Snow melt modeling (SMM) calibrated through simplex flexible algorithm: application to rainfall thresholds for landslides forecasting

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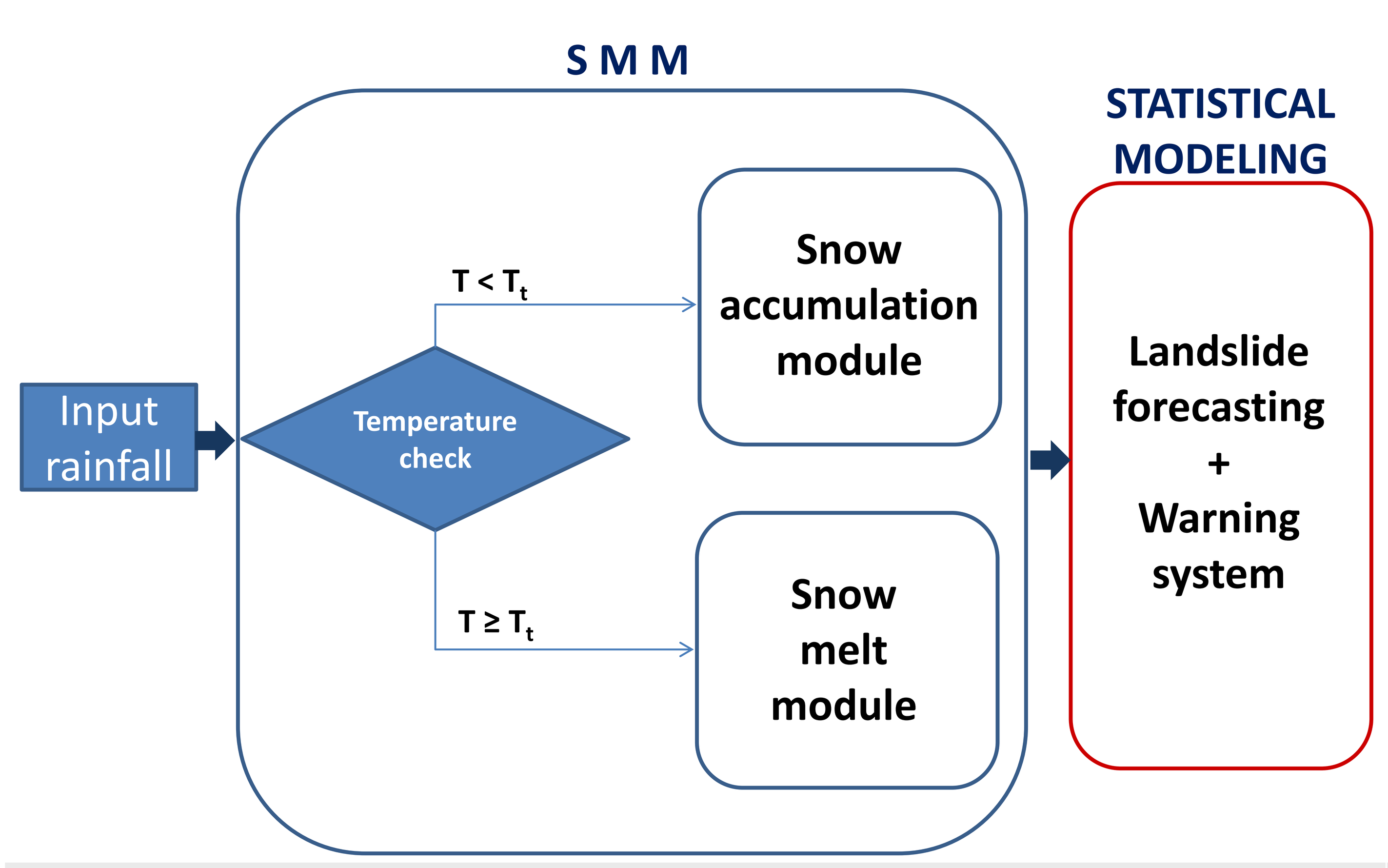
1) INTRODUCTION

In temperate or cold climate regions a not negligible part of precipitation may be withhold in a snow cover until the occurrence of a temperature change that triggers a sudden water input in the ground. Northern Apennines of Italy are no exception to this rule as demonstrated by the recent seasonal event of December 2009 in which several shallow landslides have been triggered by a rapid snow melt in Tuscany and Emilia-Romagna regions. In order to integrate snow precipitation within existing statistical models for landslide prediction, a simple conceptual dynamic model for the snow melting is proposed. The model takes into account the buildup and melting of the snow cover in time. The final objective of the work is to increase the predictive capability of the statistical models for landslide triggering based on rainfall input.

