Modeling of Borehole-Guided Waves and Reservoir Formation Reflections with a Discontinuous Galerkin Finite Element Method

Alan Schiemenz[1], Martin Käser[1], Hugues Djikpesse[2], and Heiner Igel[1]
[1] Department of Earth and Environmental Sciences (Geophysics)
Ludwig-Maximilians-Universität München, Munich, Germany
[2] Schlumberger-Doll Research, Cambridge, MA, USA

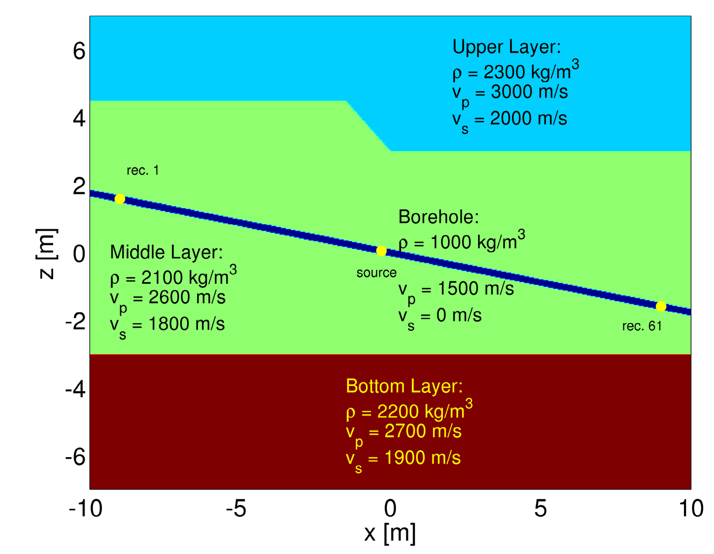
1. Introduction

We model seismic downhole sources and wave propagation in and around a wellbore with discontinuities in the surrounding rock formation. We use an abitrary high-order derivatives discontinuous Galerkin numerical scheme (ADER-DG; cf. Käser, M. and Dumbser, 2006; Dumbser and Käser, 2006) to solve the elastic wave equation.

The ADER-DG scheme allows for high approximation order in space and time using unstructured triangular and tetrahedral meshes to account for geometrically complex computational domains as typically encountered in realistic reservoir applications. We take advantage of the flexible mesh refinement strategy of the DG modeling approach to study how borehole-guided waves affect the amplitudes of seismic reflections that occur from layered media. Local time stepping is applied, greatly reducing the numerical cost that would otherwise be necessary for global time stepping in a system with largely varying element sizes (Dumbser et al., 2007).

In this study we use monopole and dipole sources to compare the synthetic wave forms with respect to their sensitivity to discontinuities and small faults in the surrounding rock formation. In comparison to the conventional, axisymmetric monopole source, dipole sources are asymmetrically oriented and are capable of S-wave velocity logging regardless of the formation type (Chen, 1988). Preliminary results suggest that a monopole source is more effective in revealing reservoir reflections.

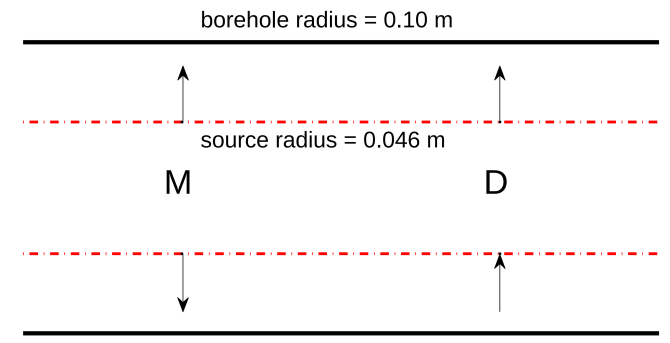
2. Physical Model : Inclined Borehole



- 20 cm-wide fluid-filled borehole inclined at 10°
- Upper and lower layers with discontinuous material properties can represent small faults or reservoir boundaries
- **Fast formation**, i.e.

