A Reduced Model for Nonlinear Interactions of Gravity Waves with Deep Convective Clouds

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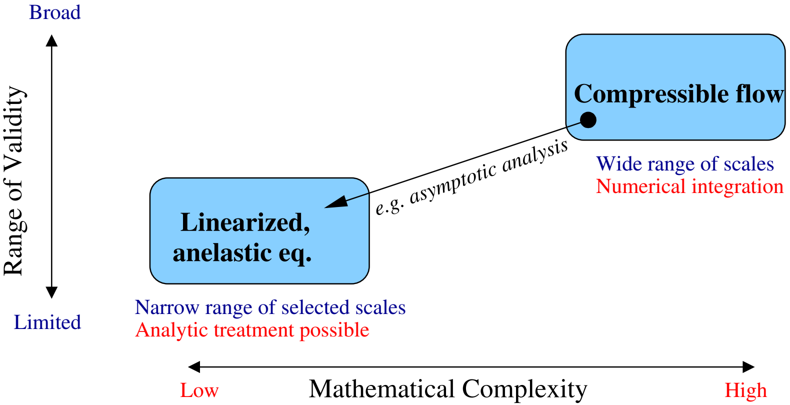
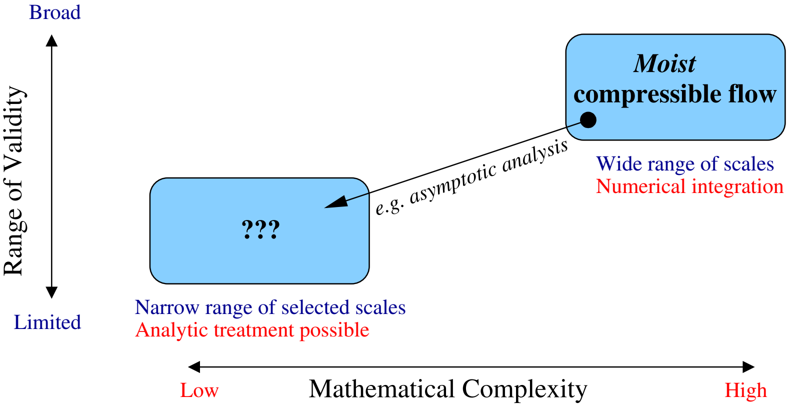
Reduced Models



Goal

Reduced model for gravity waves in atmosphere with deep convection.

Reduced Models



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Governing Equations

Conservation of mass, momentum, energy (dimensionless)

$$\rho_t + \nabla_\parallel \cdot (\rho \mathbf{u}) + (\rho w)_z = 0$$

$$\mathbf{u}_t + \mathbf{u} \cdot \nabla_\parallel \mathbf{u} + w \mathbf{u}_z + \frac{1}{\mathrm{Ro}} (\mathbf{\Omega} \times \mathbf{v})_\parallel + \frac{1}{\mathrm{Ma}^2} \frac{1}{\rho} \nabla_\parallel p = 0$$

$$w_t + \mathbf{u} \cdot \nabla_\parallel w + w w_z + \frac{1}{\mathrm{Ro}} (\mathbf{\Omega} \times \mathbf{v})_\perp + \frac{1}{\mathrm{Ma}^2} \frac{1}{\rho} p_z = -\frac{1}{\mathrm{Fr}^2}$$

$$\theta_t + \mathbf{u} \cdot \nabla_\parallel \theta + w \theta_z = \underbrace{S_\theta}_{?}$$

Bulk microphysic model

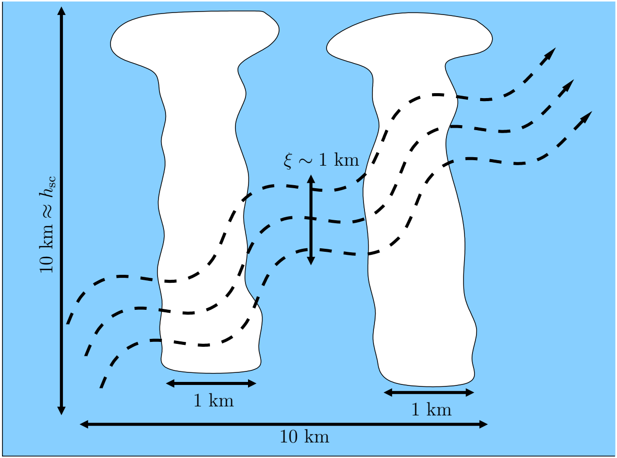
$$q_{\mathrm{v},t} + \mathbf{u} \cdot \nabla_\parallel q_\mathrm{v} + w q_{\mathrm{v},z} = C_\mathrm{ev} - C_\mathrm{d}$$

$$q_{\mathrm{c},t} + \mathbf{u} \cdot \nabla_\parallel q_\mathrm{c} + w q_{\mathrm{c},z} = C_\mathrm{d} - C_\mathrm{ac} - C_\mathrm{cr}$$

$$q_{\mathrm{r},t} + \mathbf{u} \cdot \nabla_\parallel q_\mathrm{r} + w q_{\mathrm{r},z} + \frac{1}{\rho} (\rho q_\mathrm{r} V_\mathrm{T})_z = C_\mathrm{ac} + C_\mathrm{cr} - C_\mathrm{ev}$$

