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Intrinsic low-frequency variability in a low-order model of the wind-driven ocean circulation

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A spectral quasigeostrophic model of the wind-driven ocean circulation with four degrees of freedom is derived with the aim of analyzing the low-order character of the intrinsic low-frequency variability of the midlatitude double-gyre ocean circulation, and of the related coherence resonance phenomena (Pierini, 2011). In this poster the model characteristics and the low-frequency variability are presented and discussed (for the results concerning coherence resonance see a poster of the same author in session CL4.1/NP8.2)

The model includes an exponential in the basis functions that allows for westward intensification, has two components in both horizontal directions, allows for a rectangular domain and is forced here by a steady double-gyre wind field. The dynamical systems analysis of the numerical solution, with the wind amplitude taken as the control parameter, shows several transitions that connect steady states to periodic and chaotic oscillations.

A homoclinic bifurcation, in particular, leads to intrinsic decadal relaxation oscillations similar in several respects to those obtained in a primitive equation model of the Kuroshio Extension (Pierini 2006; Pierini et al. 2009; Pierini and Dijkstra 2009) and in a previous low-order ocean model (e. g., Simonnet et al. 2005).

This result supports the hypothesis of the low-order character of the intrinsic low-frequency variability known to arise in the midlatitude double-gyre ocean problem.

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The low-order ocean model

The model is based on the evolution equation of potential vorticity in the QG reduced-gravity approximation,

$$\frac{\partial}{\partial\bar{t}}\left(\bar{\nabla}^2\bar{\psi}-\frac{\bar{\psi}}{L_R^2}\right)+\bar{J}(\bar{\psi},\bar{\nabla}^2\bar{\psi})+\beta\bar{\psi}_{\bar{x}}=-r\bar{\nabla}^2\bar{\psi}+\frac{curl_{\bar{z}}\bar{\boldsymbol{\tau}}}{\rho H}$$

that can be adimensionalized as follows:

$$\frac{\partial}{\partial t}\left(\nabla_\lambda^2\psi-F\psi\right)+\gamma J\left(\psi,\nabla_\lambda^2\psi\right)+\psi_x=-R\nabla_\lambda^2\psi-T\tau_y$$

The truncated spectral model is obtained by expanding the stremfunction ψ as:

$$\psi(\mathbf{x},t)=\sum_{i=1}^{4}\Psi_i(t)|i\rangle$$

$$|1\rangle=e^{-\alpha x}sinx\ siny\qquad |3\rangle=e^{-\alpha x}sin2x\ siny$$
$$|2\rangle=e^{-\alpha x}sinx\ sin2y\qquad |4\rangle=e^{-\alpha x}sin2x\ sin2y$$

This choice follows that of Jiang et al. (1995), who adopted $|i\rangle$, i=1,2 in their 2D model. In the Hilbert space endowed with the inner product

$$\langle f|g\rangle=\frac{4}{\pi^2}\int_0^\pi\int_0^\pi e^{2\alpha x}fg\,dxdy$$

the basis is orthonormal: $\langle i|j\rangle=\delta_{ij}$. In the following table the main features of this model are compared with those of previous low-order ocean models.

low-order models	n. Fourier modes ($x\times y$)	exp. factor in the x-modes	weighted inner product	$L_x\neq L_y$	stochastic forcing
Veronis (1963)	2×2	-	-	-	-
Jiang et al. (1995)	1×2	YES	YES	-	-
Simonnet and Dijkstra (2002)	1×2	YES	-	-	-
Simonnet et al. (2005)	1×4	YES	-	-	-
the present model	2×2	YES	YES	YES	YES

The system reduces to a set of four coupled nonlinear ODEs:

$$\dot{\Psi}_1+p\Psi_1+q\Psi_3+N_1=W_1$$
$$\dot{\Psi}_2+u\Psi_2+v\Psi_4+N_2=W_2$$
$$\dot{\Psi}_3+m\Psi_3+o\Psi_1+N_3=W_3$$
$$\dot{\Psi}_4+s\Psi_4+t\Psi_2+N_4=W_4$$

The nonlinear terms N_i are given by

$$N_i=\sum_{j,k=1}^{4}\Psi_j J_{ijk}\Psi_k$$

so that the system can be written in compact form as follows:

$$\boxed{\frac{d\mathbf{\Psi}}{dt}+\mathbf{\Psi}\mathbf{J}\mathbf{\Psi}+\mathbf{L}\mathbf{\Psi}=\mathbf{W}}$$

The rank-3 tensor **J** is given by:

$$J_{1jk}=\frac{c_1Z_{1jk}-h_1Z_{3jk}}{h_1^2+a_1c_1},\quad J_{2jk}=\frac{d_1Z_{2jk}-h_1Z_{4jk}}{h_1^2+b_1d_1}$$
$$J_{3jk}=\frac{h_1Z_{1jk}+a_1Z_{3jk}}{h_1^2+a_1c_1},\quad J_{4jk}=\frac{h_1Z_{2jk}+b_1Z_{4jk}}{h_1^2+b_1d_1}$$

where

$$Z_{ijk}\equiv\langle i|Q_{jk}\rangle$$

and where **Q** is the symmetric tensor:

$$|Q_{jk}\rangle=\frac{\gamma}{2}\Big[|j\rangle_x\big(|k\rangle_{xxy}+\lambda|k\rangle_{yyy}\big)+|k\rangle_x\big(|j\rangle_{xxy}+\lambda|j\rangle_{yyy}\big)$$
$$-|j\rangle_y\big(|k\rangle_{xxx}+\lambda|k\rangle_{yyx}\big)-|k\rangle_y\big(|j\rangle_{xxx}+\lambda|j\rangle_{yyx}\big)\Big]$$

