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Multifractality of a high-resolution rainfall time series and assessment of the zero rain rates effect

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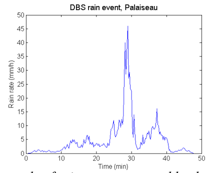
This study investigates the scaling and multifractal properties of a high-resolution (15-s) rainfall time series, recorded by the means of a spectropluviometer (DBS). This dataset enables the exploration of fine scales of rain processes that have often been neglected or underestimated in scaling/multifractal literature where the resolutions usually available are far coarser (typically, 1-h or 1-day). Therefore, the DBS dataset may provide valuable information on the underexplored internal structure of uninterrupted rain events. Moreover, it is recalled that multifractal analysis techniques are sensitive to the proportion of zeros in the data, which may be a source of errors in multifractal characterization of rainfall processes, where most dataset points contain zero values. Empirical and theoretical solutions are proposed in order to correct this kind of errors.

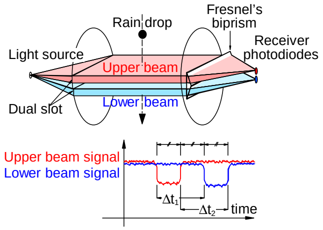
1) The dataset

- Data were collected by the means of a Dual-Beam Spectropluviometer (DBS), in Palaiseau, France
- Rain rates are estimated from arrival times, fall speeds, diameters of raindrops
- Two years of data (summer 2008- summer 2010), at 15-s resolution
- Measurement threshold: 0.1 mm/h
- Catchment surface: 100 cm²

The DBS instrument

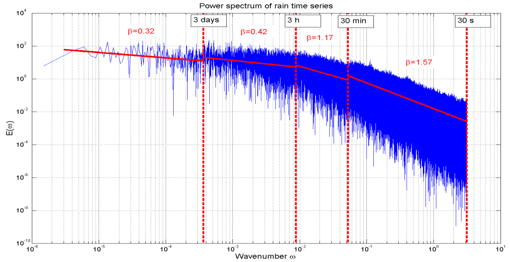


Example of rain event measured by the DBS



2) The scaling regimes of DBS series

- Scaling regimes are associated with power-law spectra, i.e. $E(\omega) \sim \omega^{-\beta}$
- At small frequences (i.e. timescales > 1 week), the spectrum is almost flat
- Scaling regimes appear at smaller scales
- Sharp transition at 30-min scale ~ average rainfall event duration
- Scales dominated by interevent variability are separated from those dominated by event internal variability



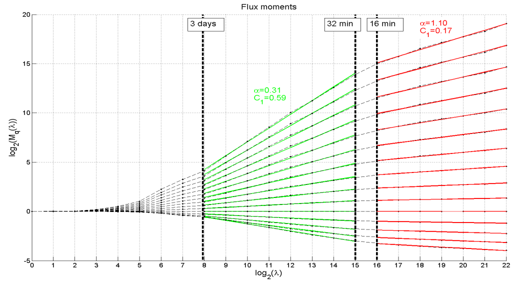
DBS series power spectrum (log-coordinates)

3) Multiplicative cascades and Universal Multifractals

- Test of multiscaling properties → multifractal analysis (MA)
- Multiscaling quantity, indexed by the resolution: Φ_λ
- Multifractality arises from multiplicative cascades schemes
- Multifractality means power-law statistical moments: $\langle \Phi_\lambda^q \rangle \propto \lambda^{K(q)}$
- Universal Multifractal model (UM) **[1]**: $K(q) = \dfrac{C_1}{\alpha - 1}\left(q^\alpha - q\right)$
- $K(q)$ is the moment scaling function (Legendre tranform of the co-dimension function)
- Observables (rain rate) may be nonstationary and are modeled as a fractionnally integrated multifractal cascade (order H)
- Three parameters: α (multifractality), C_1 (inhomogeneity), H (nonstationarity)

4) Multifractal properties of DBS series

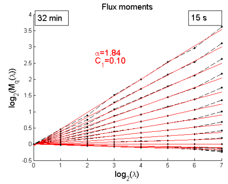
- Since DBS exhibits β > 1 at small scales, $H > 0$ (nonstationarity)
- Fractional integration inverted by taking the absolute gradient at finest scale **[2]**
- Empirical moments estimated: $0 < q < 3$; various resolutions
- Two multiscaling regimes separated by a transition at 30-min scale
- Large-scale parameters $\alpha = 0.31$, $C_1 = 0.59$, $H = 0$ close to literature



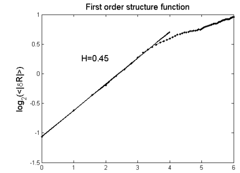
Empirical moments of the DBS absolute increments (logarithmic coordinates)

5) The zero rain rate problem (ZRRP)

- DBS series: 96% of zero values (true and instrumental)!
- Multifractal multiplicative cascades are not well designed to generate large intervals/areas of zero values
- Verrier et al. **[3]** simulated UM maps and applied a threshold to simulate zeros → MA of thresholded maps provides strongly biased UM parameters!
- Correction of the ZRRP at small scales: for DBS time series, we may consider uninterrupted rain events only
- MA of ~ 50 DBS rain events, duration ~ 30 min → corrected parameters: $\alpha = 1.84$, $C_1 = 0.10$, $H = 0.45$



Empirical moments of the rain events



First order structure function of rain events

6) Correction of the ZRRP at large scales (> 30 min)

a) Weigthed MA procedure

- The estimation of moments should overweight nonzero values in both resolution degradation procedure and moments computations
- The weighting procedure must guarantee the conservativity of Φ
- Weighted moments of DBS series seem to exhibit multiscaling properties from 1-week to 30-min scales, with $\alpha = 1.22$, $C_1 = 0.16$, $H = 0$

b) Semi-theoretical formulas for estimating the bias in MA due to ZRRP

- Suppose a normalized product of a UM cascade and of an independent monofractal support, co-dimension C_F
- Moment scaling function of the product cascade: $K_{ZRRP}(q) = K(q) + C_F(q-1)$
- Derivatives of (UM) MSF are related with the estimated parameters
- Fitting K_{ZRRP} with the two-parameter form provides biased parameters:

$$C_1' = C_1 + C_F \qquad \alpha' = \frac{C_1}{C_1 + C_F}\alpha$$

- At large-scale (> 30 min), $C_F = 0.45$ is estimated by box-counting. Using *full-rain* parameters (section 5) $\alpha = 1.84$, $C_1 = 0.10$ provides $\alpha' \sim 0.3$, $C_1' \sim 0.55$ which is in agreement with large-scale parameters (section 4).

References:

[1] Schertzer, D., Lovejoy, S., 1987: Physical modelling and analysis of rain and clouds by anisotropic scaling multiplicative processes. J. Geophys. Res. 92 (D8), 9693–9714; **[2]** Lavallée, D., Lovejoy S., Schertzer D., Ladoy P., 1993: Nonlinear variability and Landscape topography: analysis and simulation. Fractals in Geography, Eds. L. De Cola, N. Lam, 158-192, PTR, Prentice Hall; **[3]** Verrier S., L. de Montera, L. Barthès, C. Mallet (2010), Multifractal analysis of African monsoon rain fields, taking into account the zero rain-rate problem. Journal of Hydrology, 389, 111-120