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# PANDORA: Procedure for mAthematical aNalysis of lanDscape evOlution and equilibRium scenarios Assessment.

***F. Gobattoni<sup>1</sup>, R. Pelorosso<sup>1</sup>, G. Lauro<sup>2</sup>, A. Leone<sup>1</sup>, R. Monaco<sup>3</sup>***

*1 Department of Environment and Forestry, University of Tuscia, Viterbo (Italy)*

*2 Architecture Faculty, Second University of Naples, 81031 Aversa, (Italy)*

*3 Department of Mathematics, Politecnico di Torino, Torino (Italy)*



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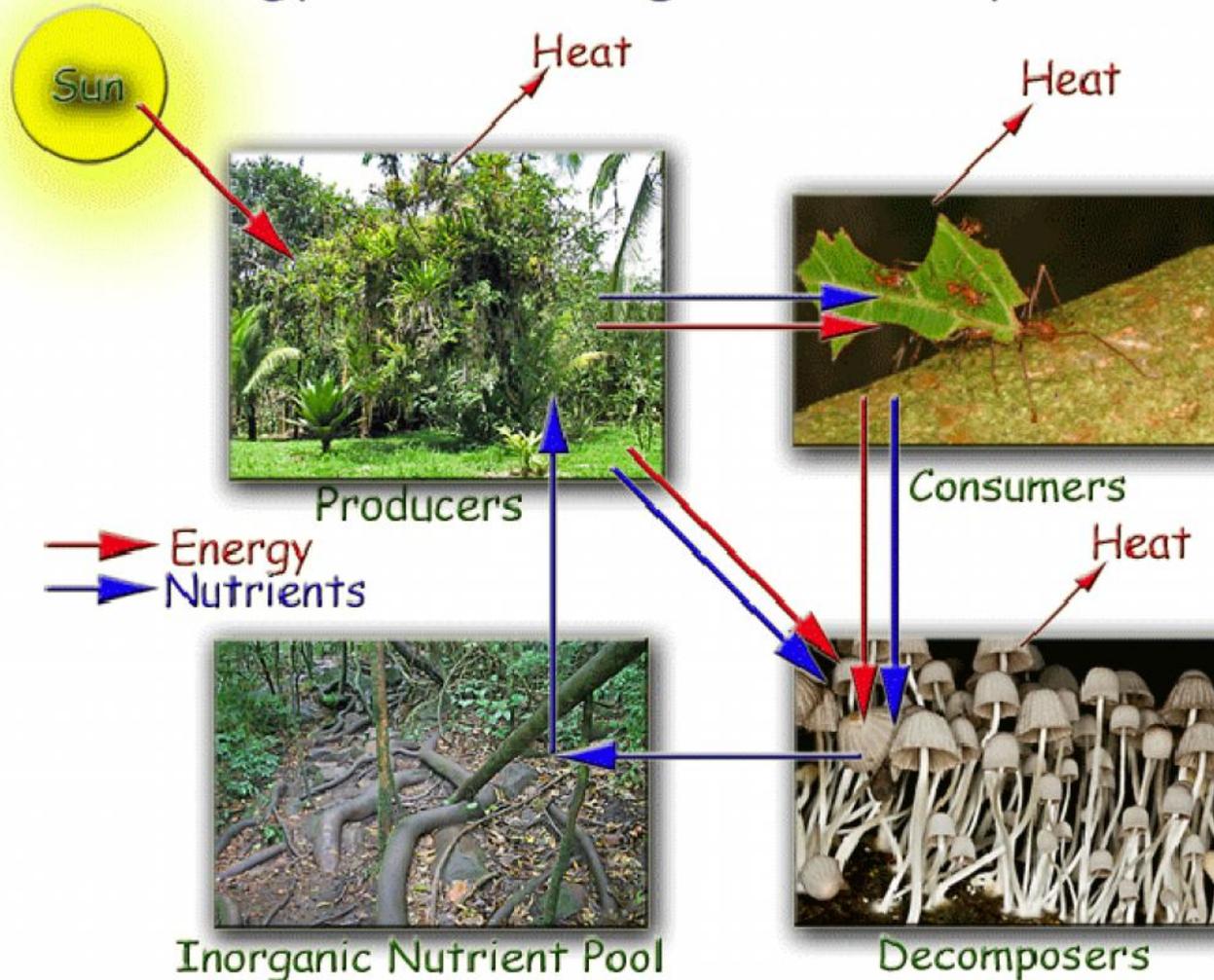
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## Introduction

*All ecosystems, as open systems, continuously exchange energy, nutrients and biomass with the environment through irreversible processes.*

## Energy Flow Through The Ecosystem



Human actions (infrastructures, urban development, natural resources exploitation etc) behave as external constraints imposed on the ecosystem, reducing flows of energy and matter, altering the dynamic equilibrium, inducing a decrease of entropy so that to make it an organized system (Chakrabarti and Ghosh, 2010).

## Introduction

Modeling the energy fluxes and variation of landscape energetic equilibrium state could allow to assess the most suitable plan strategies for natural resources conservation management and landscape functionality preservation.

Numerous physical and empirical models have been developed to simulate landscape and vegetation dynamics in time in order to explain environmental evolution and equilibrium conditions.

Although these efforts, a **macroscopic theory about landscape rules** and its variables still lacks (Chakrabarti and Ghosh, 2010; Coulthard, 2001; Jorgensen, 2004) and if some equilibrium is observed it may only be seen at certain spatio-temporal scales (Pickett et al., 1994).

## Introduction

While in mechanics sense there is a clear definition of equilibrium (see Thorn and Welford, 1994) (e.g. forces acting on object sum to zero or net torque is zero or when oscillations around a stable position are set up following a perturbation), in complex systems like landscape it is rare to find any form of equilibrium (Grimm and Wissel, 1997; Perry, 2002; Turner et al., 1993).

## Mathematical equilibrium?

*recurring to energy flows concept and species metabolism as fundamental keys in the evolution mechanisms.*

**PANDORA: Procedure for mAthematical aNalysis of lanDscape evOlution and equilibRium scenarios Assessment.**

## Introduction

Metabolic theory (**Brown et al., 2004**) aims to explain *how metabolic rate controls ecological processes*.

A simple way to apply this “metabolic” concept of energy is given by Biological Territorial Capacity (BTC) index (**Ingegnoli, 2002**).

The **Biological Territorial Capacity (BTC)**, is directly derived from metabolic characteristics of the species. It seems to sum up the energy availability in an ecosystem and, as a consequence, it can be considered as a good synthetic measure of landscape energetic levels that can lead to an equilibrium concept translated in the landscapes dynamics analysis.