Scales

Non-hydrostatic gravity waves modulated by deep convective towers



Coordinates:

- Wave-scale:
  $\mathbf{x} \sim 10$ km
- Tower-scale:
  $\eta \sim 1$ km
- Vert. scale:
  $z \sim 10$ km
- Time scale:
  $\tau \sim 100$ s

$\Rightarrow$ Vert. Disp.:
$\xi \sim 1 \text{ km} = \mathcal{O}(\varepsilon)$

$\Rightarrow$ Condensate:
$q_{\text{released}} \sim \mathcal{O}(\varepsilon)$

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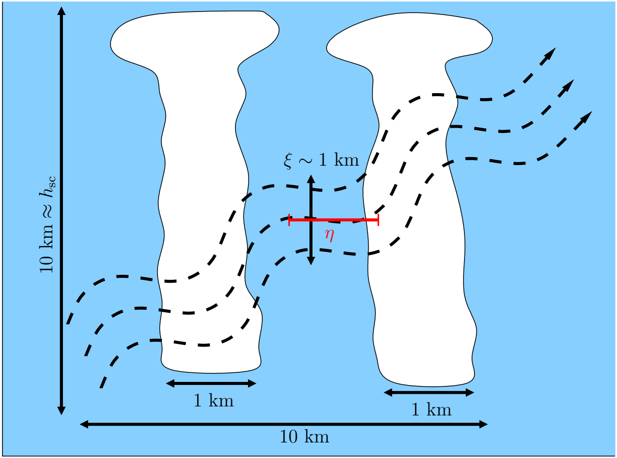

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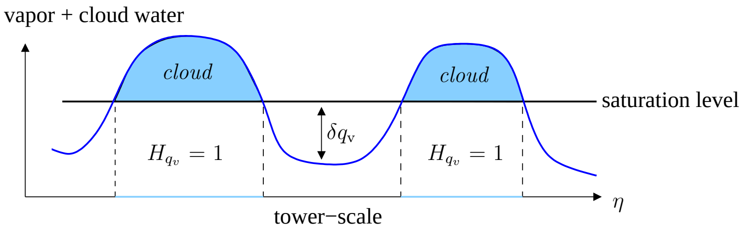
Leading Order Saturation Deficit



Switching Function

$$H_{\mathrm{q_v}} := \begin{cases} 1 & : \text{Saturation at } \mathcal{O}(1) \quad \text{(Cloud)} \\ 0 & : \text{Under-saturation} \quad \text{(No cloud)} \end{cases}$$

Saturated Area Fraction

$$\sigma := \overline{H_{\mathrm{q_v}}}^{\eta}, \quad \text{for } \delta q_{\mathrm{v}} = \mathcal{O}(1) \text{ due to short time-scale:} \quad D_\tau \sigma = 0.$$

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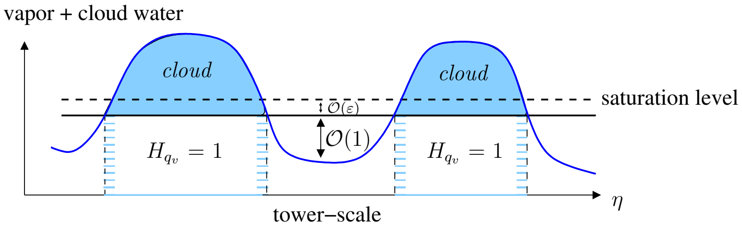

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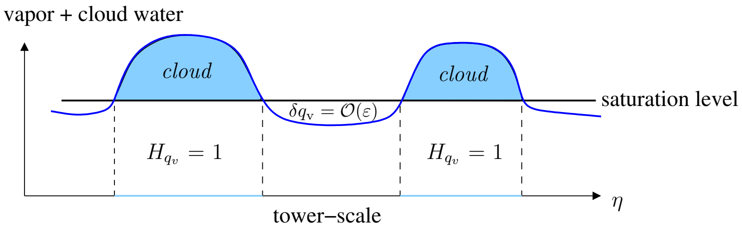
Systematically Small Saturation Deficit



