Ensembles of Ground Motion Models

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1. Motivation

Here we explore ground motion model (GMM) aggregation techniques. In particular we...

- look beyond the classical Logic Tree framework,
- avoid the assumption of models being mutually exclusive and collectively exhaustive, and
- borrow methods from the field of Machine Learning.

We assume we have several GMMs, $i = 1, \dots, M$ from which ground motion is distributed:

$$\log PGA(\boldsymbol{x}) \sim \mathcal{N}(Y|\mu_i(\boldsymbol{x}), \sigma_i^2(\boldsymbol{x}))$$

AIM: Data guided/driven GMM aggregation.

2. Ensembles in Machine Learning

ENSEMBLE: A collection of models where each model is *trained* to solve the same problem:

- The combined model can be a more powerful model that any of the constituents (*Figure A*).
- They exhibit improved generalization ability over single models (*Figure B*).
- Fuse different sources of information: members may be trained on different data (*Figure C*).

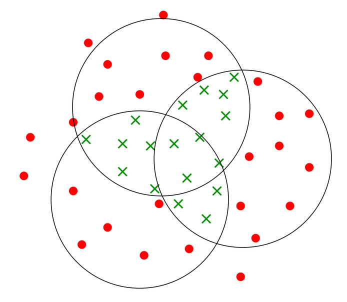


Figure A

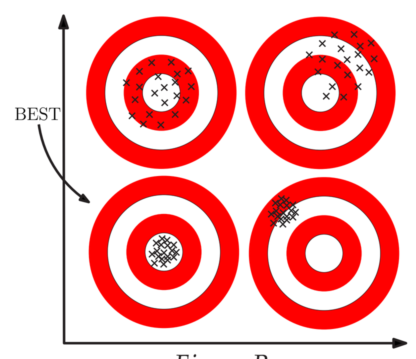


Figure B

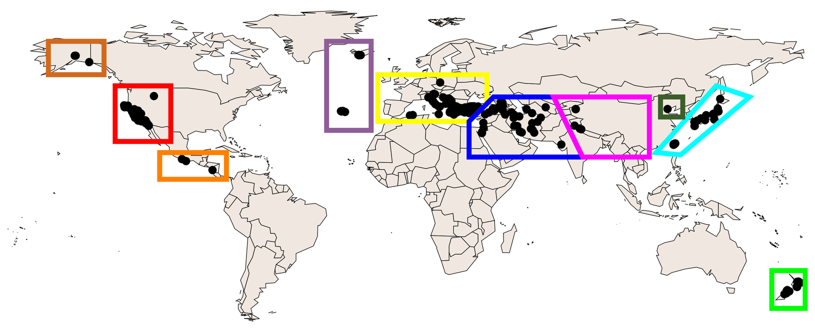


Figure C

Exploiting/inspired by these points, we pursue "ensembles" of GMMs. However:

- The GMMs are fixed ("pre-trained") and ensemble learning can't be fully data-driven.
- We are restricted from combination operations that violate what is "physically plausible".

3. Mixtures

Combination in not uni-modal or Gaussian:

Standard Mixture: A convex combination of Gaussian GMMs, where means and variances are given for each GMM (s.t. $\sum_i w_i = 1$, that is, weights are probabilities) ,

$$\Pr(Y|\boldsymbol{x}) = \sum_i w_i \cdot \mathcal{N}(Y|\mu_i(\boldsymbol{x}), \sigma_i^2(\boldsymbol{x}))$$

Conditional Mixture: A convex combination as before, but the weights are dependent on *magnitude and distance* (as before, s.t. $\sum_i w_i(\boldsymbol{x}) = 1$),

$$\Pr(Y|\boldsymbol{x}) = \sum_i w_i(\boldsymbol{x}) \cdot \mathcal{N}(Y|\mu_i(\boldsymbol{x}), \sigma_i^2(\boldsymbol{x}))$$

4. Linear Gaussians

Combination is forced to remain Gaussian:

Independent: Gaussian is closed under linear transformation when $\mathcal{N}(Y|\mu_i(\boldsymbol{x}), \sigma_i^2(\boldsymbol{x}))$ are independent (with w_i unconstrained),

$$\Pr(Y|\boldsymbol{x}) = \mathcal{N}(Y|\sum_i w_i \mu_i(\boldsymbol{x}), \sum_i w_i^2 \sigma_i^2(\boldsymbol{x}))$$