In literature, several snow accumulation and melting models are proposed. The nature and possible applications of snow melting models are varied and dependent from the purpose of use: for example in the hydrological modeling they are used for the analysis of runoff generated by the melting of the snowpack. In the prediction and study of avalanches some very sophisticated models are used which can provide a detailed representation of the internal structure of the snowpack. Essentially they are spatially distributed models based on equations of mass and energy balance. These models, based on digital elevation data (DEM) accounting for topography, generally require complex meteorological data: precipitation, air temperature, wind speed and direction, humidity, downwelling shortwave radiation, downwelling longwave radiation, cloud cover, surface pressure. Simplified approaches as degree-day or temperature-index models show good results and it has been shown that "only little additional improvement in model performance is achieved when adopting an energy balance approach".

2) SMM AND STATISTICAL MODELING FOR LANDSLIDES FORECASTING



The proposed model is built for the need of its integration with a statistical model for landslides prediction to take into account the correct computation of the rainfall and water resulting from snow melting. The snow melt modeling proposed is based on two equations: the conservation of mass (input-output balance) and an empirical equation for modeling the snow density variation. Also melting process is modeled with empirical function based on chemical kinetics. From the conservation of mass, a differential equation of snow cover depth, depending on density and average temperature of the air, can be obtained. The second equation is an empirical function for the average density variation and depends on the snow cover depth (gravity effect) and the temperature of the air, which is variable in time. In synthesis the SMM is divided in two modules: snow accumulation and snow melt.

3) METHODOLOGY OF CALIBRATION

DISCRETIZED EQUATIONS OF THE MODEL

Principle of mass conservation

Snow cover (H, ρ)
H = snowpack depth
ρ = average snow density

Accumulation $T_a(t) < T_t$

$$\begin{cases} \rho_0(t+1) = k_{\rho 0}\cdot \exp\left(k_{\exp}\cdot\left(T_a(t+1)-T_s\right)\right) \\ \rho(t+1) = \dfrac{H(t)\cdot\left(\rho(t)+k_{\rho 1}\dfrac{H(t)}{k_{\rho 2}+H(t)}\cdot\dfrac{k_\rho}{k_\rho+\rho(t)}\right)+H_w(t+1)\cdot\rho_w}{H(t)+\dfrac{H_w(t+1)\cdot\rho_w}{\rho_0(t+1)}} \\ H(t+1) = \dfrac{1}{\rho(t+1)}\left(H(t)\rho(t)+H_w(t+1)\cdot\rho_w\right) \end{cases}$$

Melting $T_a(t) \geq T_t$

$$\begin{cases} \rho(t+1) = \rho(t)+k_{\rho 1}\dfrac{H(t)}{k_{\rho 2}+H(t)}\cdot\dfrac{k_\rho}{k_\rho+\rho(t)}\cdot\dfrac{T_a(t+1)}{k_t+T_a(t+1)} \\ H_{ww}(t+1) = \left(k_2\left(T_a(t+1)-T_s\right)^{k_1}+k_3\dfrac{H_w(t+1)}{k_w+H_w(t+1)}\right)\cdot\dfrac{k_{s1}}{k_{s1}+H(t)\rho(t)} \\ H(t+1) = \dfrac{1}{\rho(t+1)}\left(H(t)\rho(t)-H_{ww}(t+1)\right) \end{cases}$$

13 parameters for modeling identification

The output of the model H depends on independent variable x and n parameters $\mathbf{P}$

$$H = f(x,\mathbf{P}) \qquad \mathbf{P} = p_1,\ p_2,\ \dots,\ p_n \in \Re^n$$

Using an appropriate optimization algorithm (**Flexible Simplex**) we can calculate the optimum parameters that minimize the quadratic difference between experimental data and output of the model:

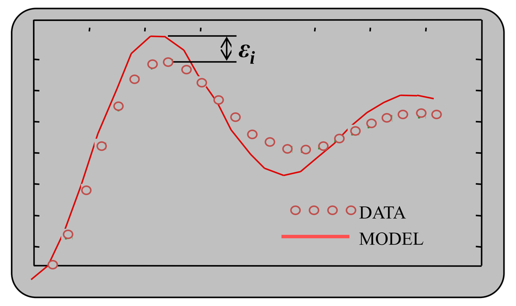
$$E(\mathbf{P}) = \frac{1}{N}\sum_{i=1}^{N} w_i\cdot\left(y_i^{\exp}-y_i\right)^2 = \frac{1}{N}\sum_{i=1}^{N} w_i\cdot\left(y_i^{\exp}-f(x_i,\mathbf{P})\right)^2$$

$$\left\{y_i^{exp} \quad i=1,2,\dots,N\right\}$$

Experimental data were very noisy → Moving average filter

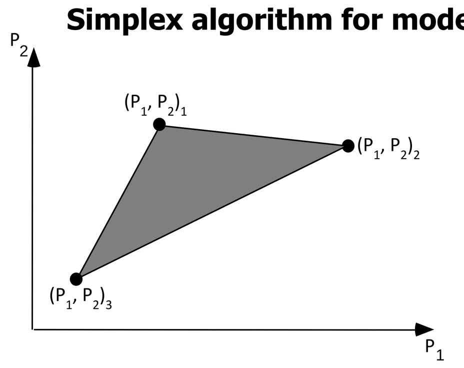
w_i = data weight, if necessary

$$\hat{\mathbf{P}} = \arg(\min(E(\mathbf{P})))$$



The model is divided in two modules depending on whether a thresholds temperature is exceeded or not. The first module accounts for the accumulation of solid rainfall in the snowpack and the second one for the snow melting. The main originality of the model is the use of an empirical functional, for melting process, based on chemical kinetics depending on air temperature, rainfall amount and depth and density of the snowpack, while other factors like wind, air humidity, atmospheric pressure and radiation are not considered since not available in our case study. In the present form, the model depends on 13 empirical parameters including a threshold temperature between snowfall and rainfall and the density of newly fallen snow.

Simplex algorithm for modeling identification: definitions



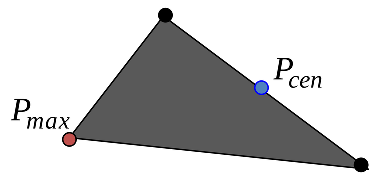
In the space of parameters, where the functional error is defined, the simplex is a n-dimensional polytope with the smallest number of vertices (n+1): for example in R^2 the simplex is a triangle.

The worst point (P_{max}) and the best point (P_{min}) are defined as:

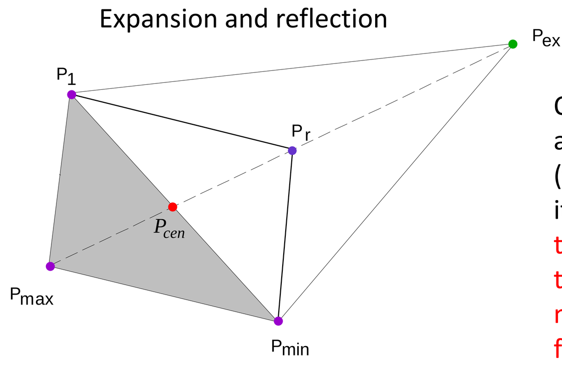
$$\mathbf{P}_{max} = arg\left[\max_{\mathbf{P}}\{E(\mathbf{P})\}\right] \qquad \mathbf{P}_{min} = arg\left[\min_{\mathbf{P}}\{E(\mathbf{P})\}\right]$$

The centroid of simplex is defined as its barycentre without P_{max} :

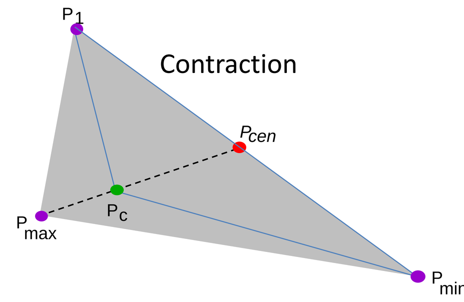
$$\mathbf{P}_{cen} = \frac{1}{n}\left[\left(\sum_{i=1}^{n+1}\mathbf{P}_i\right)-\mathbf{P}_{\max}\right]$$