$$a = \left( \frac{R}{GP} \right)_i / \left( \frac{R}{GP} \right)_{\max}$$

$$b = \left( \frac{ds}{S} \right)_{\min} / \left( \frac{ds}{S} \right)_i$$

$$\Omega = (a_i + b_i)R_i$$

**GP= gross primary production and R= respiration**

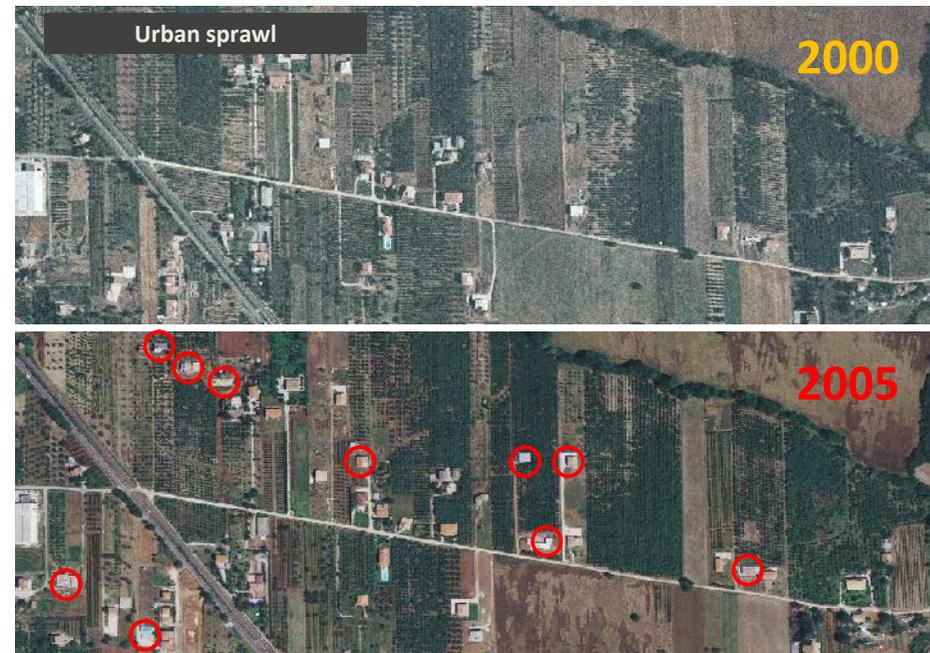
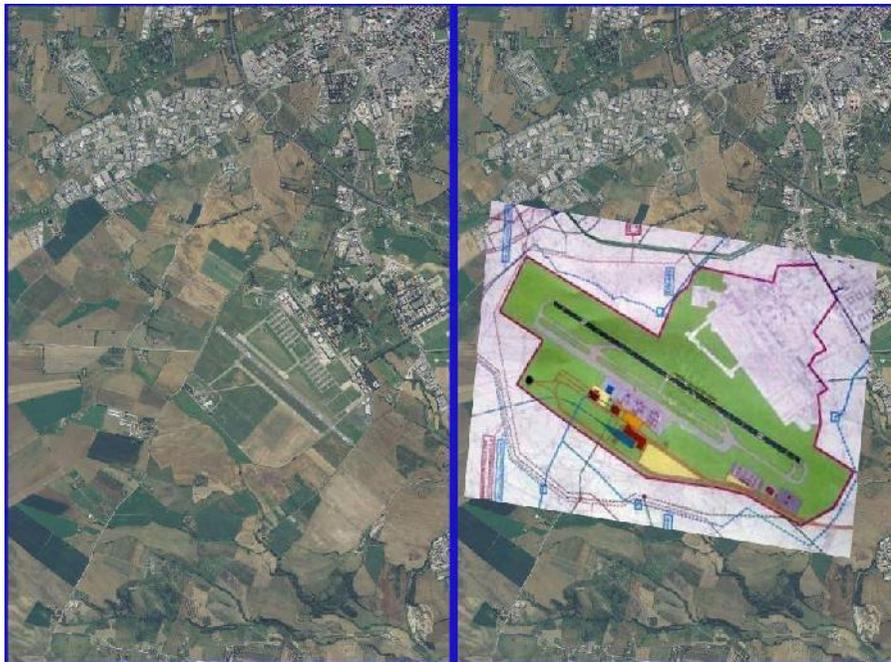
**ds/S, almost equal to R/B, is the maintaining ratio (or a thermodynamic function) of the structure and i represents the number of the main ecosystems in the ecosphere.**

$$\rightarrow BTC_i = 0.89\Omega - 0.0054\Omega^2 \quad (Mcal / m^2 / anno)$$

## Aims

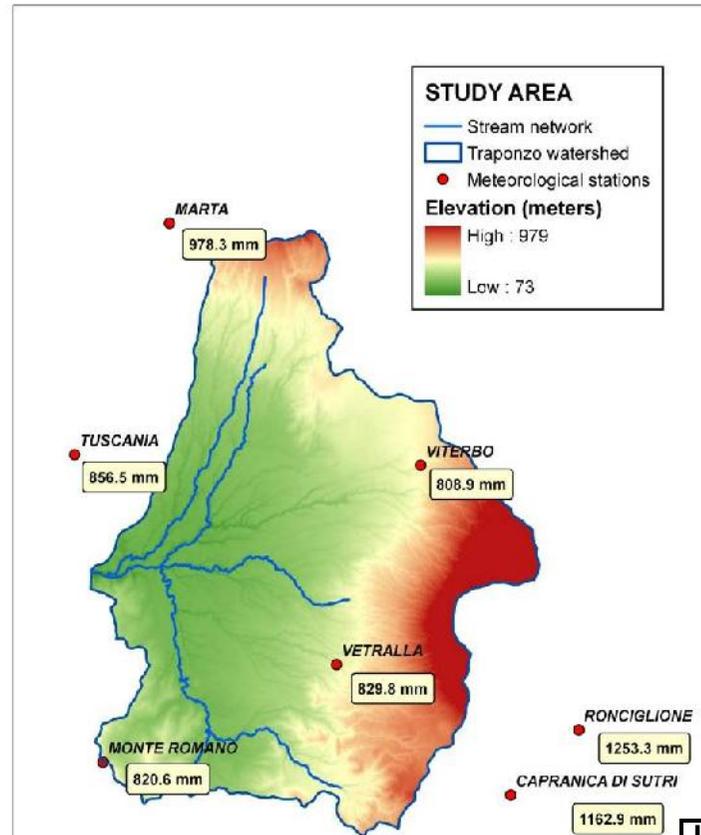
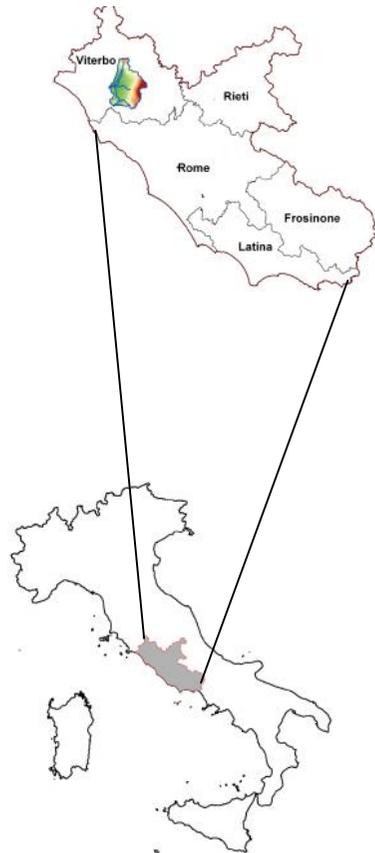
An innovative procedure, called PANDORA, Procedure for mAthematical aNalysis of lanDscape evOlution and equilibRium scenarios Assessment, is presented to assess the effects of different planning strategies on final hypothetical stable energetic equilibrium states.

An application model is proposed here as a Decision Support System for choosing among possible urban sprawl planning strategies in a Mediterranean watershed of Central Italy.

