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NONLINEAR ANALYSIS OF THE NORTH ATLANTIC (NAO), ATLANTIC MULTIDECADAL (AMO) AND WESTERN MEDITERRANEAN (WeMO) OSCILLATIONS FOR THE COMMON PERIOD 1856-2009

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Three monthly indices, all of them Gaussian distributed, concerning sea-level pressure differences (NAO and WeMO) and sea-level temperatures (AMO) are analysed by means of algorithms based on the rescaled analysis (Hurst exponent H), the reconstruction theorem (correlation μ and embedding d_E dimensions, and Kolmogorov entropy κ), predictive instability (Lyapunov exponents λ_i and Kaplan-Yorke dimension D_{KY}) and correlation and power spectrum. Whereas WeMO and especially AMO indices are characterised by notable persistence (H >> 0.5), NAO index depicts clear signs of randomness (H ≈ 0.5). The complexity of a predictive scheme is manifested by the high number of nonlinear equations (μ from 6.8 to 10.1) which would be required to reproduce the monthly series. The loss of memory of the mechanisms governing the indices is especially relevant for NAO and WeMO (κ exceeding 1.0) in comparison with AMO (κ close to 0.15). The predictive instability is quite similar for the three indices, with the highest λ_1 varying within a narrow interval (0.13-0.15). D_{KY} ≈14.5 is similar for the three indices. Power and cross-power spectra are characterised by a reduction of the spectral power amplitude when increasing the analysed frequency, especially for monthly AMO and monthly AMO-WeMO cross-power spectra. Assuming a decaying power-law, the exponents β of this law are 0.07, 0.17 and 1.06 for NAO, WeMO and AMO indices respectively. Relevant spectral power amplitudes, exceeding white-noise and 95% significant levels of Markovian red-noise contents, are found for WeMO (19 and 51 years), AMO (9, 51 and 77 years) and WeMO-AMO cross-power (51 years). As expected, the one-year periodicity is always detected, being also observed a significant six-month periodicity for the monthly NAO. After the computation of the Hausdorff dimension Ha and the comparison with the exponent of the decaying power spectrum when increasing analysed frequency, some signs of fractional Gaussian noise behaviour are detected for the three time series. A time behaviour based on fractional Brownian noise should be discarded after revising the results of the corresponding simulations.

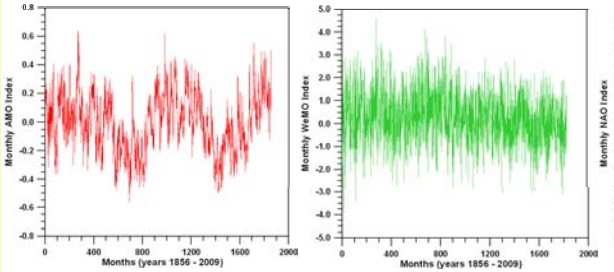


Fig. 1 Time series of AMO, WeMO and NAO indices

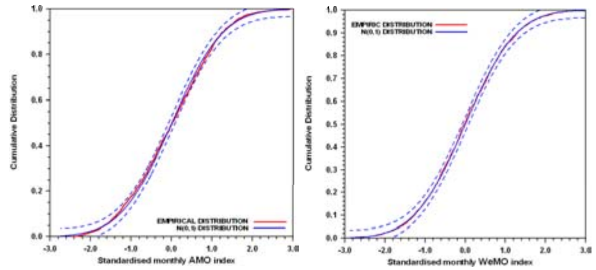


Fig. 2 Gaussian distribution of monthly AMO and WeMO indices

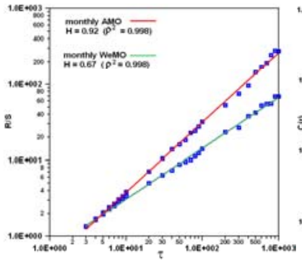


Fig.3 Hurst exponents for AMO and WeMO (NAO is characterised by H ≈ 0.5)