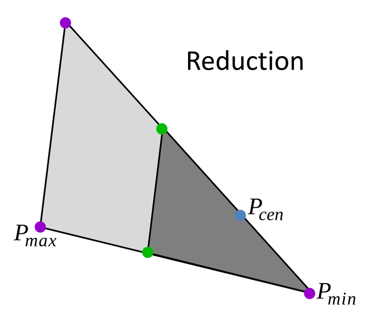


Simplex algorithm: operations



Once defined an initial simplex, the algorithm replaces the worst point (the higher functional error) at each iteraction. The criterion for stopping the algorithm is based on a flatness test of simplex: all vertices of simplex must have the same functional error for less then tolerance ε.



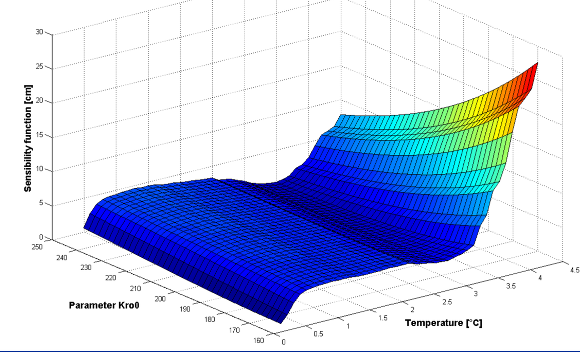


To assess the optimum values of the empirical parameters, we used an heuristic optimization algorithm (optimized flexible simplex) to minimize the errors between outputs of the model and experimental measures retrieved from a network of sensors located in the study area.

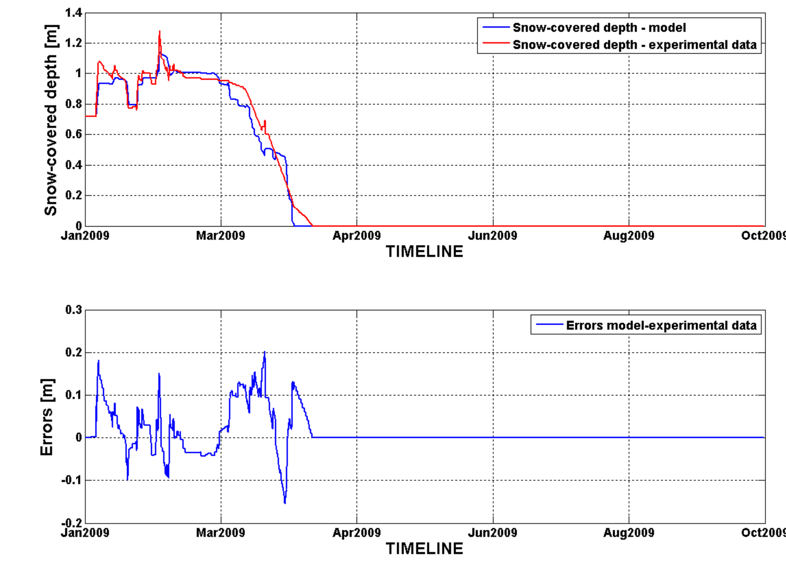
Example of static sensitivity analysis of the Model

Sensibility function is evaluated as the difference between a nominal trajectory and every trajectory with a perturbed parameters on a time interval of interest.

$$S(\mathbf{P}) = \frac{1}{N}\sum_{t=1}^{N}\left|H_t(\mathbf{x},\mathbf{P}) - H_t(\mathbf{x},\mathbf{P}^{nom})\right|$$