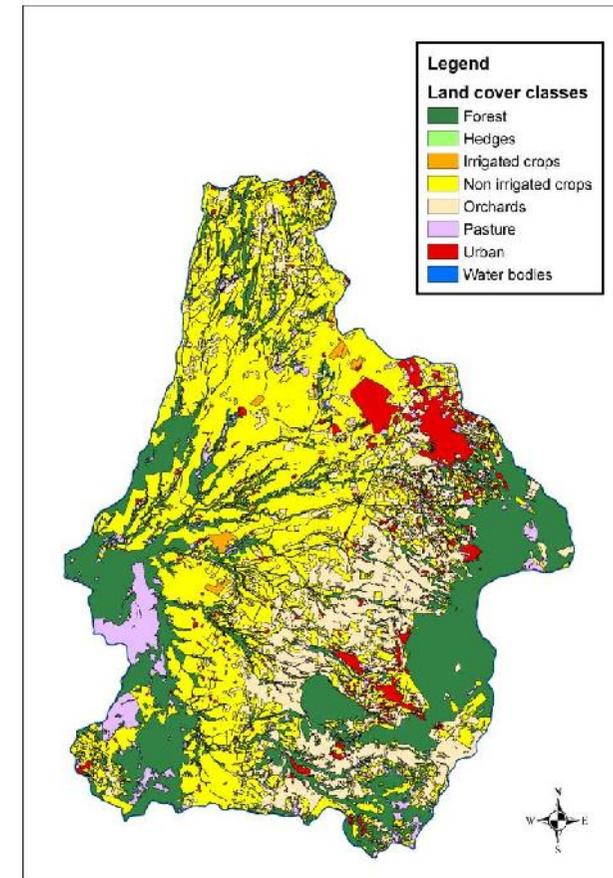


# Study area

## Lazio Region



Study area statistics	
Total area (km <sup>2</sup> )	475
Mean slope (%)	9.8
Maximum elevation (m)	979
Minimum elevation (m)	73
Mean elevation (m)	301.72
Mean annual precipitation (mm)	971.6
Mean annual temperature (°C)	15



Land cover class	Area (km <sup>2</sup> )	%
Urban	21.02	4.4
Forest	126.36	26.6
Non irrigated crops	203.04	42.8
Pasture	22.01	4.6
Orchards	88.17	18.6
Irrigated crops	2.49	0.5
Hedges	11.77	2.5
Water bodies	0.02	0.0
<b>Total area</b>	<b>474.89</b>	<b>100.0</b>

**Methods**

# Methods

- 1. Landscape Units Individuation**
- 2. Landscape Energy and Fluxes**
- 3. Model Implementation and Scenarios**
- 4. Analysis of Results and Strategy Definition**

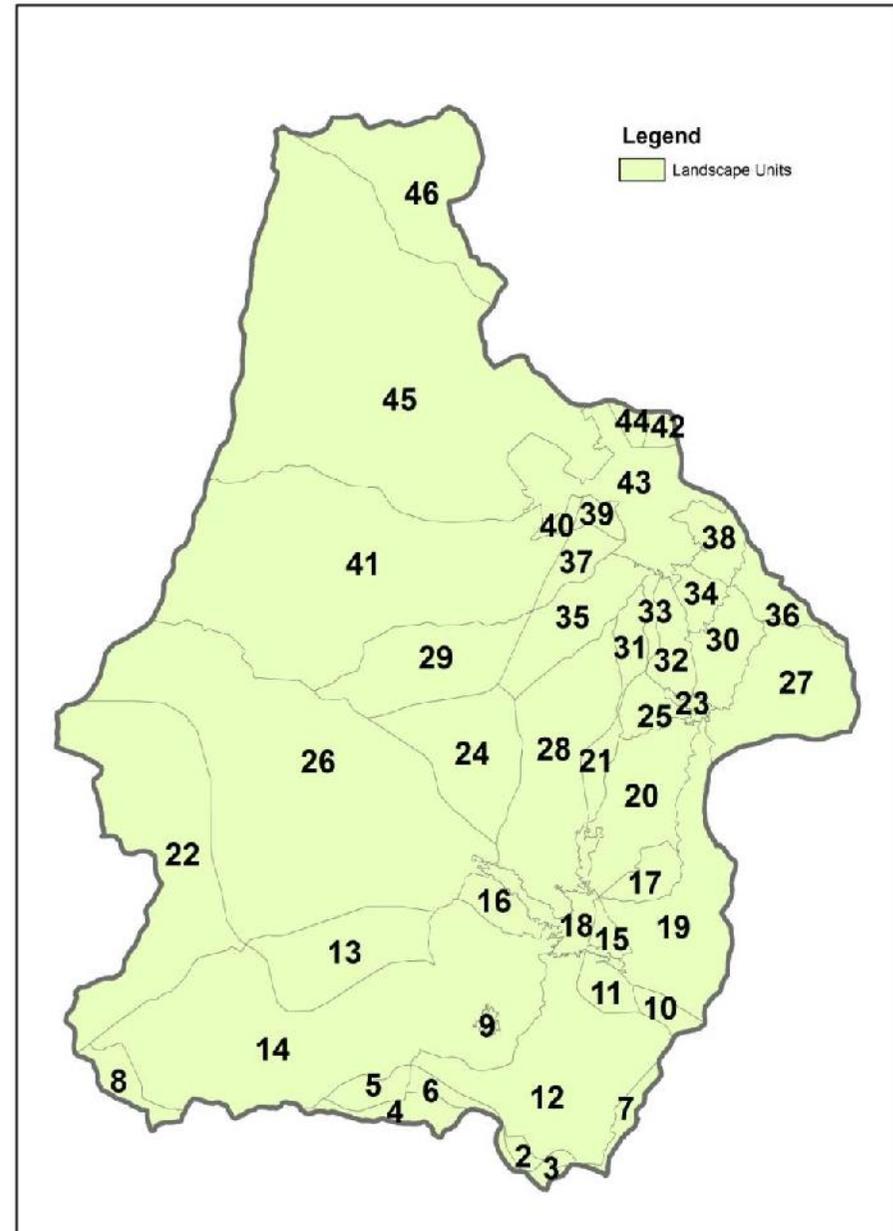
## 1. Landscape Units Identification

LUs were pointed out by means of holistic classification method (Van Eetvelde and Antrop, 2009).

The most important factors, representing barriers to energy fluxes, were weighted (Saaty matrix) and finally used to individuate LUs.

In order of importance the used barriers were:

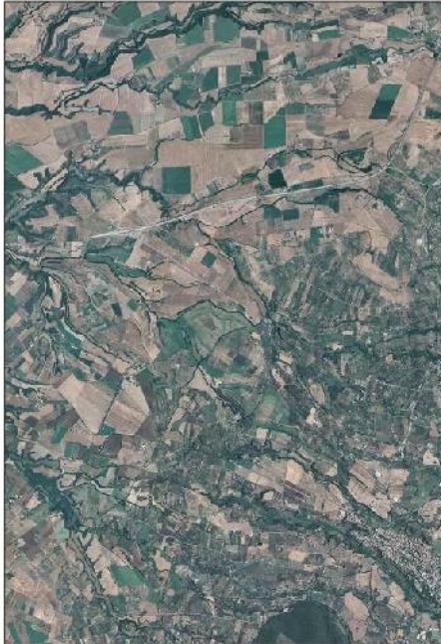
- 1) Main roads and railways
- 2) Lines of change between very different soil types
- 3) Limits between hill and mountain areas



46 Landscape Units  
Minimun LU 0.36 Km<sup>2</sup>  
Maximun LU 29.26 Km<sup>2</sup>

## 2. Landscape Energy and Fluxes

*Land cover*



The energy flow between LUs can be derived from Biological Territorial Capacity, BTC, (Ingegnoli, 2002) through the definition of a **Generalized Biological Energy** or **Bio-Energy** as the available energy for each LU.

