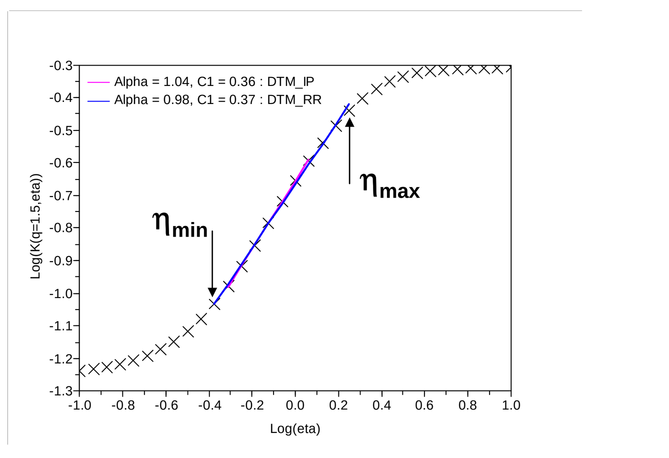

Fig. 2 Illustration of α-parameter estimation by the DTM-IP and DTM-RR methods

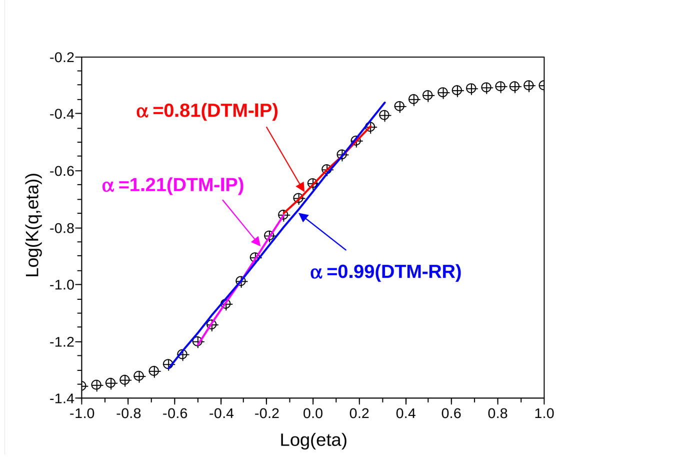

Fig. 3 Uncertainty in α-parameter estimation by the DTM-IP method, with an error δα = 0.4

The CG-94 database — The MF-P5 database

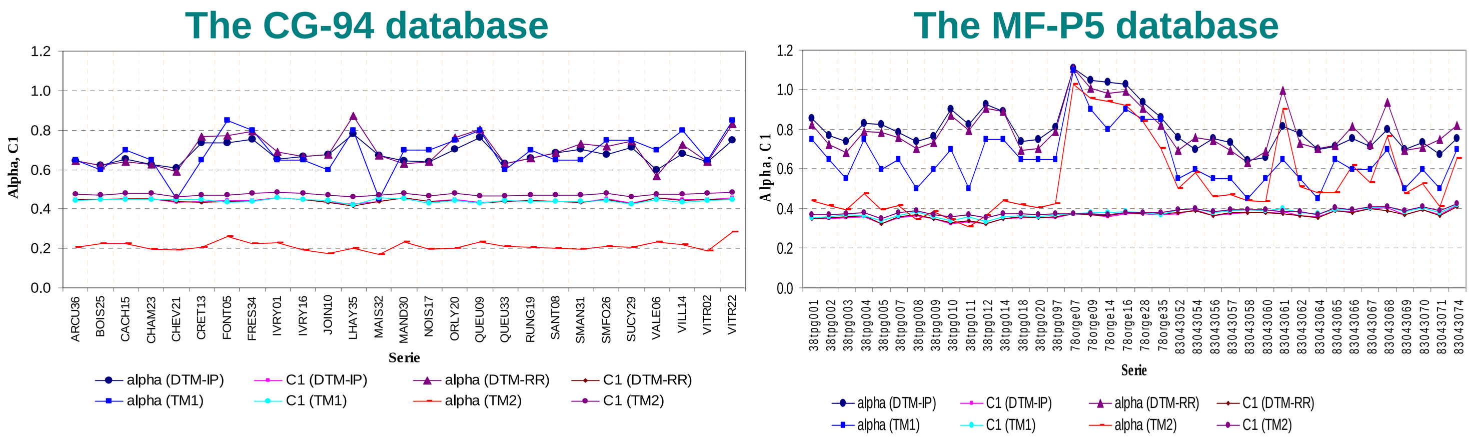

Fig. 4 Results of α and C1 for time scales ranging from 5 min to 28 days, estimated by different methods: TM1, TM2, DTM-IP and DTM-RR.