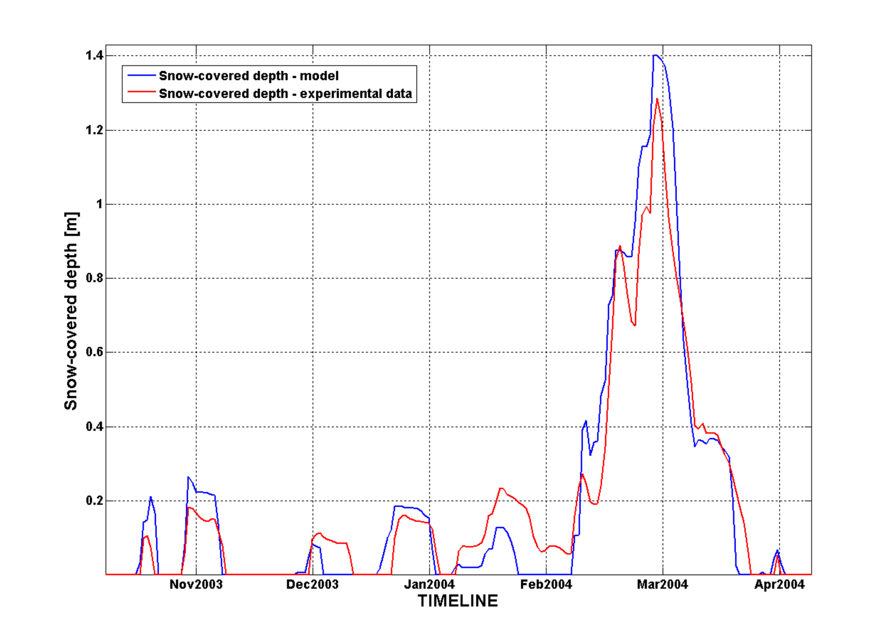


Calibration results with validated data (2009) of Doccia di Fiumalbo rain Gauge (1371 s.l.): by ARPA Emilia-Romagna region