**BTC, Biological Territorial Capacity  
Ingegnoli (2002)**

$$M_j = (1 + K_j) \cdot B_j$$

**Generalized Biological  
Energy or Bio-Energy**

**LU characteristics  
(energy diversity, shape, climatic conditions,.....)**

$$K_j = (K_j^S + K_j^P + K_j^D + K_j^C + K_j^E) / 5$$

***M* is the energy available for exchange between LUs** and it depends on several intrinsic characteristics of each LU such as energetic diversity inside it, barriers in it, shape, climatic conditions, permeabilities of the boundaries and so on.

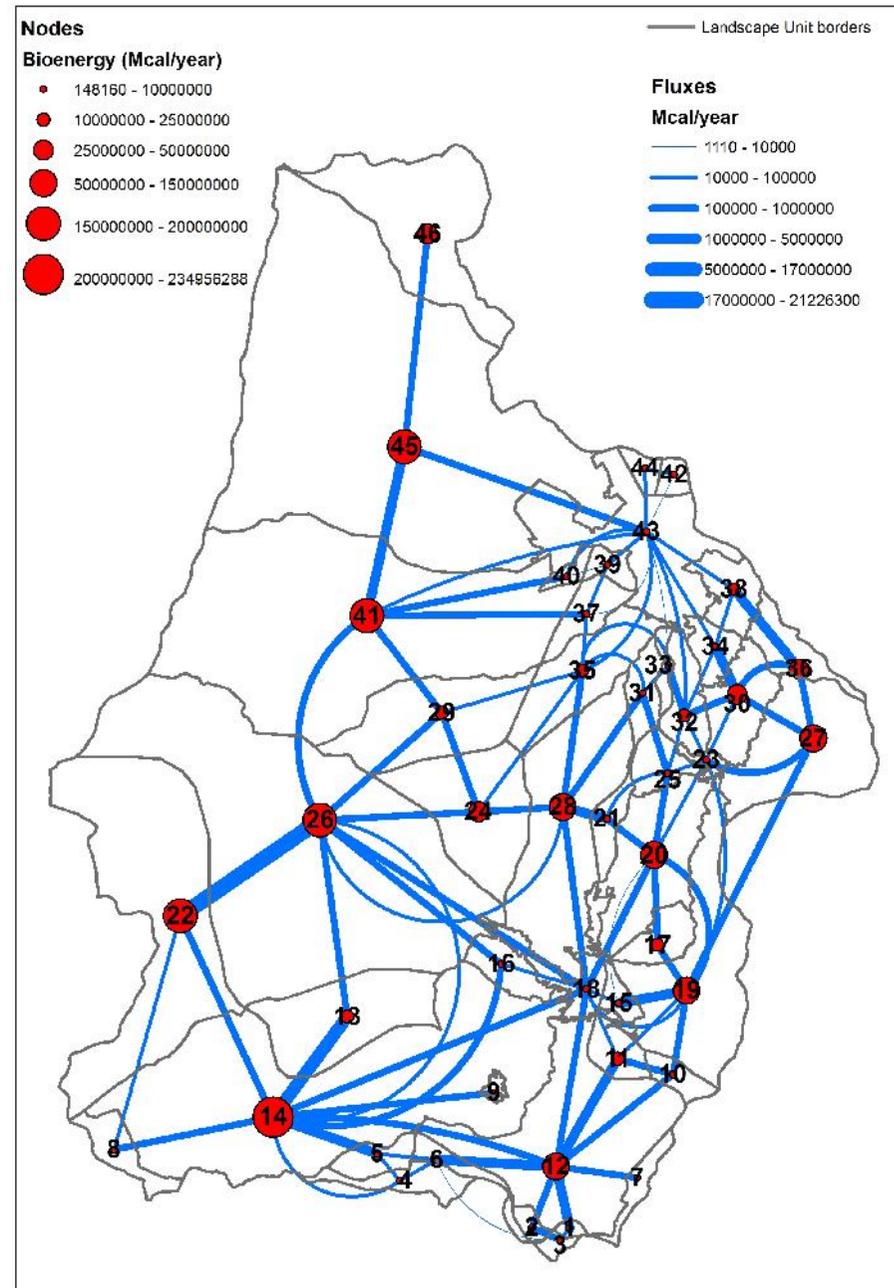
## 2. Landscape Energy and Fluxes

### Landscape Graph

- Barriers with different degrees of permeability to Bio-Energy flux.
- Bio-Energy  $M$  for each LU represented by proportional **nodes**.
- Energy exchange between LUs depends on the degree of permeability of the barriers.
- Connection between LUs represented by **arcs**, whose thickness is proportional the magnitude of the energy flux between LUs

$$F_{ij} = \frac{M_i + M_j}{2} \cdot \frac{L_{ij} p_{ij}}{P_i + P_j}$$

$M_i$  and  $M_j$  and are the Generalized Biological Energies respectively correspondent to LU- $i$  and LU- $j$ ,  $L_{ij}$  is the length of the boundary between LU- $i$  and LU- $j$  and  $P_i$  and  $P_j$  are respectively the perimeters of LU- $i$  and LU- $j$ .  $p_{ij} \in [0;1]$  is the mean permeability index of such a boundary.



**More energy exchange more biodiversity is ensured**

Analysis of M and V variation in time  $t$  by means of two logistic differential equations

$$M'(t) = cM(t) \left[ 1 - \frac{M(t)}{M_{\max}} \right] - k[1 - V(t)]M(t),$$

$$V'(t) = b_T V(t)[1 - V(t)] - h U_0 V(t),$$

$V(t)$  = fraction of the total territory occupied by areas with high values of BTC

$M(t)$  = the Generalized Biological Energy derived from BTC values and intrinsic characteristics of each LU

- $U_0$  depends on urban areas (compact and sprawl)
- $h$  depends on urban perimeters (compact and sprawl)
- $k$  depends on barriers permeabilities
- $b_t$  depends on BTC values

**Connectivity Index**

$$c = \frac{\Lambda}{\Lambda_{\max}} \cdot \frac{\bar{F}}{\bar{F} + \sigma_F} = \frac{1}{\Lambda_{\max}} \sum_{l=1}^{\Lambda} \frac{F_l}{\bar{F} + \sigma_F}$$

Where  $\Lambda$  is the number of existent fluxes,  $\Lambda_{\max}$  is the maximum number of possible fluxes calculated as  $3 \cdot (m-2)$  (Ingegnoli, 2002; Finotto et al. 2010) where  $m$  is the number of LUs,  $F_l$  represents the fluxes ordered by  $l$  index,  $\bar{F}$  is the mean energy flux of the system and  $\sigma_F$  is the standard deviation for the statistical distribution of fluxes.

