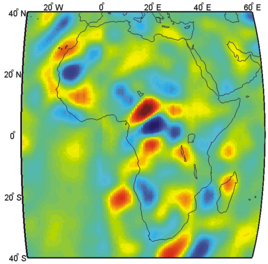
Technische Universität Kaiserslautern



Modeling the Earth's magnetic field by local multiscale methods

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EGU General Assembly 2011, Session on Planetary Magnetism

GeoMathematik

Scaling- and Wavelet-Kernels:

$$\Phi_J(\xi,\eta) = \sum_{n=0}^{\infty}\sum_{k=0}^{2n+1} (\Phi_J)^{\wedge}(n) Y_{n,k}(\xi) Y_{n,k}(\eta),$$

$$\Psi_J(\xi,\eta) = \Phi_{J+1}(\xi,\eta) - \Phi_J(\xi,\eta).$$

Scaling- and Wavelet-Kernels:

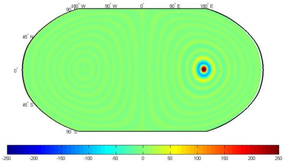
$$\Phi_J(\xi,\eta) = \begin{cases} \text{weak sing.,} & 1-\xi\cdot\eta \geq 2^{-J}, \\ \text{regularization,} & 1-\xi\cdot\eta < 2^{-J}, \end{cases}$$

$$\Psi_J(\xi,\eta) = \Phi_{J+1}(\xi,\eta) - \Phi_J(\xi,\eta).$$

Multiscale Representation:

$$F(\xi) = \int_{\Omega} \Phi_{J_0}(\xi,\eta) G(\eta) d\omega(\eta) + \sum_{J=J_0}^{\infty} \int_{\Omega} \Psi_J(\xi,\eta) G(\eta) d\omega(\eta).$$

Shannon Wavelet (J=4):



1. Bayer, Freeden, Maier: J. Atm. Solar-Terr. Phys. 63 (2001)
2. Freeden, Schreiner: J. Geod. 78 (2006)
3. Holschneider, Chambodout, Mandea: Phys. Earth Plant. Int. 135 (2003)

Radial Currents

Mie-Decomposition:

$$b = p_b + q_b = \nabla \wedge LP_b + LQ_b,$$
$$j = p_j + q_j = \nabla \wedge LP_j + LQ_j.$$

Toroidal Scalar:

$$Q_b(x) = -\int_\Omega L^*_\eta G(\Delta^*;\xi\cdot\eta)\cdot b(r\eta)d\omega(\eta).$$

Helmholtz-Mie-Kombination:

$$j(x) = \xi \underbrace{\frac{\Delta^*_\xi Q_b(x)}{r}}_{=J_{rad}(x)} + \nabla^*_\xi\left(\frac{1}{r}\frac{\partial}{\partial r}(rQ_b(x))\right) + L^*_\xi \Delta_x P_b(x), \quad r = |x|, \xi = \frac{x}{|x|}.$$

1. Maier: SIAM J. Appl. Math. 65 (2005)
2. Olsen: J. Geophys. Res. 102 (1997)

Radial Currents

Mie-Decomposition:

$$b = p_b + q_b = \nabla \wedge LP_b + LQ_b,$$
$$j = p_j + q_j = \nabla \wedge LP_j + LQ_j.$$

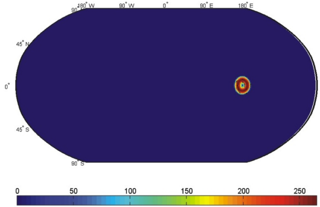
Toroidal Scalar:

$$Q_b(x) = -\int_\Omega L^*_\eta G(\Delta^*;\xi\cdot\eta)\cdot b(r\eta)d\omega(\eta).$$

Regularized Green Function:

$$G^J(\Delta^*;\xi\cdot\eta) = \begin{cases} \frac{1}{4\pi}\ln(1-\xi\cdot\eta) + \frac{1}{4\pi}(1-\ln(2)), & 1-\xi\cdot\eta \geq 2^{-J}, \\ T_3^J(\xi\cdot\eta), & 1-\xi\cdot\eta < 2^{-J}, \end{cases}$$

$$J_{rad}(x) = \lim_{J\to\infty}\int_\Omega \underbrace{\left(-\frac{1}{r}L^*_\eta\Delta^*_\xi G^J(\Delta^*;\xi\cdot\eta)\right)}_{=\phi_J(\xi,\eta)} \cdot b(r\eta)d\omega(\eta).$$



