Institut für Chemie und Biologie des Meeres

On wave motions at the steep slope of a coral reef

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I. Introduction

Wave-driven circulation in coral reef systems is an important mechanism for water renewal behind the reefs and for maintaining a complex and fragile habitat. Coral reefs transform arriving waves and thus create special currents that are important for the biodiversity and distribution of larval fishes, phyto- and zooplankta.

Figure 1: The island of Moorea in the southern Pacific (Photographer: Jacques Beauregard)

In 2007/2008 an in-situ deployment took place in Paopao Bay, Moorea (Fig. 1). The isolated location of the island combined with unique characteristics, like small tidal and wind influences, has various advantages for the analysis of the main physical characteristics. In this investigation the Radiation Stress, particle velocities and Stokes drift are discussed.

II. Background

Coral-reef Circulation:
All coral-reef circulation models (e.g. Gourlay and Colleter 2005, Hearn 1999) are based on the idea of an equilibrium between the spatial gradients of the wave setup (deviation of the mean sea level), the Radiation Stress, bottom friction and convective acceleration. The depth integrated and steady momentum equation is presented in eq. 1:

$$\underbrace{g\frac{\partial\bar{\eta}}{\partial x}}_{\text{Wave-Setup}} + \underbrace{\frac{1}{\rho(\bar{\eta}+h)}\frac{\partial S_{xx}}{\partial x}}_{\text{Radiation Stress}} + \underbrace{U\frac{\partial U}{\partial x}}_{\text{Convective Acceleration}} + \underbrace{\frac{\tau^B}{\rho(\bar{\eta}+h)}}_{\text{Friction}} = 0 \qquad (1)$$

Here g is the gravity acceleration, ρ the density of seawater, $\bar{\eta}$ the wave-setup, τ^B the wave- averaged bottom friction and U the vertically integrated velocity in x-direction. In figure 2 the wave-induced flow is illustrated.

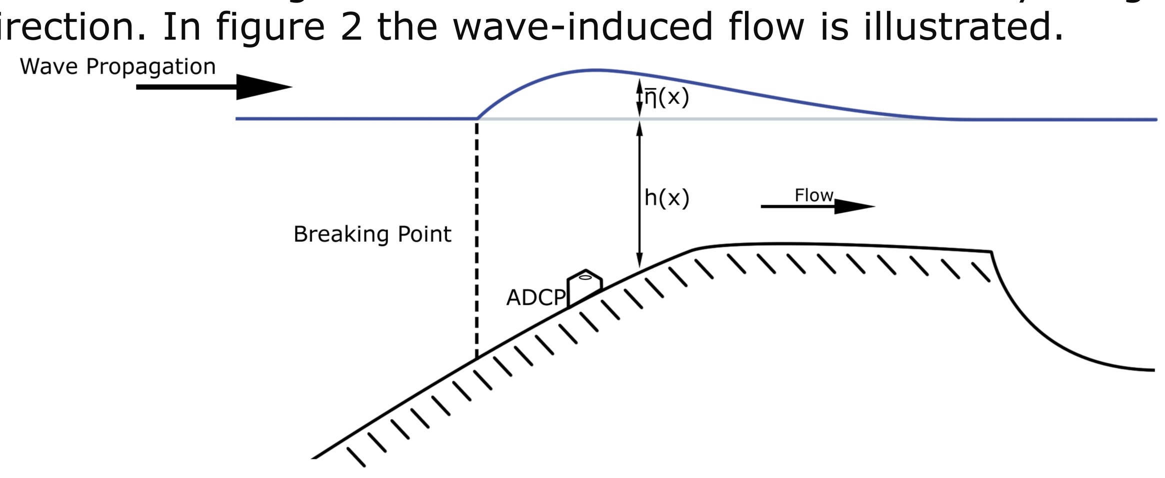

Figure 2: Illustration of the wave-induced flow in a coral reef system (after Hearn 1999, Monismith 2007)

Radiation Stress:
To date, models use a linear wave theory approach for the Radiation Stress, which was derived by Longuet-Higgins and Stewart (1964). In the mathematical ansatz they calculate the mean total flux of momentum minus the mean flux in absence of waves, that is:

$$S_{xx} = \overline{\int_{-h}^{\eta}(p+\rho u^2)dz} - \int_{-h}^{\eta} p_0 dz = E\left(\frac{2kh}{\sinh(2kh)} + \frac{1}{2}\right) \approx \boxed{\frac{3}{2}E} \qquad (2)$$

where k is the wave number , E=1/2 $\rho g a^2$ the wave energy and a the wave-amplitude. The approximate sign indicates shallow water wave theory.