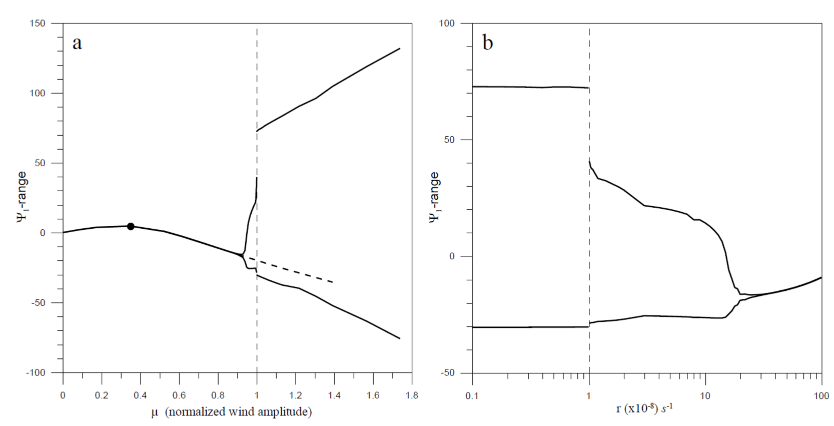
For all the details and parameter values see Pierini (2011).

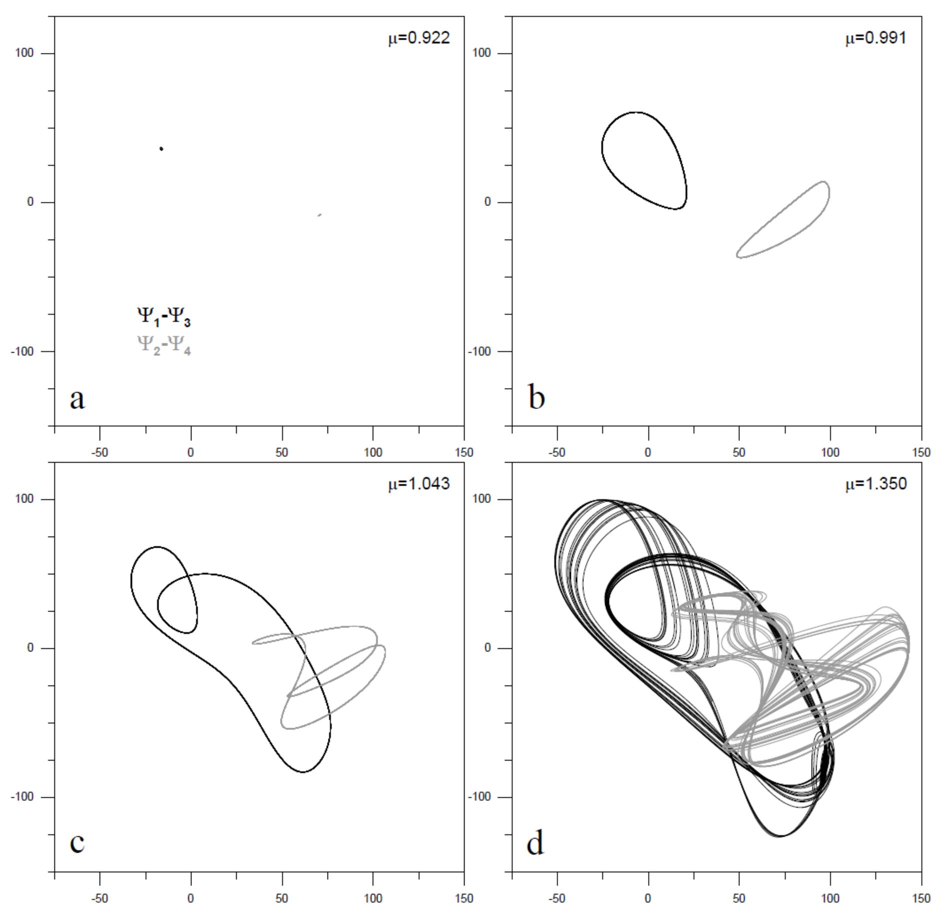
Intrinsic low-frequency variability

In order to reveal intrinsic oscillations due to nonlinear mechanisms internal to the ocean system, the equations are forced by the following *time-independent* climatological wind stress curl derived from *ECMWF* data:

$$curl_{\bar{z}}\bar{\boldsymbol{\tau}}=-\frac{d\bar{\tau}_0(\bar{y})}{d\bar{y}}=-\theta\bar{a}_w\sin\left(\bar{k}_w\bar{y}\right)$$

with $\bar{a}_w=6\times10^{-8}\ N\ m^{-3}$, $\bar{k}_w=1.7\ rad\ m^{-1}$ and θ=1. In the panels **below-left** bifurcation diagrams for $10^{-5}\cdot\Psi_1$ versus the normalized wind amplitude $\mu=\theta/\theta_0$ (θ_0=574.85 marks the homoclinic bifurcation) with $r=1\times10^{-8}\ s^{-1}$ (a), and versus the friction coefficient r with μ=1 (b), are shown. In (a) the dot denotes the first Hopf bifurcation and the thick dashed line the branch of the unstable steady state.





Right panel: Orbits in the Ψ_1-Ψ_3 (black) and Ψ_2-Ψ_4 (gray) planes for 4 values of μ. **Below-left**: time series of $10^{-5}\cdot\Psi_1$ for 12 values of μ. **Below-right**: sequence of snapshots of $10^{-5}\cdot\psi$ for 2 values of μ.