$$v_{s,\text{middle layer}} > v_{p,\text{borehole}}$$

- 5000 Hz Ricker pulse source
- 61 receivers spaced 0.3048 m apart, located on both sides of source
- Schematic of monopole (M) and dipole (D) sources :



3. Numerical Method : ADER-DG

Elastic wave equations in an isotropic, non-attenuating medium in velocity-stress form are written as a linear hyperbolic system:

$$\frac{\partial \mathbf{Q}_p}{\partial t} + A_{pq}\frac{\partial \mathbf{Q}_q}{\partial x} + B_{pq}\frac{\partial \mathbf{Q}_q}{\partial y} + C_{pq}\frac{\partial \mathbf{Q}_q}{\partial z} = \mathbf{s}_p. \quad (1)$$

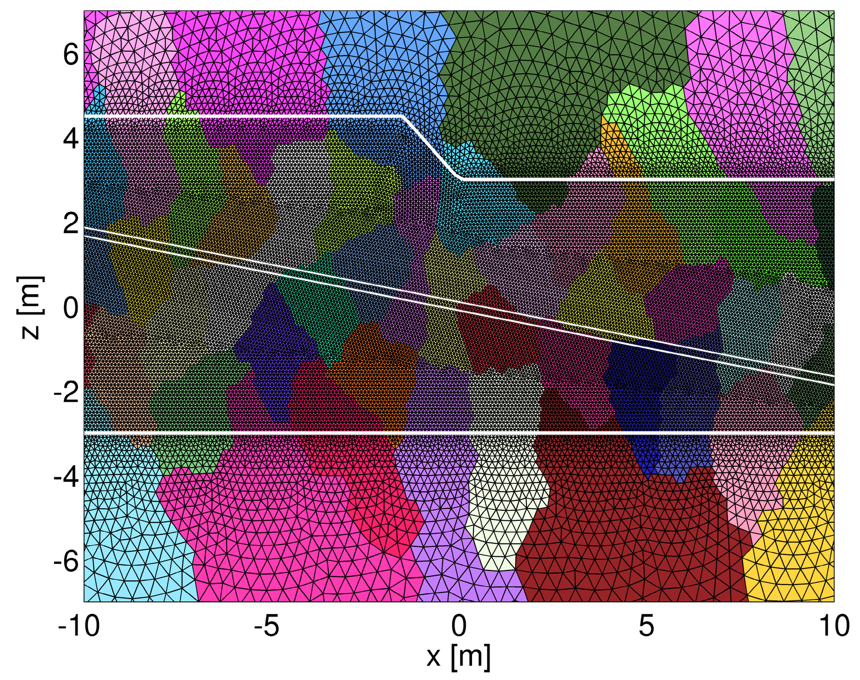
- $\mathbf{Q}$ is the solution array: 6 independent stresses and 3 independent velocities in 3-D; 4 stresses and 2 velocities in 2-D
- A, B, C are matrices containing medium properties (v_p,v_s,ρ)
- $\mathbf{s}_p$ are source terms (monopole, dipole, quadrupole)

Assuming a local coordinate system $\overrightarrow{\xi}$, the discrete solution representation to component p on element m is expanded in modal form as

$$\left(\mathbf{q}_h^{(m)}\right)_p\left(\overrightarrow{\xi},t\right) = \hat{\mathbf{Q}}_{pl}^{(m)}(t)\Phi_l\left(\overrightarrow{\xi}\right). \quad (2)$$