Stokes Drift:
Considering particles in the water column an important motion is the Stokes drift. This movement occurs in shallow water due to unclosed wave orbits. Using linear wave theory an analytical expression can be derived by integrating the Langrangian velocity u_L over the depth and the wave period:

$$Q_S = \frac{1}{T}\int_T\int_h u_L\,dz\,dt = \frac{a^2 k}{2}\frac{\cosh(kh)}{\sinh(kh)}\sqrt{gh} \approx \boxed{\frac{a^2}{2}\sqrt{\frac{g}{h}}} \qquad (3)$$

III. Results

In figures 3-4 analytical models describing particle velocities, Stokes drift, and Radiation Stress are compared to ADCP observations (4.5m depth). The measurements are processed by using a 2nd order Butterworth low-pass filter. The Stokes drift is approximately determined by calculating the wave transport.

Radiation Stress **Stokes Drift**

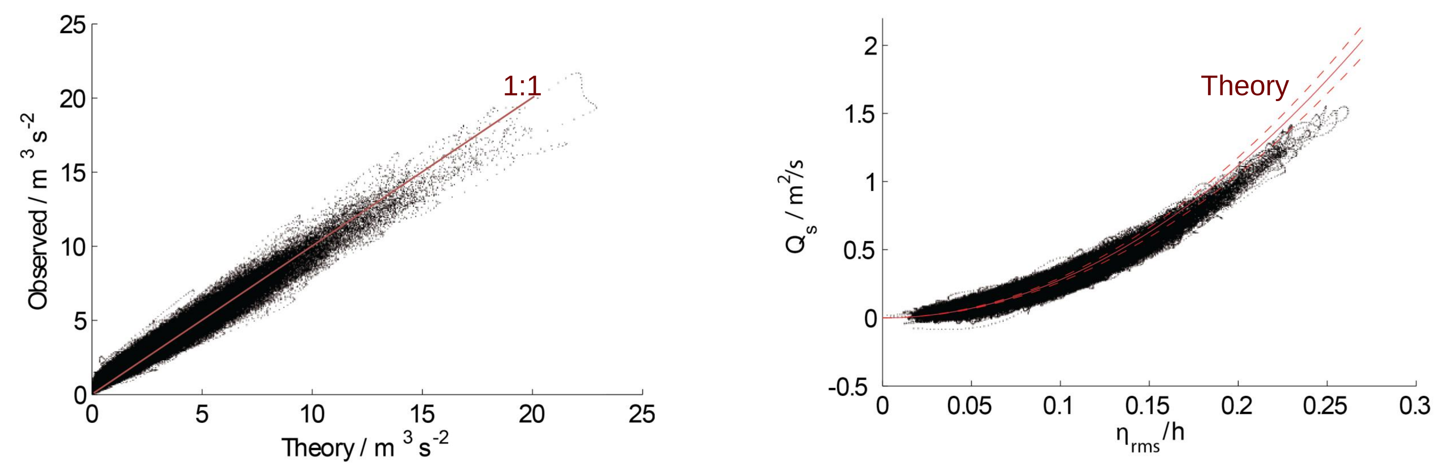

Figure 3: Radiation Stress (S_{XX} / ρ) and Stokes Drift compared to observations

Radiation Stresses computed from the ADCP values scatter slightly around the theoretical curve, especially for small amplitude waves the theoretical values underpredict the observations. The prediction of the Stokes drift compares well with observations, although for high amplitude waves the theory overpredicts the drift.

