Formula for Motion Threshold per Grain Size for Graded Sediments in Steady Flows

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Introduction

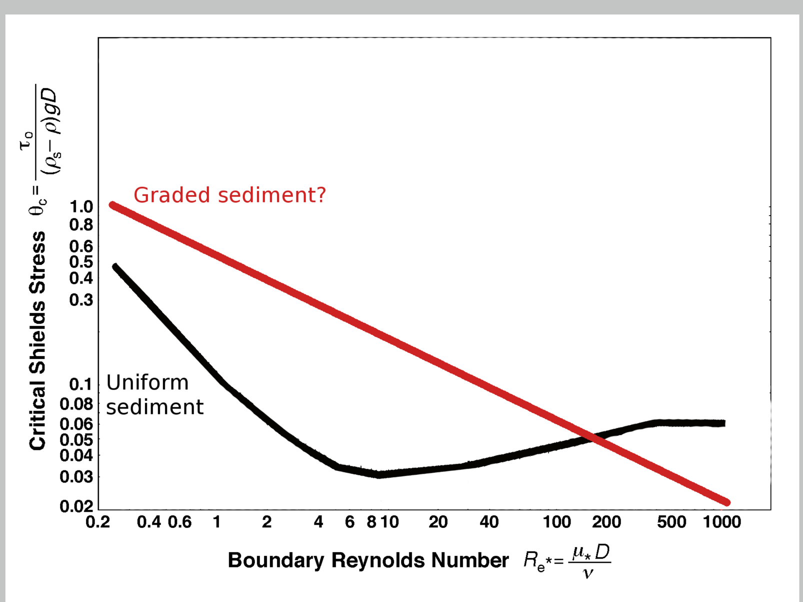
- The response of grain size at a given shear stress different from what it would be if the bed were uniform
- Most sediment mobilisation models assume a single 'representative' grain size
- Those parameterisations based on a distribution of sizes are unsatisactory

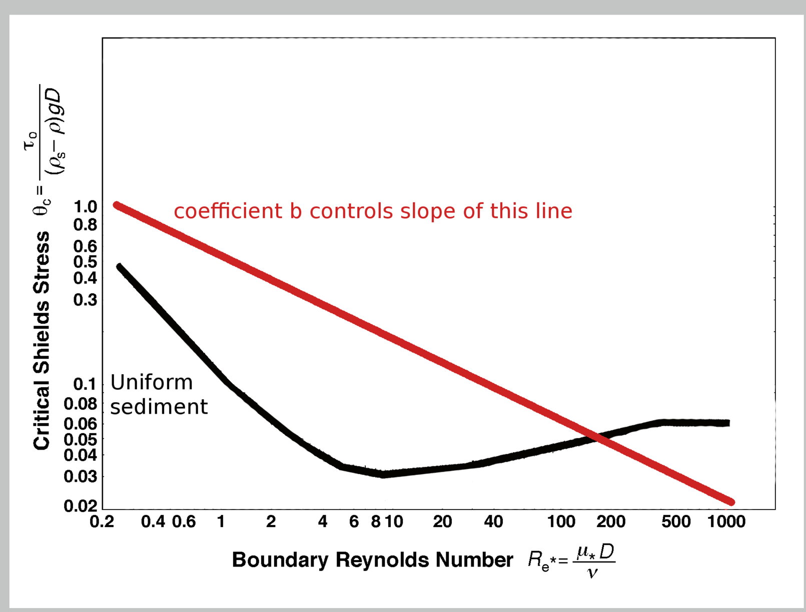
Different Approaches

- Physical: balance of all forces
- Complicated to formulate/measure, especially mixtures
- Engineering: $\theta > \theta_c$ for sediment to move
- $\mathbf{Q} = \mathbf{F}\left[\theta - \theta_c\right]$ = nonlinear function of excess shear stress
- $\theta = \mathbf{F}\left(\frac{u_*^2}{D}\right)$, $\theta_c = \mathbf{F}\left(\frac{u_* D}{\nu}\right)$