### 3. Model Implementation and Scenarios

$V(t)$ = percentage of areas with high values of BTC

$M(t)$ = Generalized Biological Energy or Bio-Energy

EQUILIBRIUM SOLUTIONS		STABILITY ANALYSIS		ENVIRONMENTAL SETTLEMENT	
$V_e$	0	$c < k$	&	$b_t < h \times U_0$	This equilibrium configuration corresponds to a territorial settlement with a progressive lack of areas at high value of bio-potentiality ( $V_e=0$ ) without bio-energy production and diffusion.
$M_e$	0				
$V_e$	$(b_t - hU_0)/b_t$	$h \times k \times U_0 > c \times b_t$	&	$b_t > h \times U_0$	The system keeps areas at high value of bio-potentiality but they are isolated in landscape pattern and the fluxes of bio-energy between them are limited.
$M_e$	0				
$V_e$	0	$k < c$	&	$b_t < h \times U_0$	This equilibrium configuration corresponds to a territorial settlement with a lack of areas at high value of bio-potentiality ( $V_e=0$ ) but the low impermeability of barriers allows a positive flux of bio-energy.
$M_e$	$(M_{\max}[c - k(1 - V_e)])/c$				
$V_e$	$(b_t - hU_0)/b_t$	$c \times b_t > h \times k \times U_0$	&	$b_t > h \times U_0$	The environmental system is characterized by a great presence of areas at high value of bio-potentiality with a high production of bio-energy.
$M_e$	$(M_{\max}[c - k(1 - V_e)])/c$				

- $U_0$  depends on urban areas (compact and sprawl)
- $h$  depends on urban perimeters (compact and sprawl)
- $k$  depends on barriers permeability
- $b_t$  depends on BTC values

$$c = \frac{\Lambda}{\Lambda_{\max}} \cdot \frac{\bar{F}}{\bar{F} + \sigma_F} = \frac{1}{\Lambda_{\max}} \sum_{l=1}^{\Lambda} \frac{F_l}{\bar{F} + \sigma_F}$$

**Connectivity Index**  $c \in [0; 1]$

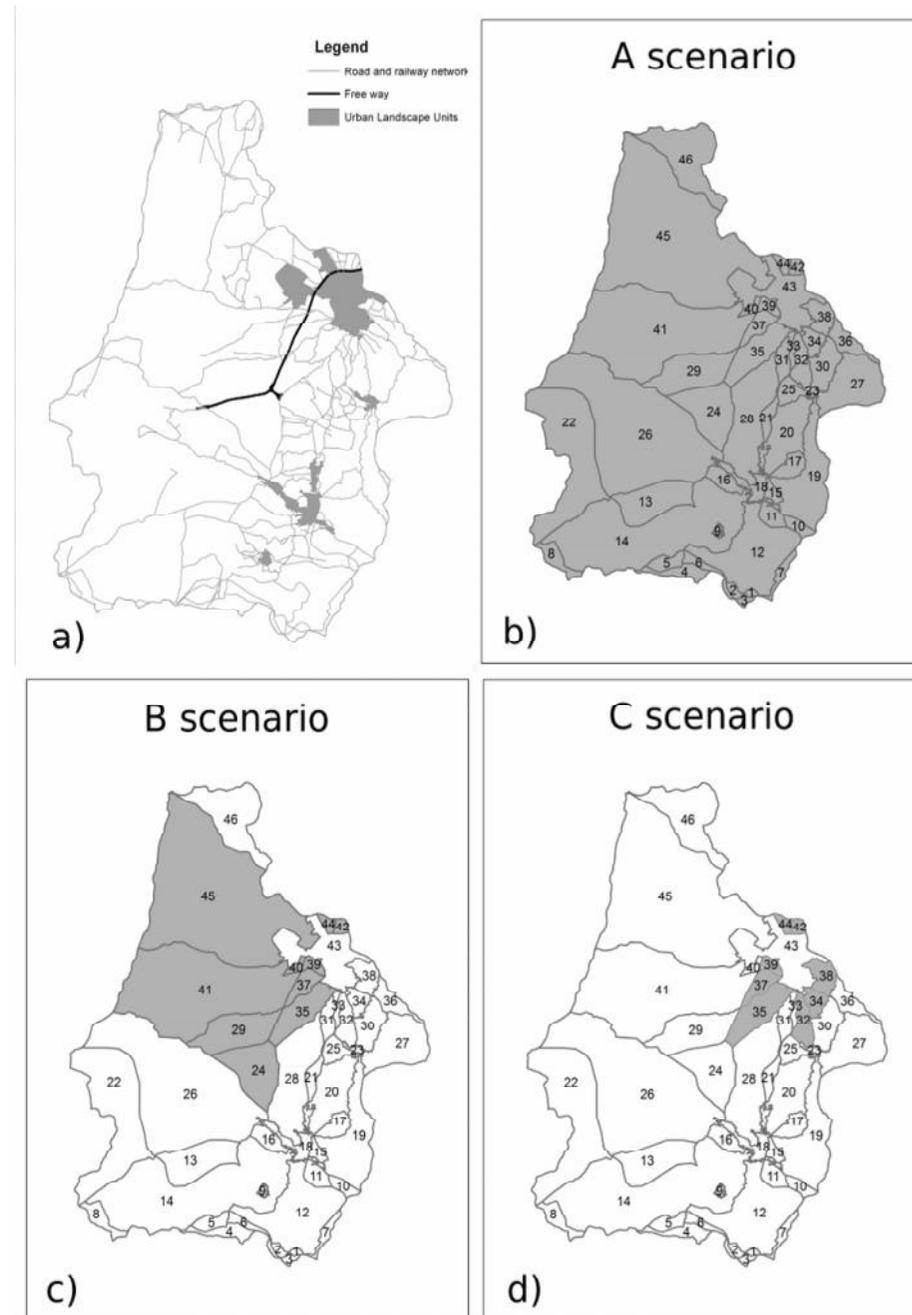
### 3. Model Implementation and Scenarios

**Scenario A:** a urban development spread all over the whole study area (the urban sprawl proceeds in each landscape unit on the basis of the area occupied by the actual urban sprawl)