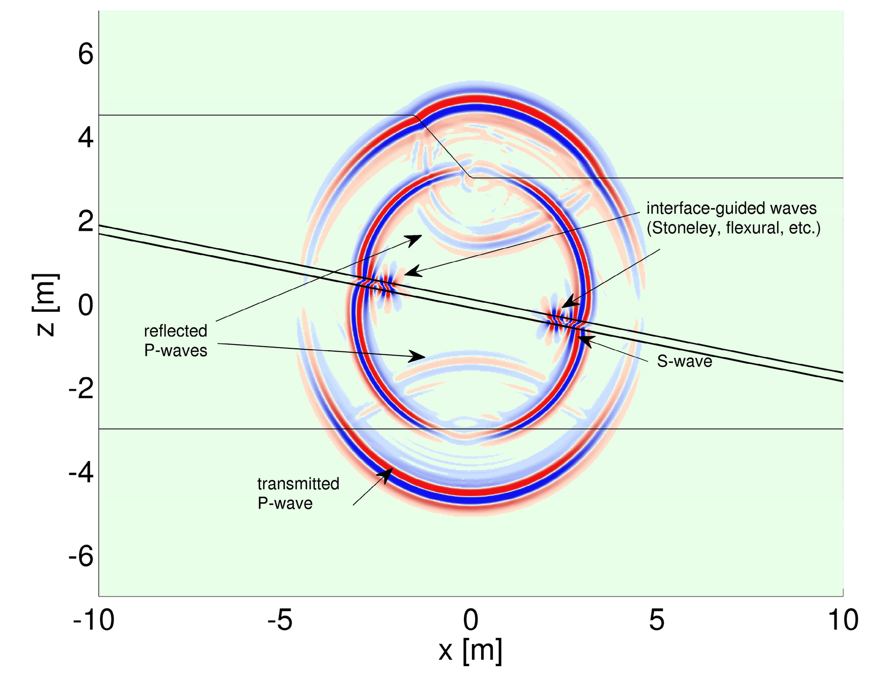
- $\hat{\mathbf{Q}}_{pl}$ are the (unknown) coefficients and Φ_l are orthogonal Legendre polynomials
- "ADER" : time derivative in Eq. (1) is replaced by spatial derivative operators, **achieving the same order accuracy in time that is selected in space**
- "DG" : solution expansion in Eq. (2) is discontinuous across elements, and solutions are coupled across elements through a numerical flux as in finite volume schemes.

Discretization



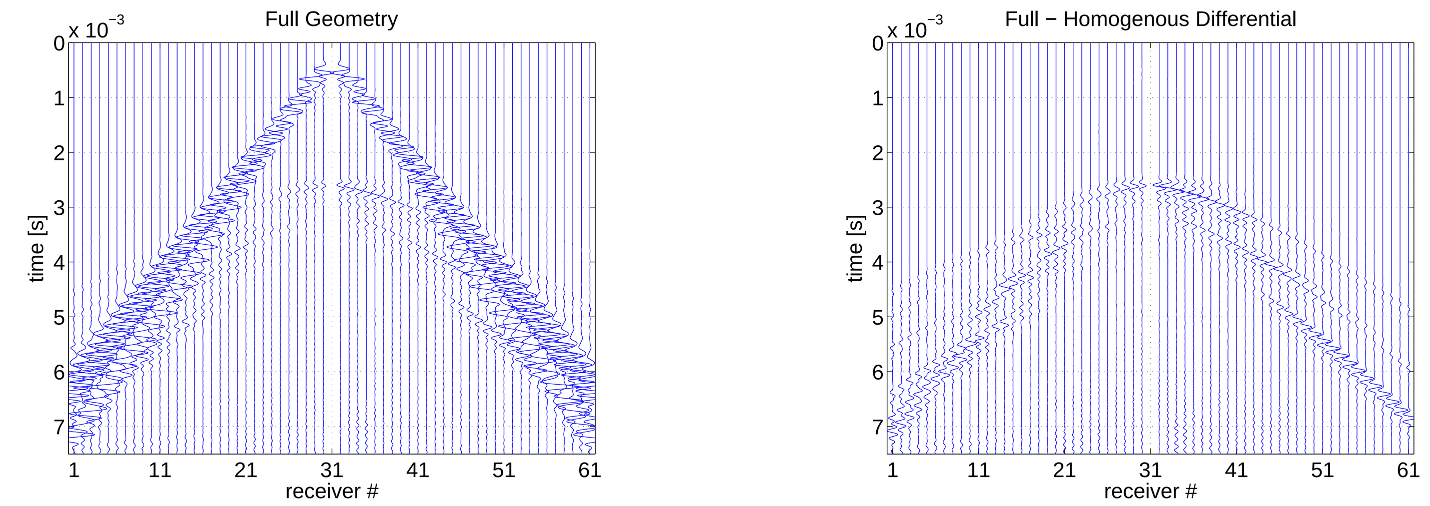
- Local time stepping for strongly varying triangular/tetrahedral element sizes
- Load balancing for parallelization requires weighted mesh partitioning (PMETIS; see Karypis and Kumar, 1998)
- Smaller element sizes in and around the borehole, with coarsening away from the layered interfaces
- Different colors represent different CPU partitions
- Total simulation domain size is 30 m wide by 20 m tall