Particle Velocities

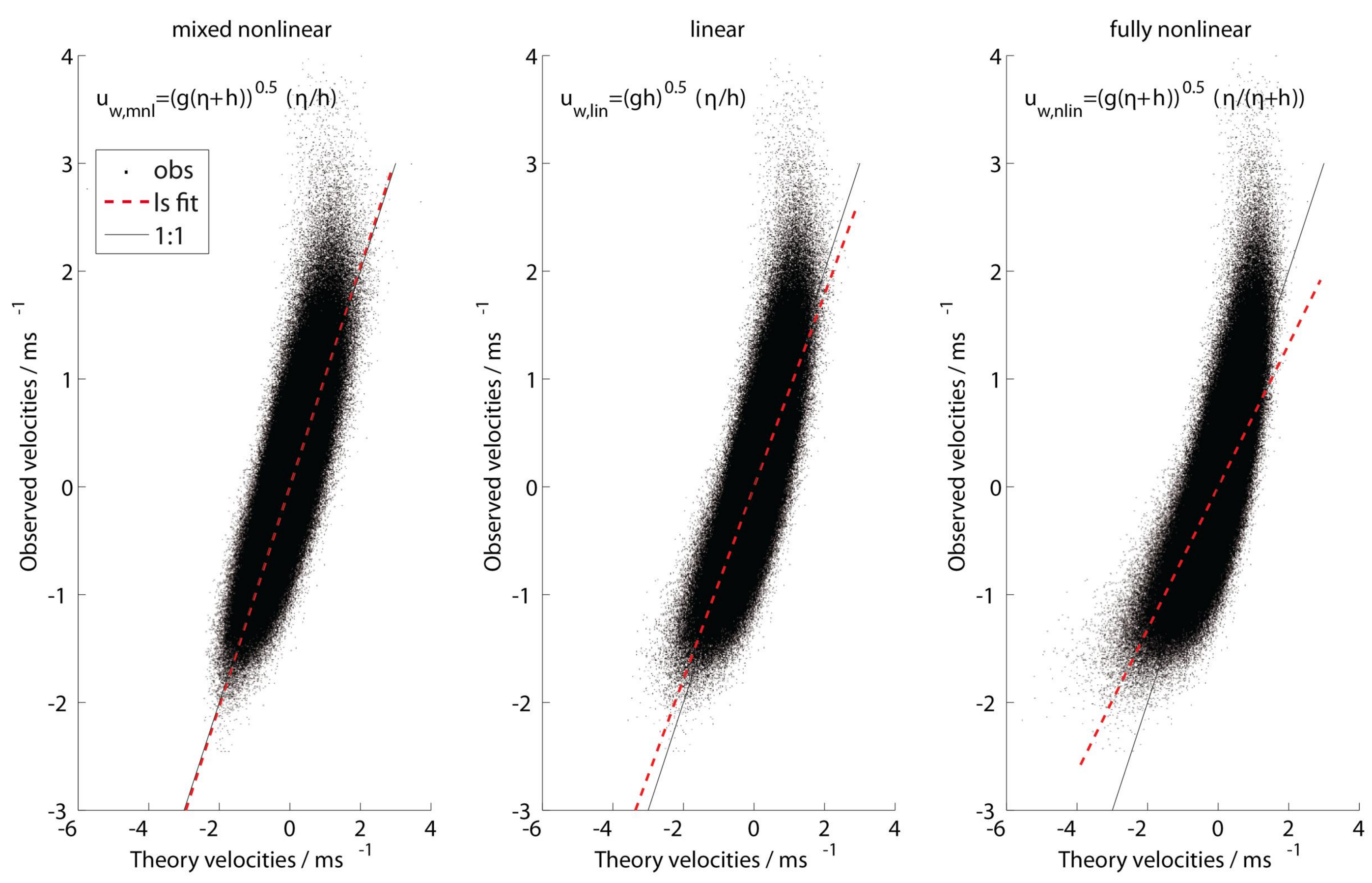

Figure 4: Particle velocities compared to different model-approaches

The unfiltered particle velocities are compared to theoretical values derived by nonlinear, linear and mixed approaches. For small to medium waves the mixed non-linear approach yields best results. Considering higher values the linear approach fits best. Breaking waves lead to the tale for high positive values in the direction of higher observations.

IV. Discussion

This investigation shows that linear wave theory is an appropriate way to describe the wave motions above the steep slope of a coral reef. It is shown that the Radiation Stress and Stokes Drift from in-situ observations are very close to theoretical values. Furthermore, the analyzed particle velocities indicate that linear approaches yield best results. Counter-intuitive, the results provided by non-linear theory show significant deviations from the observations. Considering the results of this study it seems promising to just implement a simple wave model into more complex 3-D hydrodynamical models to investigate the full circulation in coral reef systems. These model studies would be very useful from a biological perspective giving plankton distribution pattern and Lagoon residence times.

V. Outlook

At present more detailed analyzes are underway for observations above the reef flat and for ADCPs moored at different depths. Future investigations will also focus on the wave-induced alongshore current.

References and Acknowledgements

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