Apparent Threshold for i Grain Sizes

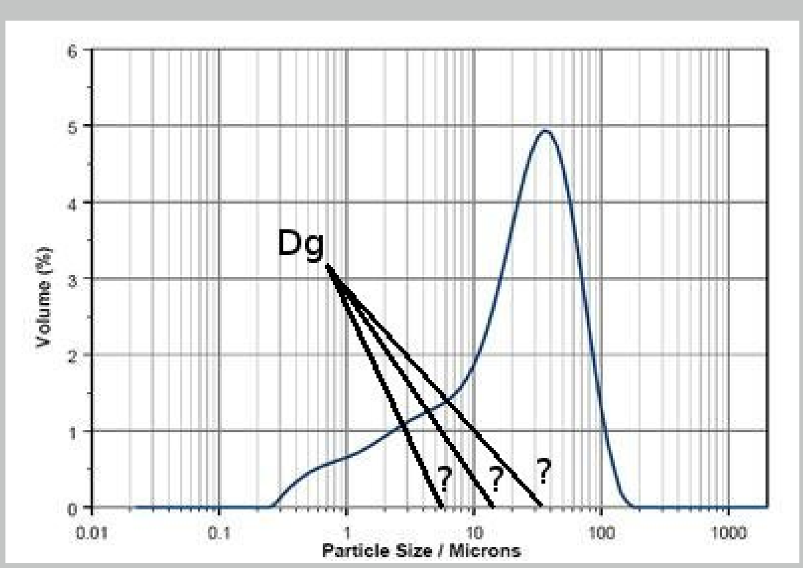
- $\mathbf{Q_i} = \mathbf{F}\left[\theta_g - \theta_{ci}\right] \mathbf{P_i}$ typically over-predicts fines
- Instead, threshold for fines raised and for coarse lowered
- Einstein (1950): $\frac{\theta_{c1}}{\theta_{c2}} \approx \frac{D_2}{D_1}$
- Define apparent threshold:
- $\theta'_{ci} = \theta_{cg}\left(\frac{D_i}{D_g}\right)^{b-1}$

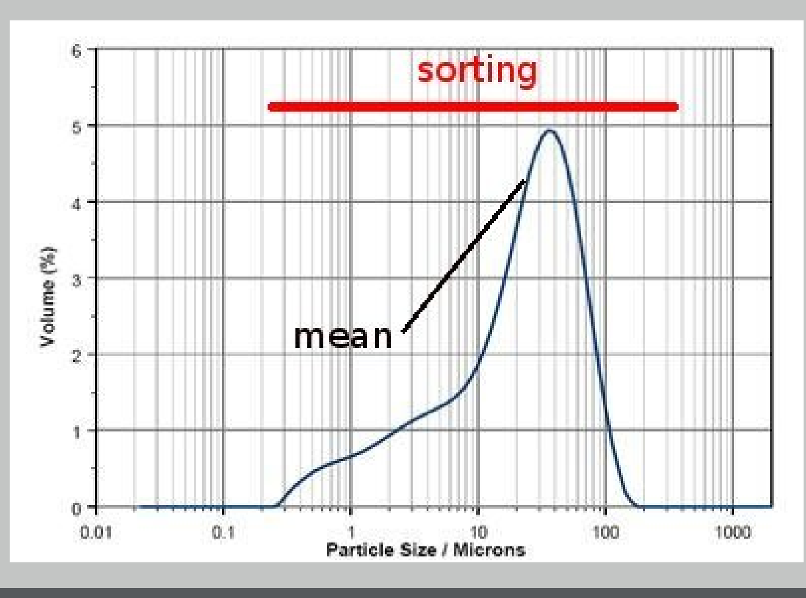




Problems with 'b'

- b determined by matching modelled and measured $\mathbf{Q_i}$
  - This introduces an unnecessary level of abstraction
- all include arbitrary breakpoints at some **D**
- 3 known parameterizations, none universally applied. All are based on matching predicted and observed sediment transport rates
- Large degree of empiricism
- 2 do not include sorting, σ





A New Approach

- We propose an approach based on the inferred measured particle size-distribution of the mobilized sediment, $\mathbf{P_{mi}}$, rather than transport rate per fraction $\mathbf{Q_i}$
  - This avoid uncertainties in direct modelling of $\mathbf{Q_i}$ (specification of **F**)
  - Depth-integrated transport rate $\mathbf{Q} = \mathbf{U_s L}$
  - Depth-integrated transport load $\mathbf{L} = \int_0^h \mathbf{C(z)dz}$
  - Assume **L** is linearly proportional to $(\theta - \theta_c)$
  - Rate per fraction $\mathbf{Q_i} = \mathbf{U_s}\left(\theta_g - \theta'_{ci}\right)\mathbf{P_i}$
- Infer $\mathbf{P_{mi}} = \frac{\int \mathbf{U_s}\left(\theta_g - \theta'_{ci}\right)\mathbf{P_i dt}}{\int \sum \mathbf{U_s}\left(\theta_g - \theta'_{ci}\right)\mathbf{P_i dt}}$
- Given θ_g, $\mathbf{L_i}$, $\mathbf{P_i}$ and $\mathbf{D_g}$, we fit **b** using:
- $$= \frac{\theta_g - \theta_{cg}\left(\frac{D_i}{D_g}\right)^{b-1} P_i}{\sum\left(\theta_g - \theta_{cg}\left(\frac{D_i}{D_g}\right)^{b-1} P_i\right)}$$