Wavefield



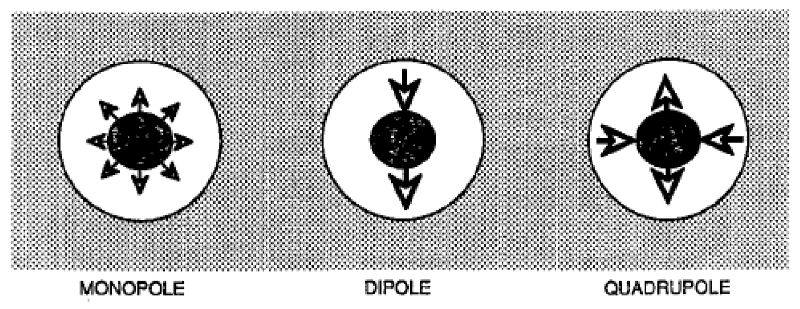
- $z-$velocity component at time $t = 2$ ms for M source
- Large-amplitude interface waves travel along borehole wall : tube (Stoneley) wave for monopole source, and flexural wave for dipole source
- Interface waves travel with velocity slightly less than the formation S-wave velocity
- P-wave reflections observed due to layered geometry ⇒ hyperbolic travel-time curve
- Complexity in top layer leads to a skewness in the reflected wave

Seismograms



- Seismograms for z-velocity, M source, **using order-8 convergence in space and time.**
- Left : Full geometry (i.e. top, middle and bottom layers). Λ-shaped pattern represents borehole-guided waves. Curved arrivals in the interior of this shape are reservoir reflections.
- Right: Subtraction of seismograms for full geometry and a homogeneously layered case, where top and bottom layers are set equal to middle layer. **The resulting differential highlights the reservoir-reflected waves.**

