



# Multi-scaling properties of remotely sensed oceanic chlorophyll maps

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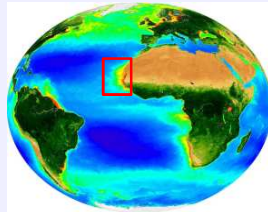
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**Abstract:** The analysis of chlorophyll maps measured by the SeaWiFS orbiting sensor shows multifractal properties that are consistent with the statistical theory of turbulence. Therefore, within this scale range (1-128km), turbulent mixing seems to be the dominant effect explaining phytoplankton variability. Moreover, multifractal patchiness can cause significant biases in the nonlinear terms involved in biogeochemical numerical models.

## Experimental dataset ①

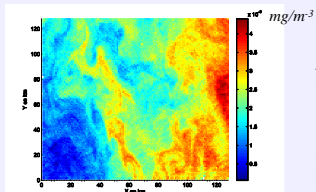
### The Senegalo-Mauritanian Upwelling

- High chlorophyll concentration  
=> low relative noise
- Very sunny weather  
=> large zone of available data



10-26°N / 14-26°W

**100 SeaWiFS chlorophyll maps with no cloud cover**  
(128\*128km, resolution 1\*1km, product L2, period: 07/2003-06/2004)

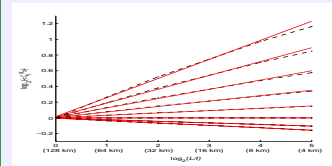


All maps were checked manually because of isolated unrealistic pixels.

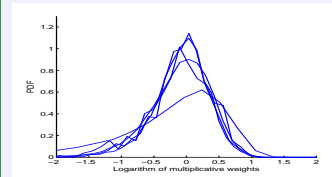
Is it possible to describe these chaotic and highly heterogeneous data in a simple way?

## Multifractal model and validation ②

**FIF model** (Fractionally Integrated Flux, Schertzer & Lovejoy, 1987)



Scaling of statistical moments



PDF of log of cascade weights for each resolution

Kolomogorov-like law for chlorophyll concentration:

$$\Delta Chl_l \approx \zeta l^H$$

Multifractal intermittency:

$$\langle \zeta_l^q \rangle \approx (L/l)^{K_\zeta(q)}$$

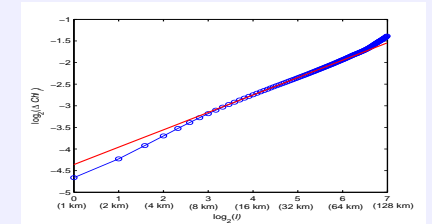
$$K_\zeta(q) = \frac{C_{1\zeta}}{\alpha_\zeta - 1} (q^{\alpha_\zeta} - q)$$

**Multifractal parameters:**  
 $H=0.4 \quad \alpha=1.92 \quad C_1=0.12$

## Multifractal analysis technique

1- Plot 1st order structure function and verify it is linear

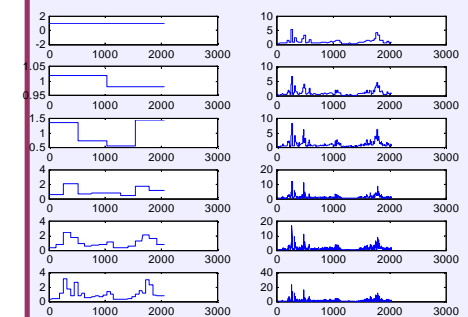
2- Average to cut-off these noisy high frequencies



First order structure function of the 100 SeaWiFS images

3- Fractionally differentiate the process (or take the norm of the gradient)

4- Reconstitute the cascade by successive averaging of contiguous data



Example of the reconstitution of a multiplicative cascade

5- Estimate the multifractal parameters with the scaling of the moments for each order and scale

## Link with Kolmogorov-Obukhov-Corrsin theory ③

### Semi-theoretical prediction of parameter H

If variability is due to turbulent mixing, then it is like a passive scalar:

$$\Delta Chl_l \approx \chi^{1/2} \varepsilon^{-1/6} l^{1/3} \Rightarrow H = 1/3 + K_\varepsilon(1/6) - K_\chi(1/2)$$

$K_\varepsilon(1/6)$  is due only to turbulence and is well-known (equal to -0.05)

$K_\chi(1/2)$  has to be estimated using multifractal parameters of  $\zeta$ :

$$\alpha_\chi \approx \alpha_\zeta \text{ and } C_{1\chi} \approx 2^{\alpha_\zeta} C_{1\zeta} \Rightarrow K_\chi(1/2) = -0.11$$

Finally  $H=0.33-0.05+0.11=0.39$  which is consistent with the empirical value

**In the studied scale range, turbulence seems to be the main cause explaining the observed variability**

## Possible application in numerical simulations ④

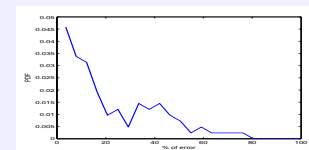
The hypothesis of homogeneity at the pixel scale biases the nonlinear terms of the equations in numerical simulations

**Parameters obtained in laboratory are not valid at a larger scale!**

**Therefore it is necessary to correct the parameters involved in numerical simulations by using empirical multifractal parameters.**

Example:

- 1/ Suppose a term of the form  $\beta \cdot Chl^2$
- 1/ Compute  $Chl^2$  at 1\*1km resolution
- 2/ Compute  $Chl^2$  at 128\*128km resolution
- 3/ Plot the distribution of the difference



**Error on parameter  $\beta$ : 22%**