$|\phi_6(\varepsilon^2,\cdot)|$

1. Freeden, Gerhards: Math. Geosc. (2010)

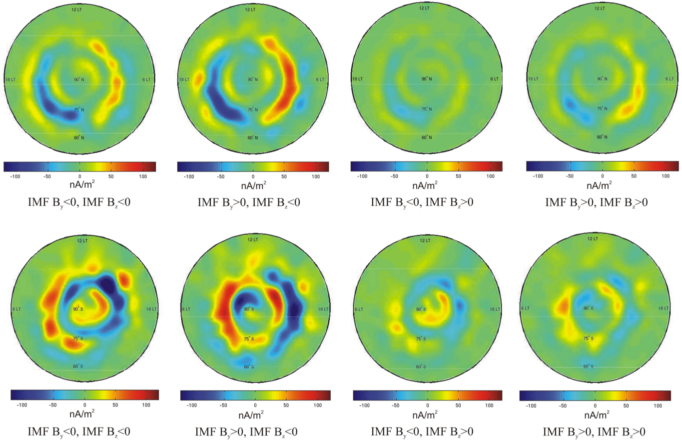
Radial Currents

CHAMP Data: Jan. 2001 - Dec. 2007

Substraction of CHAOS-3 model, Binning via Huber's M-estimation

N-Winter

S-Summer



1. Gerhards: PhD Thesis, Geomathematics Group, TU Kaiserslautern
2. Papitashvili, et al.: Geophys. Res. Lett. 29 (2002)
3. Weimer: J. Geophys. Res. 106 (2001)

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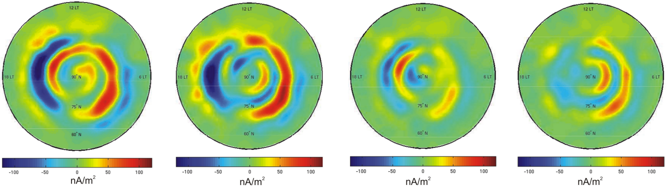
Radial Currents

CHAMP Data: Jan. 2001 - Dec. 2007

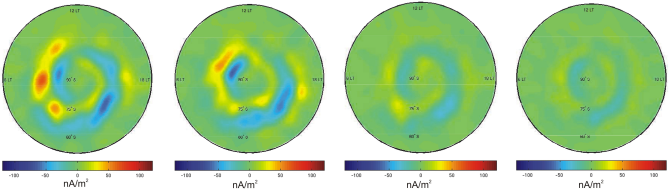
Substraction of CHAOS-3 model, Binning via Huber's M-estimation

N-Summer



IMF B_y<0, IMF B_z<0 | IMF B_y>0, IMF B_z<0 | IMF B_y<0, IMF B_z>0 | IMF B_y>0, IMF B_z>0

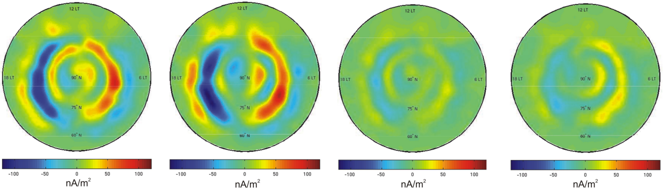
S-Winter



IMF B_y<0, IMF B_z<0 | IMF B_y>0, IMF B_z<0 | IMF B_y<0, IMF B_z>0 | IMF B_y>0, IMF B_z>0

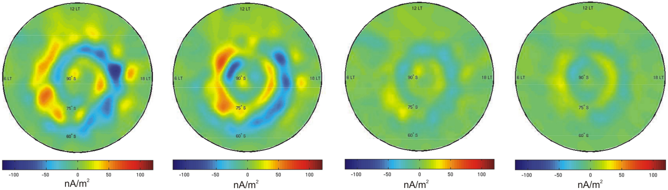
CHAMP Data: Jan. 2001 - Dec. 2007
Substraction of CHAOS-3 model, Binning via Huber's M-estimation