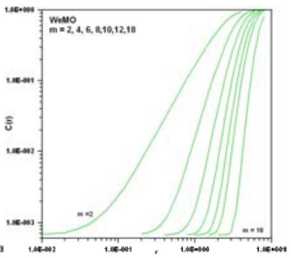


Fig.4 Correlation integral C(r) up to dimension 18 for WeMO index

H ≈ 0.5 randomness

H >> 0.5 persistence

H << 0.5 anti-persistence

{X} : time series of indices

$Z_k = \{X_{k+1}, \ldots, X_{k+m}\}$

$$C(r) = (1/N^2) \sum H[r - |Z_i - Z_j|] \quad (i,j = 1, \ldots, N;\ N \to \infty)$$

$$C(r) = A(m)\, r^{\mu(m)}\, e^{-m\kappa}$$

$$\ln\{C(r)\} = \ln\{A(m)\} - m\kappa + \mu(m) \ln\{r\}$$

$\alpha(m) = \ln\{A(m)\} - m\kappa$, provided that:

$A(m+1) \approx A(m)\ ;\ m \to \infty$

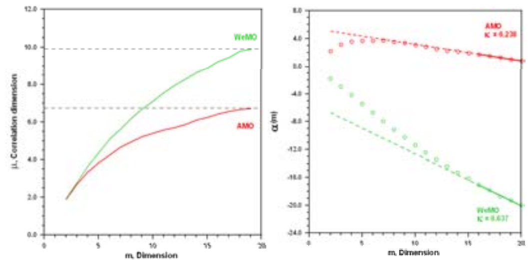


Fig.5 Correlation dimensions (for an embedding dimension of 19) and Kolmogorov entropies for AMO and WeMO indices

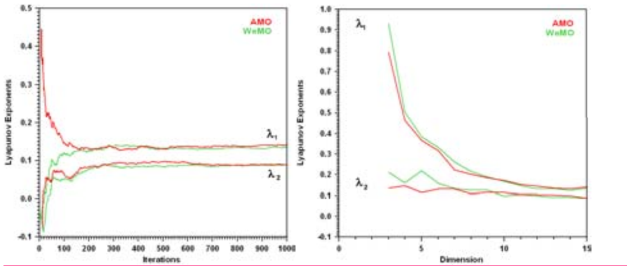


Fig.6 The first two positive exponents for AMO and WeMO indices (d_E=15) after 1,000 iterations

	H	μ (d_E)	κ	λ_1, λ_2	Ha	β
AMO	0.92	6.8(20)	0.24	0.14, 0.09	0.10	1.08
WeMO	0.67	10.1(20)	0.64	0.13, 0.09	0.15	0.17
NAO	0.54	10.1(19)	1.37	0.13, 0.10	0.17	0.07

	AMO-WeMO	AMO-NAO	WeMO-NAO
β	0.81	0.73	0.14

Table 1. Fractal parameters for the three time series

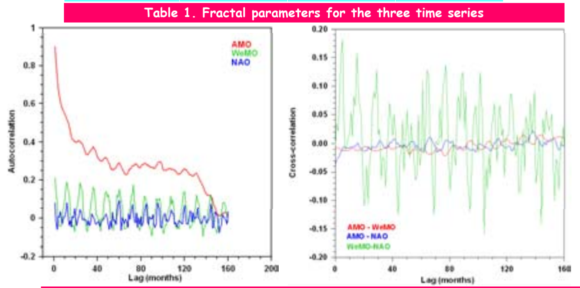


Figure 7. Autocorrelation and cross-correlation coefficients for the three series