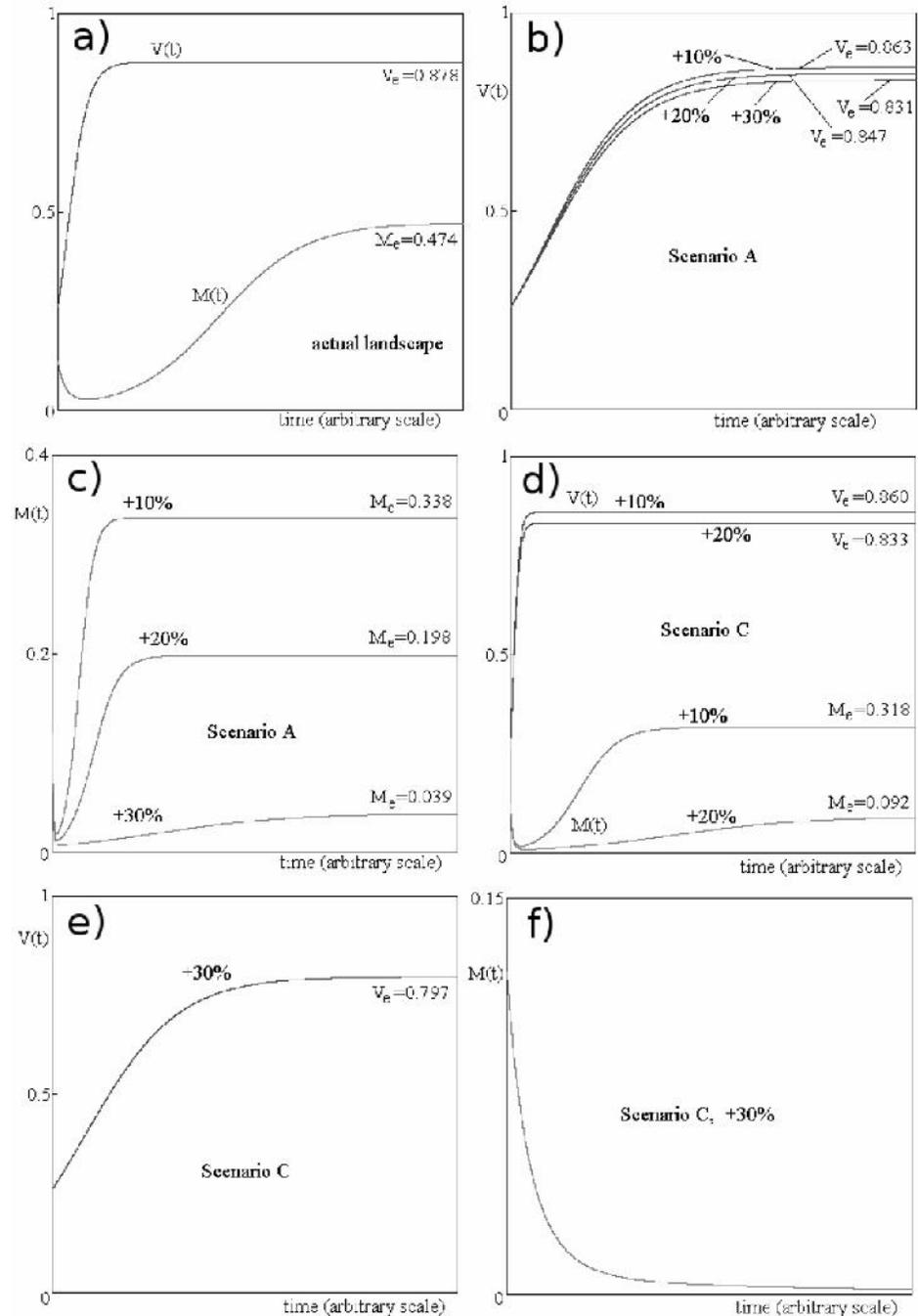
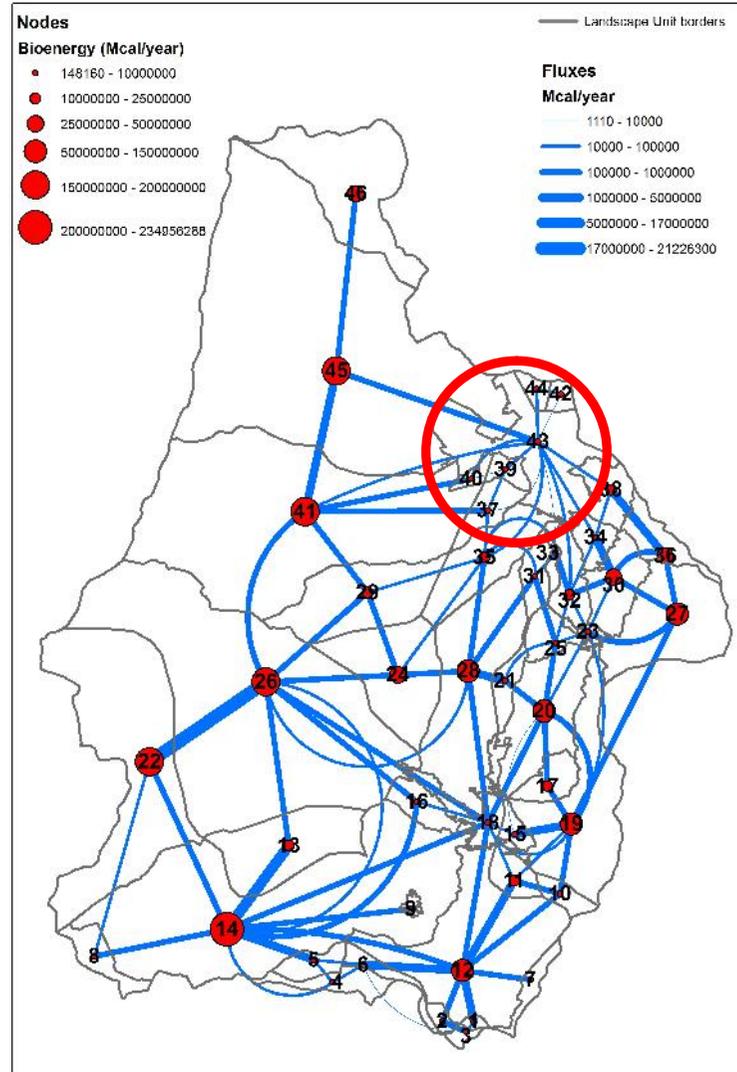
**Scenario B:** a future urban sprawl concentrated in the LUs that are closest to the main roads such as the Orte-Civitavecchia freeway.

**Scenario C:** a urban development only around the bigger city of Viterbo.

*Three different (10, 20 and 30 %) urban growth increasing rates were hypothesized for each scenario (A, B, C) giving place to 9 sub-scenarios.*



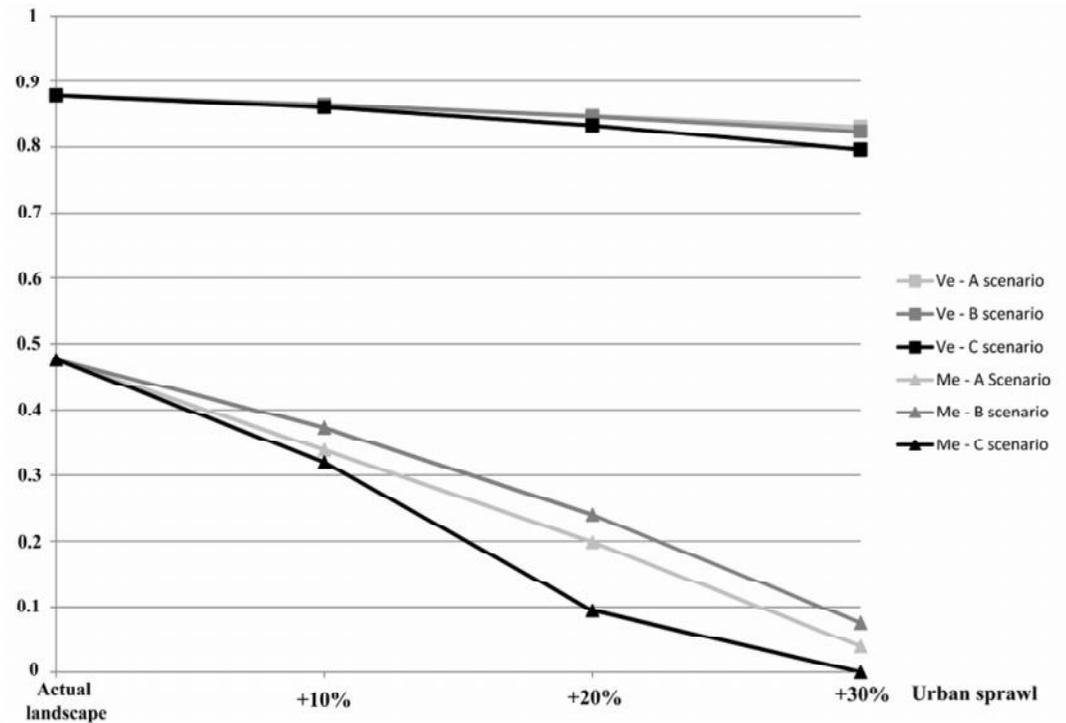
## 4. Analysis of Results and Strategy Definition



$M_e^*$  is the equilibrium value for  $M$  normalized by the maximum value of  $M$  ( $M_{max}$ ).