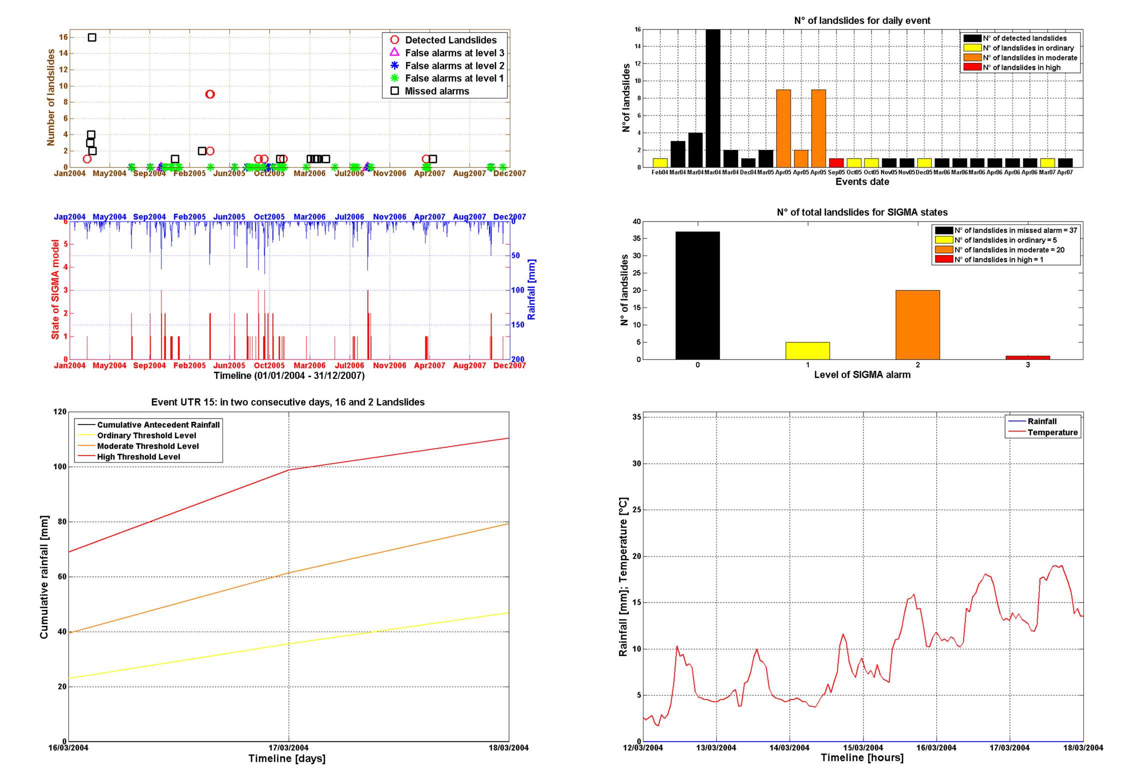


4) VALIDATION AND APPLICATION TO STATISTICAL MODEL

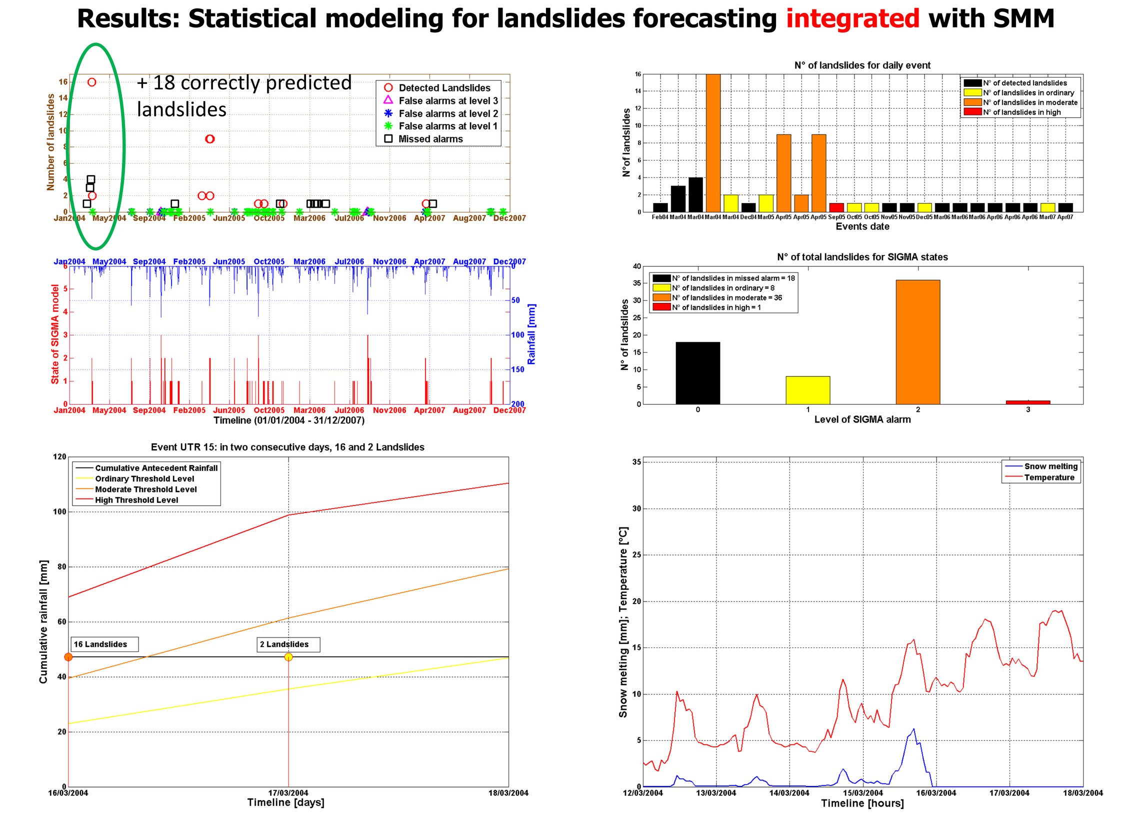
The model is validated with data (11/2003 – 4/2004) of Febbio rain Gauge (1148 s.l.). As this experimental data is very noise, it is filtered with moving average. In the next figure an example of validation period is reported: the model shows a good behavior in the matching of experimental data. Then it is applicable to statistical modeling for landslides forecasting: in particular the SMM is integrated with SIGMA model (see EGU2011-3395; NH3.7; Poster Area Hall XY556 - 05 April 2011)



Results: Statistical modeling for landslides forecasting not integrated with SMM



Results: Statistical modeling for landslides forecasting integrated with SMM



5) CONCLUSION

The SMM results are quite satisfactory: the absolute mean error is 5.7 cm in calibration and 11.8 cm in validation, below the measurement errors of the rain gauge sensors. The SMM shows a good robustness and it improves the statistical model with the detection of 36 landslides from snow melting. In addition, false alarms were reduced due to re-distribution of water input in the ground.

EGU2011-3322; HS3.2; Poster Area Hall A348 - 04 April 2011