# On the role of non-normality in the overreflection of gravity waves


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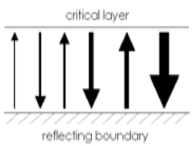
## 1. Introduction

- The big picture — comprehensive understanding of stratified shear instability

- Instability is a result of overreflection
  - gravity waves propagating towards their critical layer are overreflected
  - waves are reflected back by a containing surface
  - continuous overreflection leads to instability
  - accurate prediction of the growth rate [1]



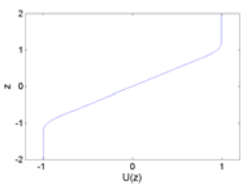

- Overreflection
  - necessary conditions are identified [1]
  - mechanism involves transient growth but is not known

- Our purpose — identify the mechanism underlying overreflection

- Our method — Generalized Stability Theory (GST) [2]

## 2. Model description

- We use a 2d linear model in the Boussinesq approximation of a finite shear layer




$$\left(\partial_t + ikU(z)\right)\nabla^2\psi - ik\psi\partial^2_{zz}U = ikRi\rho + \frac{1}{\mathrm{Re}}\nabla^4\psi$$

$$\left(\partial_t + ikU(z)\right)\rho + ik\psi = \frac{1}{\mathrm{Re}}\nabla^2\rho$$

$$\vec{U} = U(z)\hat{i}$$

$$[u,w] = \left[\partial_z\psi, -\partial_x\psi\right]$$

*Fig. 1 : Velocity profile of the shear layer.*

## 3. Overreflection: a three stage process

- We launch an upward propagating wavepacket and study its evolution

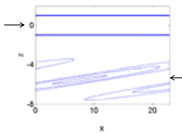



*Fig. 2 : Initial wavepacket (contours of streamfunction).*

- Stage one — incident wave excites non-locally perturbations inside the shear layer

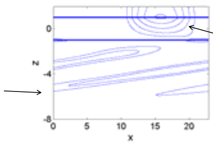



*Fig. 3 : Snapshot of streamfunction at t=120.*

- Stage two — perturbation energy increases due to the modified Orr mechanism

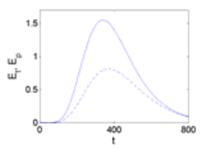



*Fig. 4 : Kinetic (solid) and potential (dashed) energy evolution.*

- Stage three — excitation of reflected wave at the lower region of the shear layer

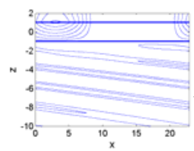



*Fig. 5 : Snapshot of streamfunction at t=700.*

## 4. Non-normal mode projection

- We write the system in the form: $\frac{d\mathbf{x}}{dt} = \mathbf{A}(U)\mathbf{x}$, where $\mathbf{x} = [\psi\ \rho]^T$

- Spectrum of linear operator **A**

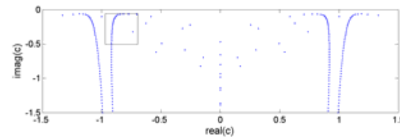



*Fig. 6 : Real and imaginary part of the eigenvalues of* **A** *for $k=1$, $Ri=0.2$, $Re=10^3$*

- Overreflection: projection of the adjoint modes on the analytic modes of **A**

$$\mathbf{x}(t) = \Sigma a_i e^{-ikc_i t}\mathbf{u}_i \text{ , } \mathbf{u}_i \text{ eigenmodes , } a_i = \frac{\left[\mathbf{v}_i^+, \mathbf{x}(0)\right]}{\left[\mathbf{v}_i^+, \mathbf{u}_i\right]} \text{ , } \mathbf{v}_i \text{ eigenmodes of } \mathbf{A}^+$$

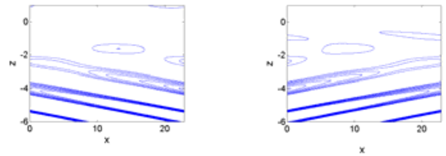



*Fig. 7: Streamfunction of one of the leading normal modes (left) and its adjoint (right)*

## 5. Conclusions

- Overreflection exhibits the characteristics of stimulated emission: non-local excitation of perturbations by incident wave, growth of perturbations and excitation of reflected wave in the lower region of the shear layer

- Modified Orr mechanism produces the observed growth

- Overreflection is a result of the non-normal interaction of the analytic normal modes


References

[1] Lindzen, 1988, *Pure Appl. Geophys.*, **126**, 103-121

[2] Farrell and Ioannou, 1996, *J. Atmos. Sci.*, 53, 2025-2041

**Bakas and Farrell, 2010, *J. Atmos. Sci.*, 67, 2547-2558**



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