Summary of experimental conditions for the data used in this study

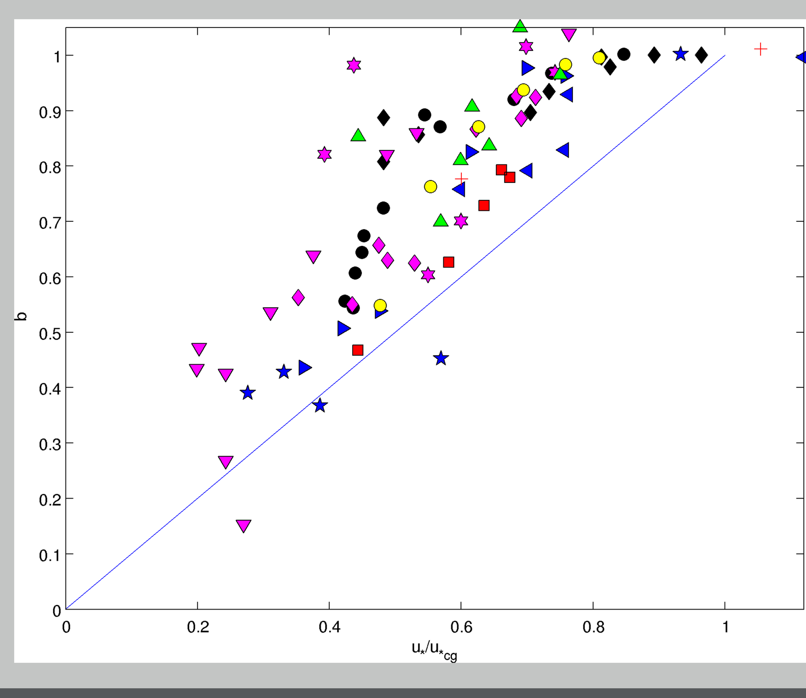
- Non-linear optimization of this simple equation is used to find the optimal form of a weighting (so-called 'hiding') function used to modify critical entrainment criteria to provide the best possible fit with data.
- 12 sets of published data from flume experiments. Total of 81 different mixed sand and gravel beds and flow conditions (grain Reynolds numbers between 1 and 10,000)

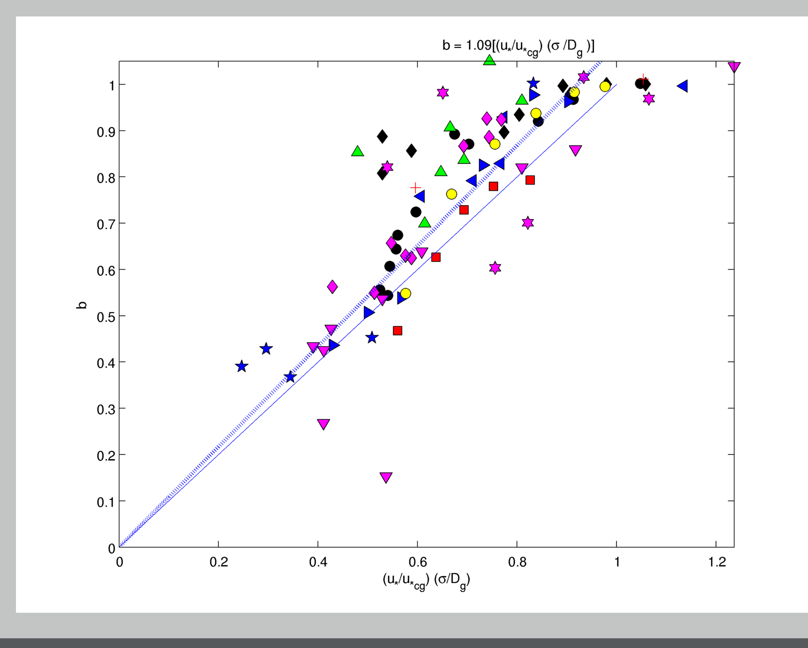
Experiment *Published*	$\theta_g \times 10^{-3}$	D_i/D_g	D_g *(mm)*	σ *(mm)*	*mean* **B**
Day (1980)	5.2→20.8	0.02→3.49	2.5	2.48	3.36
Day (1980)	7.07→28.3	0.03→3.03	1.84	1.51	9.62
Blom & Kleinhans (1999)	8.36→20.9	0.055→4.97	2.8→3.5	3.55→3.95	5.49
Kuhnle (1993)	4.7→53.47	0.22→8.35	0.97	1.52	0.52
Kuhnle (1993)	17.06→59.67	0.12→4.79	1.7	2.14	1.24
Kuhnle (1993)	3.92→17.34	0.081→3.21	2.6	2.45	3.65
Wilcock & McArdell (1993)	0.53→7.86	0.017→9.12	5.1→6	7.97→13.42	6.34
Wilcock et al (2001)	4.77→20.91	0.0058→3.11	14.5→17.1	9.95→12	0
Wilcock et al (2001)	4.6→18.81	0.0062→3.19	14.2→16.2	11.2→11.9	0
Sun & Donahue (2000)	6.07→17.32	0.18→7.9	0.7→1.08	0.55→1.16	0
Tait et al. (1992)	11.22→32.46	0.03→2.05	3.1→3.4	1.48→1.57	0
Kuhnle (1992)	12.34→35.51	0.0079→4.26	15.72	18.96	7.91