Mixture Moment Matching: Mean and variance is forced to equal mean and variance of a mixture with the same weights (s.t. $\sum_i w_i = 1$),

$$\begin{aligned}\Pr(Y|\boldsymbol{x}) &= \mathcal{N}(Y|\sum_i w_i \mu_i(\boldsymbol{x}), \sigma^2(\boldsymbol{x})) \\ \sigma^2(\boldsymbol{x}) &= \sum_i w_i(\boldsymbol{x}) \mu_i^2(\boldsymbol{x}) \\ &+ \sum_i w_i(\boldsymbol{x})(\mu_i^2(\boldsymbol{x}) + \sigma_i^2(\boldsymbol{x})) - \mu^2(\boldsymbol{x})\end{aligned}$$

5. Learning Weights

We are learning weights from data using:

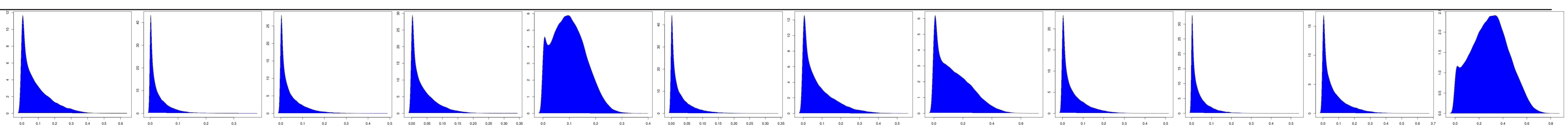
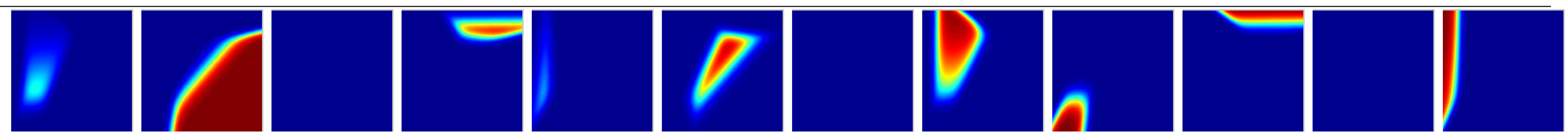
EM (Expectation Maximization) is an iterative deterministic method which in general converges towards the "correct" Maximum Likelihood point estimate.

MCMC (Markov chain Monte Carlo) is a Bayesian simulation technique:

- Prior knowledge is defined as a distribution on the weights: Prior for Mixture weights is a Dirichlet (may be informative). Prior for Independent Gaussian weights are uniform or wide Gaussian.
- Once data are considered we get the posterior distribution, which we can plot, or generate summary statistics from.

6. Example Experiments

AIM: From observations/data from the European/Middle East region and predictions from 12 GMMs create ensembles using the methods above.

GMM Name	Berge-Thierry ('03)	Campbell Bozorgnia ('08)	Boore Atkinson ('08)	Cotton ('08)	Kanno ('06)	Zhao ('06)	Abrahamson Silva ('08)	Chiou Youngs ('08)	Cauzzi Faccioli ('08)	Fukushima ('03)	Chiou ('10)	Akkar Bommer ('10)
Std. Mixture (MCMC) (weight vs. density) →*Non-Gaussian!*												
	0.0863± 0.088	0.029 ± 0.038	0.044 ± 0.055	0.037 ± 0.041	0.10 ± 0.061	0.028 ± 0.036	0.080 ± 0.084	0.14 ± 0.118	0.051 ± 0.060	0.038 ± 0.049	0.069 ± 0.081	0.29 ± 0.154
Cond. Mixture (EM) (magnitude vs. dist.) →*Non-Gaussian!*												
Mix. Mom. (MCMC) →*Forced Gaussian!*	0.132 ± 0.108	0.030 ± 0.040	0.041 ± 0.051	0.043 ± 0.046	0.074 ± 0.057	0.034 ± 0.044	0.067 ± 0.076	0.127 ± 0.112	0.050 ± 0.059	0.044 ± 0.055	0.073 ± 0.082	0.279 ± 0.154
Independent (MCMC) →*Forced Gaussian!*	0.123 ± 0.284	−0.182± 0.335	0.064 ± 0.335	0.047 ± 0.164	0.103 ± 0.180	−0.088± 0.244	−0.052± 0.328	0.243 ± 0.330	0.107 ± 0.231	0.012 ± 0.331	0.165 ± 0.360	0.539 ± 0.234

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