## 4. Analysis of Results and Strategy Definition

*V(t) and M(t) equilibrium values for the considered 9 sub-scenarios.*



**Model parameters and equilibrium solutions**

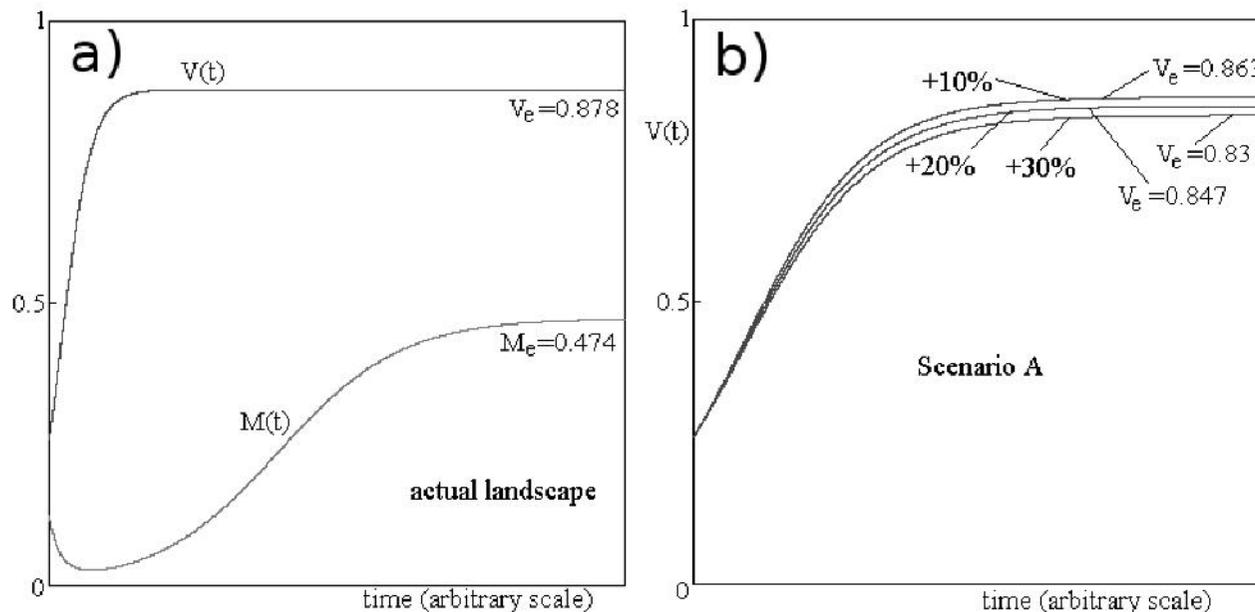
Parameter	Actual landscape	Scenario A			Scenario B			Scenario C		
		+ 10%	+ 20%	+ 30%	+ 10%	+ 20%	+ 30%	+ 10%	+ 20%	+ 30%
$V_o$	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
$b_t$	0.37000	0.36466	0.36388	0.36382	0.3699	0.36988	0.36982	0.36994	0.36988	0.36982
$M_{max}$ (Mcal/year)	304403079.3	304381538.1	304359996.9	304338455.6	304403079.3	304403079.3	304403079.3	304403079.3	304403079.3	304403079.3
$B$ (Mcal/year)	24446564.76	24093940.77	24041949.6	24037888.29	24439930.52	24438507.15	24434478.35	24442736.13	24438507.15	24434478.35
$M_o$ (Mcal/year)	37093444.05	36528628.75	36450844.93	36444699.56	37083468.38	37081218.62	37075105.91	37087338.36	37081331.44	37075275.13
$U_o$	0.02295	0.02296	0.02297	0.02298	0.02296	0.02297	0.02298	0.02297	0.02301	0.02305
$h$	1.95739	2.17789	2.41938	2.68187	2.18652	2.47953	2.83642	2.25313	2.69157	3.27269
$k$	0.71363	0.78203	0.85043	0.91883	0.76504	0.81646	0.86788	0.80447	0.89532	0.98616
$c$	0.16543	0.16205	0.16200	0.16199	0.16542	0.16542	0.16541	0.16541	0.16540	0.16539
$V_c$	0.87857	0.86286	0.84727	0.83060	0.86427	0.84601	0.82371	0.86007	0.83259	0.79605
$Me^*$	0.47618	0.33819	0.19824	0.03915	0.37228	0.23992	0.07506	0.31949	0.09381	0.00000

*Me\* is the equilibrium value for M normalized by the maximum value of M (Mmax)*

## Considerations

-M, the Generalized Biological Energy, based on BTC index and depending on landscape features, **is an attempt to quantify the efficiency in energy transmissions for ecosystems** since it accounts for flows of available energy between LUs.

-The increasing trend of V implies a natural expansion of areas characterized by high value of biological energy (i.e. high BTC values) and, as a consequence, ***high metabolic rate***: the species that are most effective in consuming energy are, therefore, naturally selected according to the 2nd law of thermodynamics (Wurtz and Annala, 2010).



## Considerations

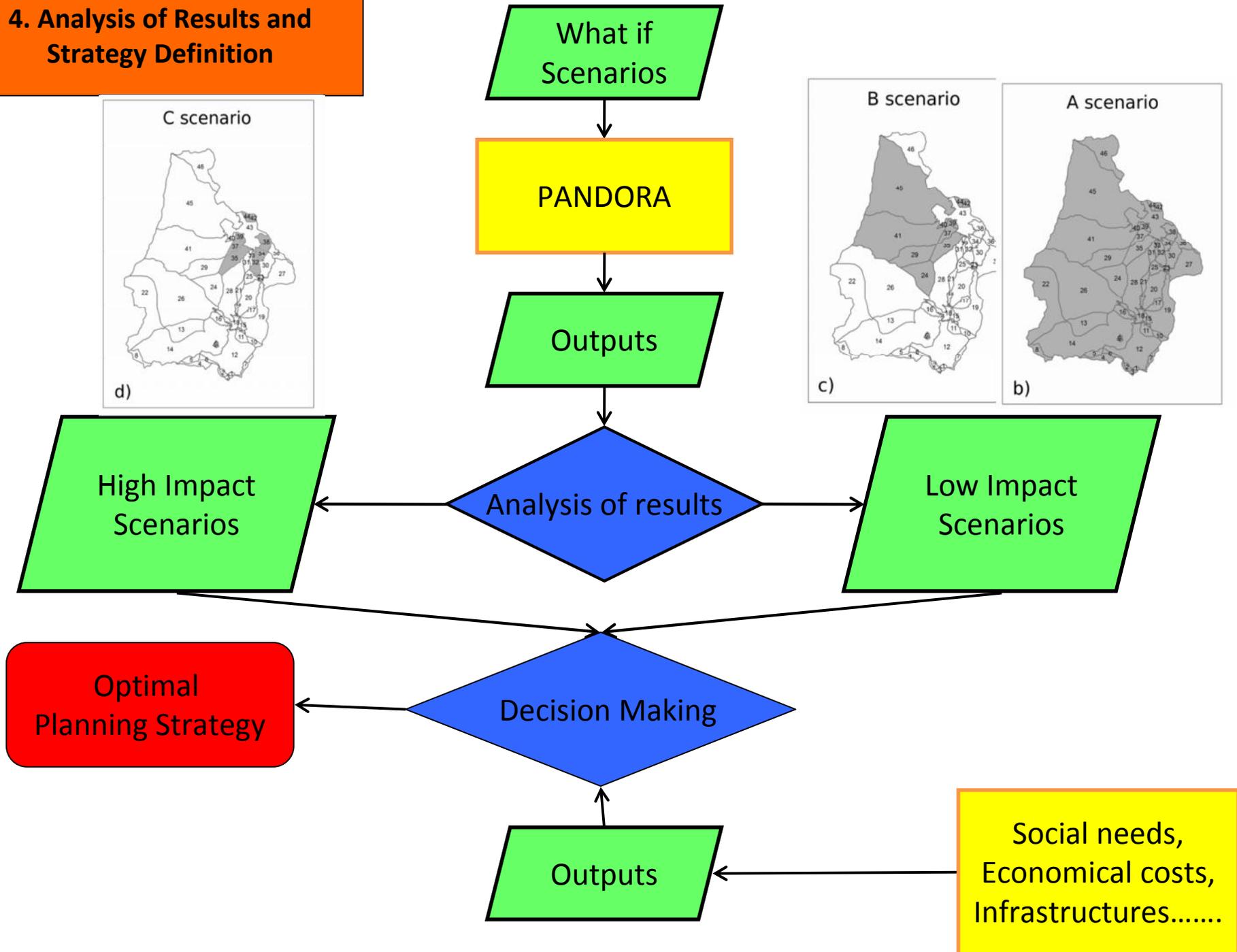
-V and M equilibrium values can be seen as ***indices to evaluate energetic equilibrium conditions of different scenarios*** and their impact on landscape.