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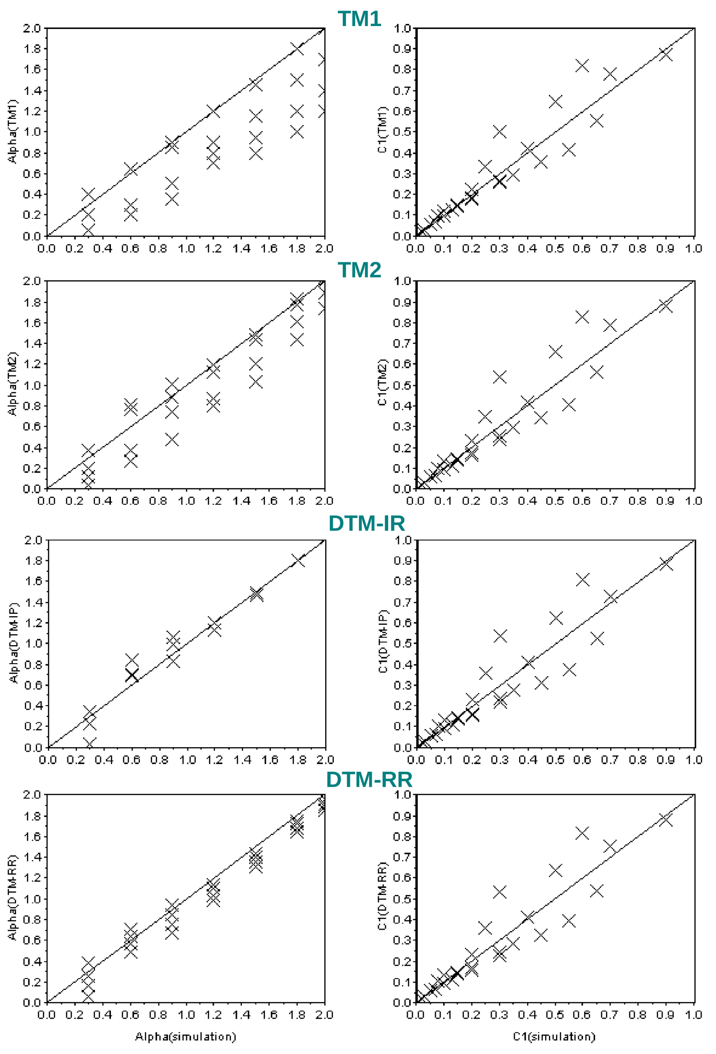

Fig. 5 Correlation between the theoretical parameters α and C1 and those estimated by different methods on simulated time series

EXTREME RAINFALL BEHAVIOUR

❖ The multifractal phase transitions (q_{crit}), corresponding to the discontinuities in the (first or second) derivative of K(q) [5,6]

- The first order phase transition, displayed by a divergence of moments order q_D : $\langle R_\lambda^q \rangle \to \infty$ for $q > q_D$, the divergence corresponds to a hyperbolic tail for the probability distribution: $P_r(R_\lambda > x) \approx x^{-q_D}$ for $x >> 1$
- The second order phase transition (q_s), the value of q_s can be estimated for UM as: $q_s = (\Delta_s/C1)^{1/\alpha}$.

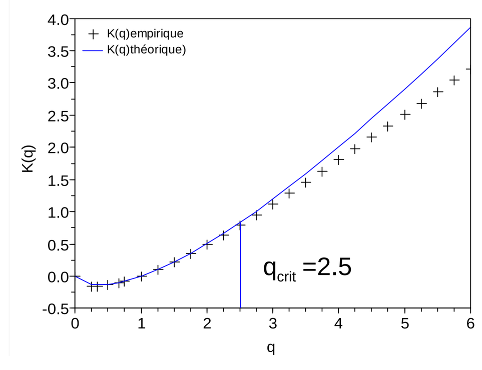

Fig. 6 A departure of the theoretical K(q) function (continuous line, the DTM-RR method) from the empirical one (crosses) allows to estimate q_{crit}