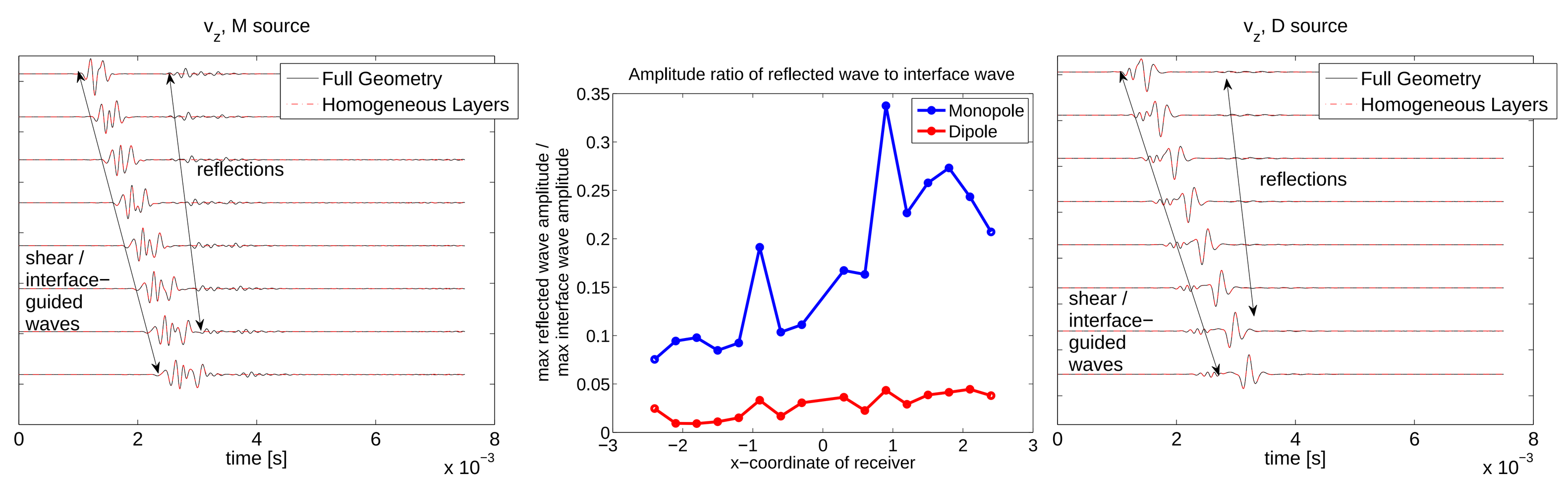
4. Source Descriptions



from Zemenek et al. (1991)

- Monopole : conventional seismic source; energy is radiated radially from tool axis (axisymmetric). Does not generate S-waves in a slow formation (Chen, 1982).
- Dipole : energy is radiated along a single direction (asymmetric). Generates strong shear waves in both fast and slow formations (Chen, 1988).
- Quadrupole : like the dipole source, can generate shear waves in both fast and slow formations, but can be operated at higher frequencies (Chen, 1989). Can only be implemented in 3-D.

5. Measuring Reflections



- Seismograms are computed with full geometry (solid black line) and with homogenous layers (dashed red line), illuminating the reservoir reflections.
- Left / right boxes : reservoir reflections easily seen with monopole source, but are much smaller with the dipole source.
- Middle box : ratio of amplitudes of reflections to interface-guided waves. **The monopole better insonifies the layered geometry.** The source is located at $x = 0$ (see Panel 2).

6. Conclusion

The high spatial and temporal accuracy and geometric flexibility of the ADER-DG is well-suited for wave propagation in a medium with discontinuous material properties. We can identify reservoir reflections with both M and D sources, but the former appears to be much better at insonifying the layered geometry.

Future work will be to implement a three-dimensional problem to examine the performance of the Q source. We also will investigate varying the source frequency, as the highly dispersive nature of the surface waves may imply that different source types are preferable in different frequency ranges.

References:

- Chen, S., 1982. The full acoustic wave train in a laboratory model of a borehole: *Geophysics*, v. 47, 1512-1520.
- Chen, S., 1988. Shear-wave logging with dipole sources: *Geophysics*, v. 53, 659-667.
- Chen, S., 1989. Shear-wave logging with quadrupole sources: *Geophysics*, v. 54, 590-597.
- Dumbser, M. and M. Käser, 2006. An arbitrary high order discontinuous Galerkin method for elastic waves on unstructured meshes II: The three-dimensional case: *Geophys. J. Int.*, v. 167, 319-336.
- Dumbser, M., M. Käser, and E. F. Toro, 2007. An arbitrary high order discontinuous Galerkin method for elastic waves on unstructured meshes V: Local time stepping and p-adaptivity: *Geophys. J. Int.*, v. 171, 695-717.
- Käser, M. and M. Dumbser, 2006. An arbitrary high order discontinuous Galerkin method for elastic waves on unstructured meshes I: The two-dimensional isotropic case with external source terms: *Geophys. J. Int.*, v. 166, 855-877.
- Karypis, G. and V. Kumar, 1998. A fast and high quality multilevel scheme for partitioning irregular graphs: *SIAM J. Sci. Comp.*, v. 20, 359-392. http://people.sc.fsu.edu/~jburkardt/c_src/pmetis/pmetis.html, last accessed 31.03.2011.
- Zemanek, J., D. M. Williams, and D. P. Schmitt, 1991. Shear-wave logging using multipole sources: *The Log Analyst*, v. 32, 233-241.