N-Equinox



IMF $B_y<0$, IMF $B_z<0$ | IMF $B_y>0$, IMF $B_z<0$ | IMF $B_y<0$, IMF $B_z>0$ | IMF $B_y>0$, IMF $B_z>0$

S-Equinox

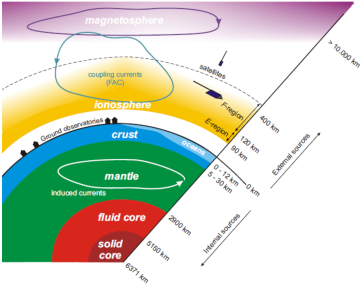


IMF $B_y<0$, IMF $B_z<0$ | IMF $B_y>0$, IMF $B_z<0$ | IMF $B_y<0$, IMF $B_z>0$ | IMF $B_y>0$, IMF $B_z>0$

Separation of Sources

Mie-Decomposition:

$$b = p_b + q_b = \nabla \wedge L P_b + L Q_b,$$
$$j = p_j + q_j = \nabla \wedge L P_j + L Q_j.$$



Separation of the Poloidal Part:

$$\nabla \wedge p_b^{int}(x) = \begin{cases} q_j(x), & x \in \Omega_R^{int}, \\ 0, & x \in \Omega_R^{ext}, \end{cases} \qquad \nabla \wedge p_b^{ext}(x) = \begin{cases} 0, & x \in \Omega_R^{int}, \\ q_j(x), & x \in \Omega_R^{ext}. \end{cases}$$

Potential Representation:

$$p_b^{int}(x) = \nabla U^{int}(x), \quad x \in \Omega_R^{ext}, \qquad p_b^{ext}(x) = \nabla U^{ext}(x), \quad x \in \Omega_R^{int}.$$

1. Backus, Parker, Constable: Foundations of Geomagnetism (1996)

Vector Spherical Harmonics:

$$\nabla_x \frac{1}{R}\left(\frac{R}{|x|}\right)^{n+1} Y_{n,k}\left(\frac{x}{|x|}\right) = -\frac{1}{R^2}\left(\frac{R}{|x|}\right)^{n+2} \tilde{o}^{(1)}_{\xi} Y_{n,k}(\xi)\,,$$

$$\nabla_x \frac{1}{R}\left(\frac{|x|}{R}\right)^{n} Y_{n,k}\left(\frac{x}{|x|}\right) = \frac{1}{R^2}\left(\frac{R}{|x|}\right)^{n-1} \tilde{o}^{(2)}_{\xi} Y_{n,k}(\xi)\,.$$

Sphärische Operatoren:

$$\tilde{o}^{(1)} = o^{(1)}\left(D + \frac{1}{2}\right) - o^{(2)}, \qquad o^{(1)}_{\xi} = \xi, \qquad D = \left(-\Delta^* + \frac{1}{4}\right)^{\frac{1}{2}},$$

$$\tilde{o}^{(2)} = o^{(1)}\left(D - \frac{1}{2}\right) + o^{(2)}, \qquad o^{(2)}_{\xi} = \nabla^*_{\xi},$$

$$\tilde{o}^{(3)} = o^{(3)}, \qquad o^{(3)}_{\xi} = L^*_{\xi}.$$

Decomposition:

$$b = p_b^{int} + p_b^{ext} + q_b = \tilde{o}^{(1)} B^{int} + \tilde{o}^{(2)} B^{ext} + \tilde{o}^{(3)} B^{q}.$$

1. Edmonds: Angular Momentum in Quantum Mechanics (1957)

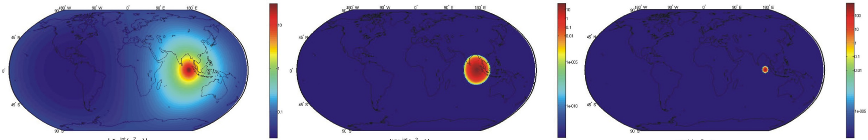
2. Mayer, Maier: Geophys. J. Int. 167 (2006)