-***The c connectivity index plays a key role in defining the equilibrium state of a landscape*** and the correspondent equilibrium values for V and M: it reflects the ***great importance of ecological connection in defining the health of an ecosystem in terms of biodiversity and available energy.***

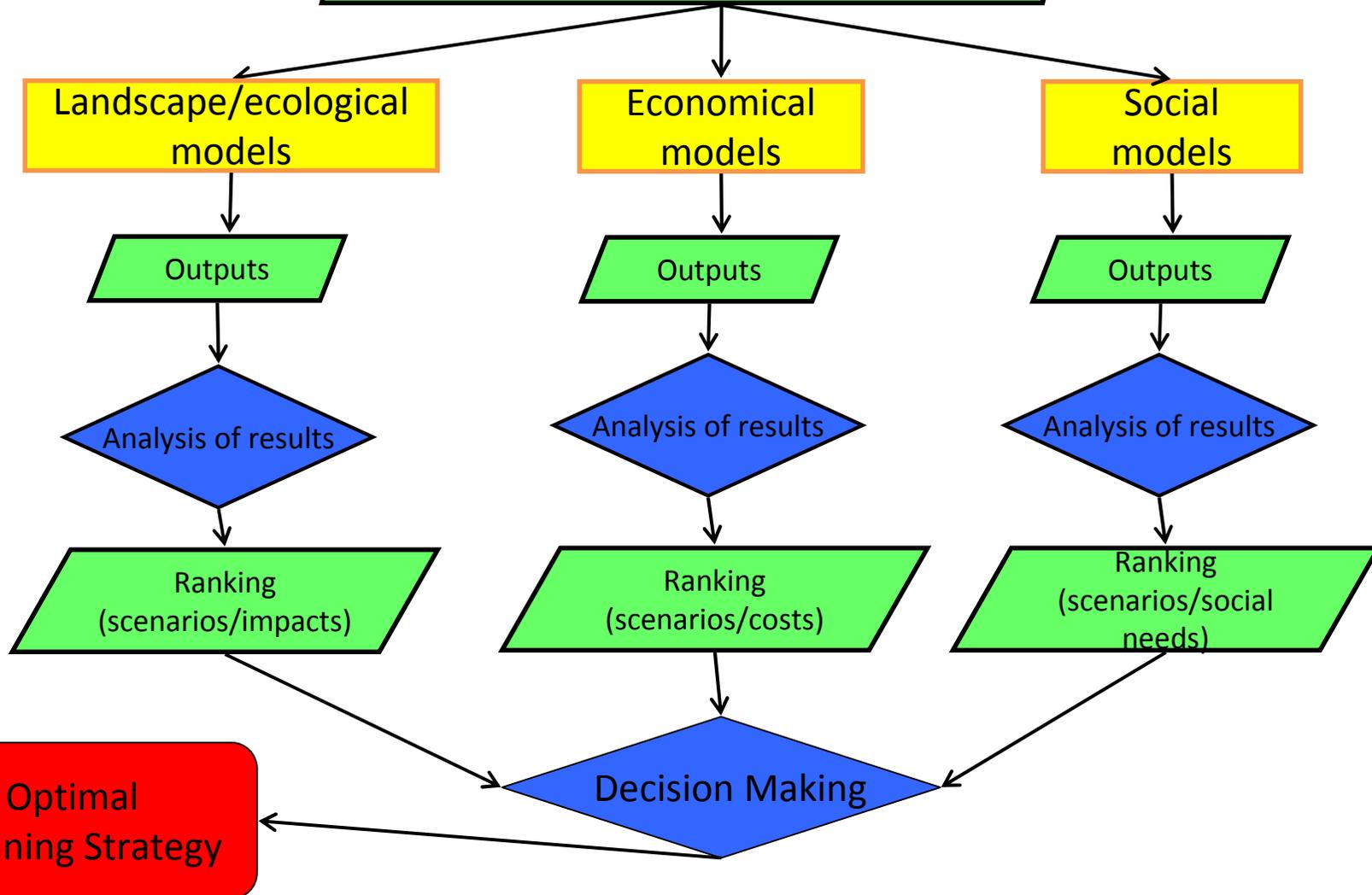
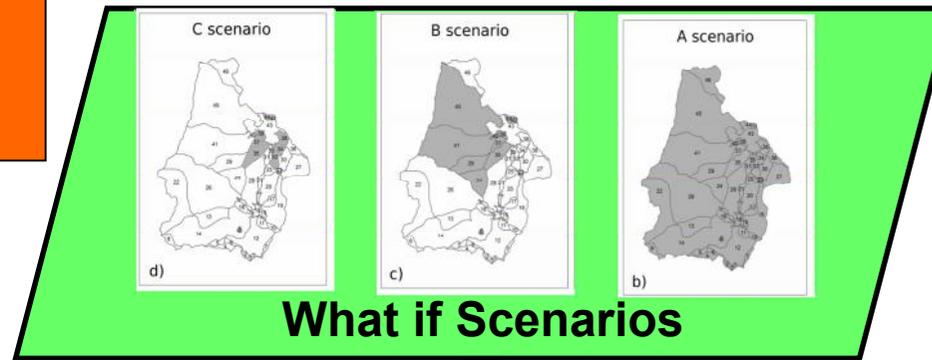
-***Infrastructure and services costs are excluded from PANDORA model*** even if the road network has been enriched proportionally to the increase in urban cover to account for new roads serving the considered new housing schemes corresponding to A, B, C scenarios.

-Model parameters (c, h, k,  $U_0$ ) can be used as ***indices to characterize landscape structure (landscape metrics).***

#### 4. Analysis of Results and Strategy Definition



#### 4. Analysis of Results and Strategy Definition



## Conclusions

- PANDORA model is an attempt to study equilibrium conditions for landscapes ***analysing spatial data***: it works with two global variables and finds mathematical solutions for landscapes evolution equations depending on parameters that can be obtained from ***GIS data***, available, usually, by land managers.
- Even if the quantitative result could not be so significant in itself, ***the comparison between PANDORA outputs*** from different management hypotheses could be a reliable tool for planning in terms of ***scenarios analysis***
- It aims to represent an innovative approach for the evaluation of “what if” scenarios for land planning ***linking together the thermodynamic concepts, mathematical equilibrium, metabolism theory and landscape metrics***.
- Improvements and adjustments have to be reached in order to make this model much more automated. Other study cases and applications have to be encouraged to refine the introduced methodology and to compare the obtained results.



*Thank you for your attention!*

**Federica Gobattoni**

*DAF-Department of Environment and Forestry  
University of Tuscia, Viterbo (Italy)*

*Email: [f.gobattoni@unitus.it](mailto:f.gobattoni@unitus.it)*

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**PANDORA: Procedure for mathematical Analysis of landscape evolution and equilibrium scenarios Assessment.**