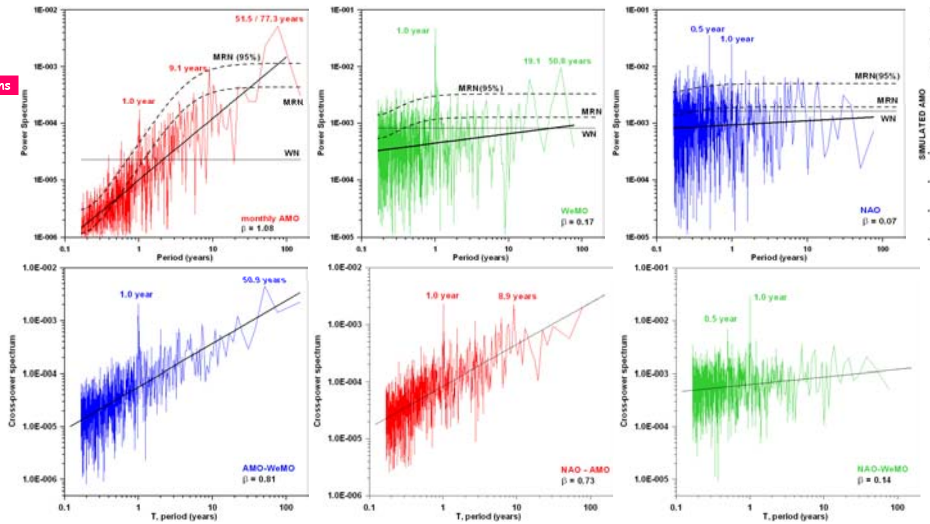
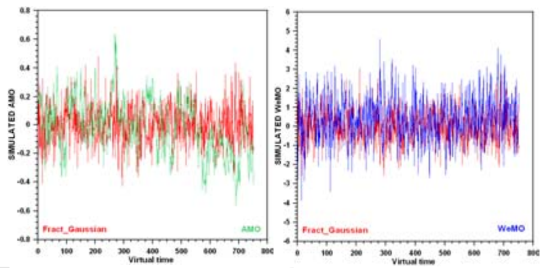


Fig.8 Power and cross-power spectra for AMO, WeMO and NAO. MRN and MRN(95%) represent Markovian red-noise components and 95% confidence bands respectively. WN designs the constant white-noise spectral content and black lines the power law $S(\omega) \sim \omega^{-\beta}$ ($\sim T^{\beta}$)



Fractional Gaussian-Noise

Gaussian-noise sequence:
$\{gn(t_j)\}$, j = 1, … , N

Discrete Fourier Transform
$SG(\omega) = DFT\{gn(t)\}$

Filtering
$SGF(\omega) = SG(\omega)[m/N]^{\beta/2}$
(m =1, …, N)

Inverse DFT:
$FGN(t) = DFT^{-1}\{SGF(\omega)\}$

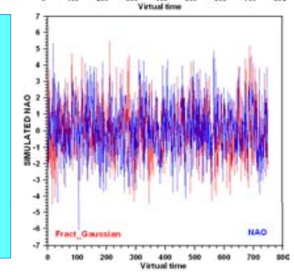


Fig.9 Fractional Gaussian-noise series compared with segments of real AMO, WeMO and NAO monthly series

CONCLUSIONS

- The three indices are Gaussian distributed.
- Strong persistence for AMO, weaker for WeMO and clear randomness for NAO.
- Indices are described by complex systems of nonlinear equations (μ from 7 to 10).
- In agreement with Hurst exponents, large, moderate and small loss of memory for NAO, WeMO and AMO respectively.
- Very similar predictive instability manifested by λ_1 and λ_2 Lyapunov exponents.
- Pairs {Ha, β} suggest fractional Gaussian-noise behavior, especially for WeMO and NAO, with β < 1.0.
- As expected, one-year period peak is detected for all power and cross-power spectra.
- An spectral peak at 0.5 years (NAO) also appears in the cross-power spectrum (NAO-WeMO).
- A peak close to 9.0 years (AMO) in the cross-power spectrum (NAO-AMO).
- Multidecadal periodicities (51 years) observed in WeMO and WeMO-AMO spectra.
- Multidecadal periodicities (WeMO, 19 years; AMO, 77 years) not detected in the corresponding cross-power spectra.
- First-lag autocorrelation in agreement with persistence/randomness suggested by H.
- Almost null cross-correlations for all analysed lags (months), except for WeMO-NAO, slightly departing from zero.
- Signs of correlation at multidecadal time scale between AMO and WeMO.
- Signs of correlation at medium time scale between NAO and AMO.
- Signs of NAO-WeMO correlation are constrained to the annual and six-month periodicities.

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