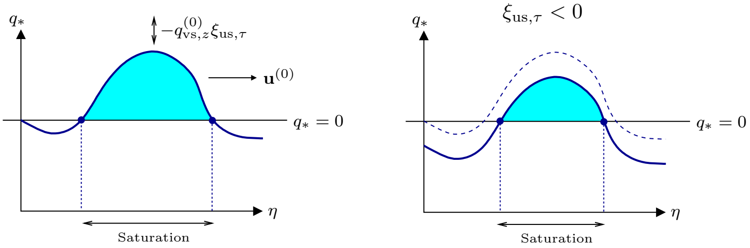
Expansion of Vapor Mixing Ratio

$$q_v = \underbrace{q_{vs}^{(0)}}_{\text{saturation}} + \varepsilon q_v^{(1)} + \mathcal{O}(\varepsilon^2)$$

Switching Function

$$H_{q_v} := \begin{cases} 1 & : \text{Saturation at } \underline{\mathcal{O}(\varepsilon)} \\ 0 & : \text{Under-saturation} \end{cases}$$

Tracking Saturated Areas



Shape of Level Set Function

$$q_*(\eta,\tau) = q_{*,0}(\eta - \int_0^\tau \mathbf{u}^{(0)} d\tau') - q^{(0)}_{\mathrm{vs},z}\xi_{\mathrm{us}}(\tau),$$ Assume: $\xi_{\mathrm{us}}(\tau) \approx \xi(\eta,\tau)$

Hence : $q_*(\eta,\tau) = 0 \Leftrightarrow q_{*,0}(\eta - \int_0^\tau \mathbf{u}^{(0)} d\tau') = q^{(0)}_{\mathrm{vs},z}\xi_{\mathrm{us}}(\tau)$

Model Equations

Closed model equations:

Wave-scale dynamics:

$$\mathbf{u}_\tau^{(0)} + \mathbf{u}^\infty \cdot \nabla_x \mathbf{u}^{(0)} + \nabla_x \pi = 0$$

$$\bar{w}_\tau^{(0)} + \mathbf{u}^\infty \cdot \nabla_x \bar{w}^{(0)} + \pi_z = \bar{\theta}^{(3)}$$

$$\bar{\theta}_\tau^{(3)} + \mathbf{u}^\infty \cdot \nabla_x \bar{\theta}^{(3)} + (1-\sigma)\Theta_z^{(2)} \bar{w}^{(0)} = \Theta_z^{(2)} w'$$

$$\nabla_x \cdot \left(\rho^{(0)} \mathbf{u}^{(0)}\right) + \left(\rho^{(0)} \bar{w}^{(0)}\right)_z = 0$$

Effective tower-scale dynamics:

$$w'_\tau + \mathbf{u}^\infty \cdot \nabla_x w' + \frac{\sigma_\tau}{1-\sigma} w' = \theta'$$

$$\theta'_\tau + \mathbf{u}^\infty \cdot \nabla_x \theta' + \sigma \Theta_z^{(2)} w' + \frac{\sigma_\tau}{1-\sigma} \theta' = \sigma \left(1-\sigma\right) \Theta_z^{(2)} \bar{w}$$

$$\sigma_\tau = \xi_{\mathrm{us},\tau} \Psi(\xi_{\mathrm{us}})$$

$$\xi_{\mathrm{us},\tau} = \bar{w} - \frac{w'}{1-\sigma}$$

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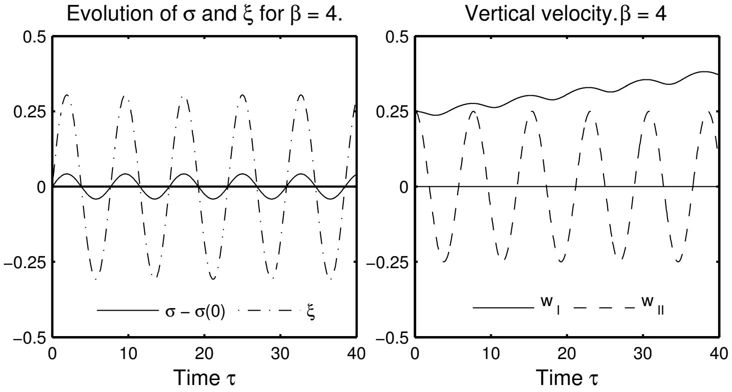
$$\sigma_\tau = \xi_{\mathrm{us},\tau} \Psi(\xi_{\mathrm{us}})$$

$$\xi_{\mathrm{us},\tau} = \bar{w} - \frac{w'}{1-\sigma}$$

Plane Wave Amplitudes



Amplitudes of plane wave

$$\phi(x, z, \tau) = \hat{\phi}(\tau) \exp\left(ikx + imz\right)$$

Summary

- Derived **reduced model** for gravity waves in deep convecting atmosphere
- Assuming **small saturation deficit** leads to **nonlinear interactions** between waves and clouds
- Tracking of saturated spots by **level set function**: Simplifying assumptions allow to derive simple evolution equation
- Final model is extension of linearized anelastic equations

Literature

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