D_g = arithmetic mean; σ = arithmetic sorting; **B** = bimodality index (0=unimodal)

Analysis reveals functional form of b

- $\mathbf{b} = \left(\frac{u_*}{u_{*cg}}\right)$
- $\mathbf{b} = 1.09\left(\frac{u_*}{u_{*cg}}\right)\left(\frac{\sigma}{D_g}\right)$



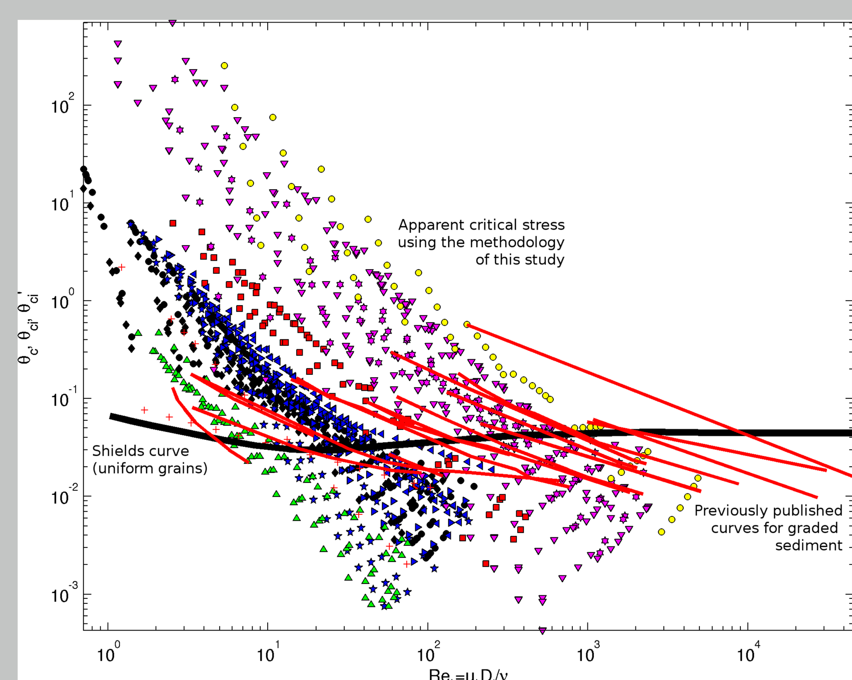


Results

- Analysis reveals that motion threshold is dependent on:
  1. excess shear stress
  2. ratio of particle (arithmetic) sorting and mean grain size
- Our new formula for the mobilization of graded sediment:
  - $\theta'_{ci} = \theta_{cg}\left(\frac{D_i}{D_g}\right)^{\left(1.09\left(\frac{u_*}{u_{*cg}}\right)\left(\frac{\sigma}{D_g}\right)\right)-1}$
  - Based on routinely measured quantities, and easily calculable based on mean grain size
- This new relation outperforms existing formulae in 11 out of 12 data sets, and with an expected error of $\pm$ 20%

Synthesis

- A simple deterministic equation, in non-dimensional form, is proposed for fraction-specific apparent critical shear stress and mobilized particle size distribution
- It predicts that θ'_{ci} varies over more than 5 orders of magnitude for graded sediment, compared to a 1 order variation in θ_{cg} for non-graded sediment (dark solid line in Figure adjacent) and 2 orders in θ_{ci} found by previous studies (the light solid lines in Figure).
- The slope of the relation between $\mathbf{R_e}$ and threshold condition is apparently steeper than previously thought



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