Multiscale Representation of the Internal Magnetic Field:

$$p_b^{int}(x) = \lim_{J\to\infty} \int_\Omega \mathbf{\Phi}_J(\xi,\eta) b(r\eta) d\omega(\eta),$$

with

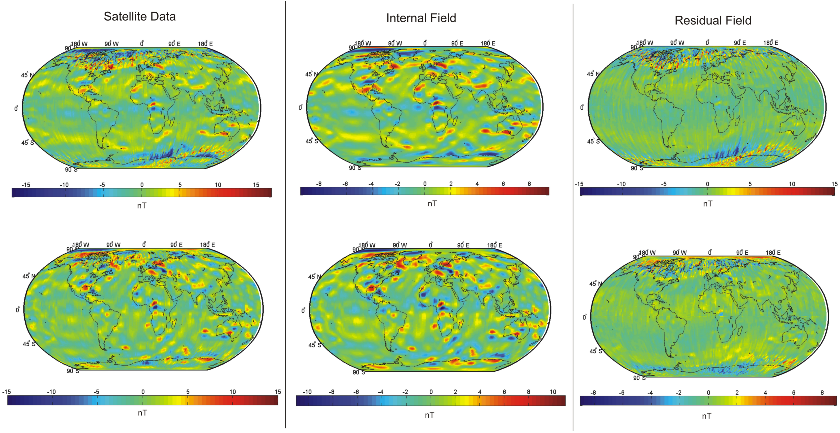
$$\mathbf{\Phi}_J(\xi,\eta) = \xi \otimes \eta \left(\frac{1}{2}\Delta_\xi^* G^J(\Delta^*;\xi\cdot\eta) + \frac{1}{8\pi} S^J(\xi\cdot\eta)\right) - \frac{1}{4\pi}\xi \otimes \nabla_\eta^* S^J(\xi\cdot\eta)$$
$$+ \frac{1}{4}\nabla_\xi^* \otimes s_{\nabla^*}^J(\eta,\xi) - \frac{1}{4\pi}\nabla_\xi^* S^J(\xi,\eta)\otimes\eta - \frac{1}{2}\nabla_\xi^* \otimes \nabla_\eta^* G^J(\Delta^*;\xi\cdot\eta).$$



$|\Phi_4^{int}(\varepsilon^2,\cdot)|$ $|\Psi_4^{int}(\varepsilon^2,\cdot)|$ $|\Psi_8^{int}(\varepsilon^2,\cdot)|$

1. Gerhards: PhD Thesis, Geomathematics Group, TU Kaiserslautern

CHAMP Data: June 2001 - Dec. 2001
Binning via Huber's M-estimation



1. Mayer, Maier: Geophys. J. Int. 167 (2006)

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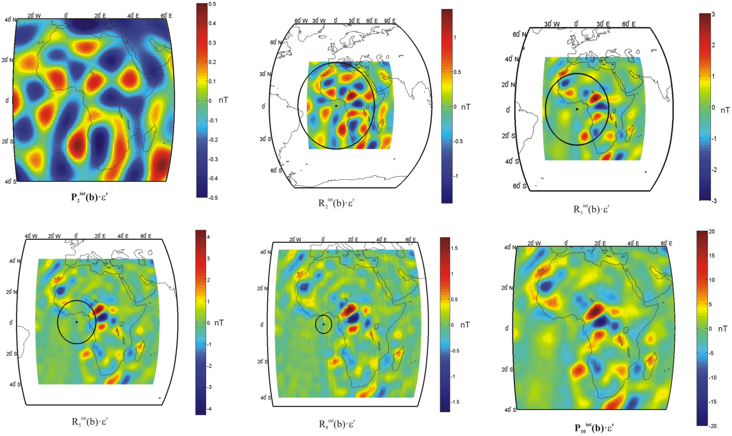
Separation of Sources

CHAMP Data: Jan. 2009 - Apr. 2010

Substraction of CHAOS-4 model, Binning via Huber's M-estimation



1. Gerhards: PhD Thesis, Geomathematics Group, TU Kaiserslautern

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