The CG-94 database — The MF-P5 database

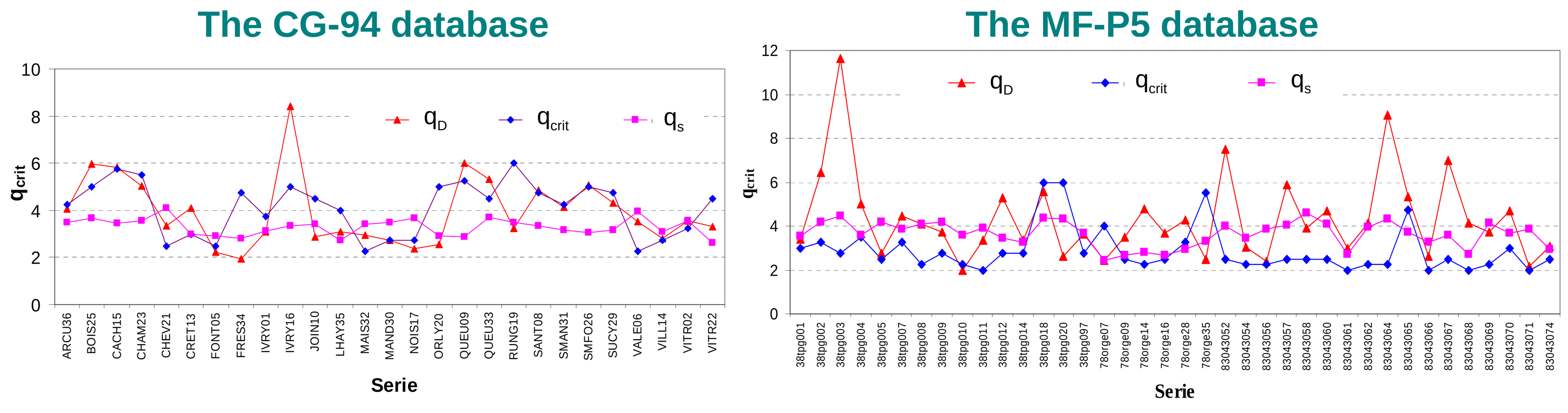

Fig. 8 Estimates of q_D, q_{crit} and q_s on the MF-P5 and CG-94 databases, the fluctuations illustrate difficulties to distinguish between multifractal phase transitions for $q_{crit} > q_s$ or for $q_{crit} < q_s$ and $q_D >< q_{crit}$.

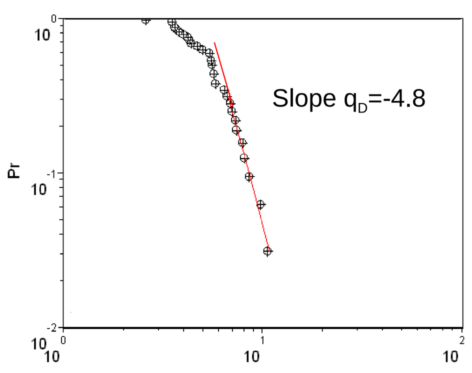

Fig. 7 The probability distribution of annual maxima rainfall, the value of q_D estimated by the slope of a hyperbolic tail.

❖ Detection of hydrological impacts of climate change

The evolution of the maximal observable singularity $\gamma_s=C1(C1^{(1-\alpha)/\alpha}-1/\alpha)\alpha/(\alpha-1)$ allows to evaluate a tendency of rainfall evolution.

The serie at Orly — The serie at Orgeval

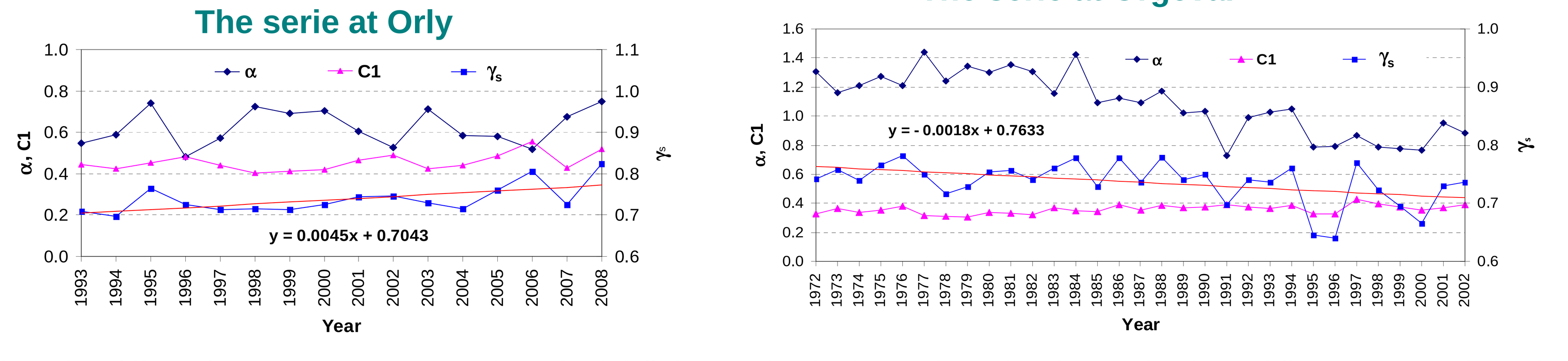

Fig. 9 Time evolution of multifractal parameters α, C1 and γ_s of rainfall time series in the Paris region (France). The red line corresponds to the linear regression of the γ_s estimates, a positive slope indicates an increasing tendency (e.g., series at Orly) and a negative one indicates a decreasing tendency (e.g., series at Orgeval) of the maximal rainfall intensity.

DISCUSSION OF THE RESULTS

- The TM1 and TM2 methods underestimate the α parameter (Fig.5). Nash values (Tab.1) indicate a superiority of the DTM-IP and DTM-RR methods. However, estimates of α by DTM-IP do not remain robust due to the fact that the linearity of the interval [η_{min} η_{max}] is not always satisfactory (Fig.2), then the DTM-RR allows to reduce uncertainties.

- The uncertainties of parameter estimates introduce significant uncertainties to the estimates of q_{crit} and q_s that define the multifractal phase transitions. The sensitivity analysis results for q_{crit} and q_s are displayed in Table 2.

Table 2: The sensitivity analysis

δα	δC1	δq_s	δq_{crit}
± 0.01	± 0.01	0.3	3.8
± 0.05	± 0.05	2.1	4.8

- The evolution of the multifractal parameters is useful for detection of hydrological impacts of climate change.

- A small variation (Fig.9) of singularities could result in a significant variation of the maximal rainfall intensity, for larger return periods of high resolution rainfall.

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