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Dependence structure in the weights of a multiplicative random cascade for rainfall

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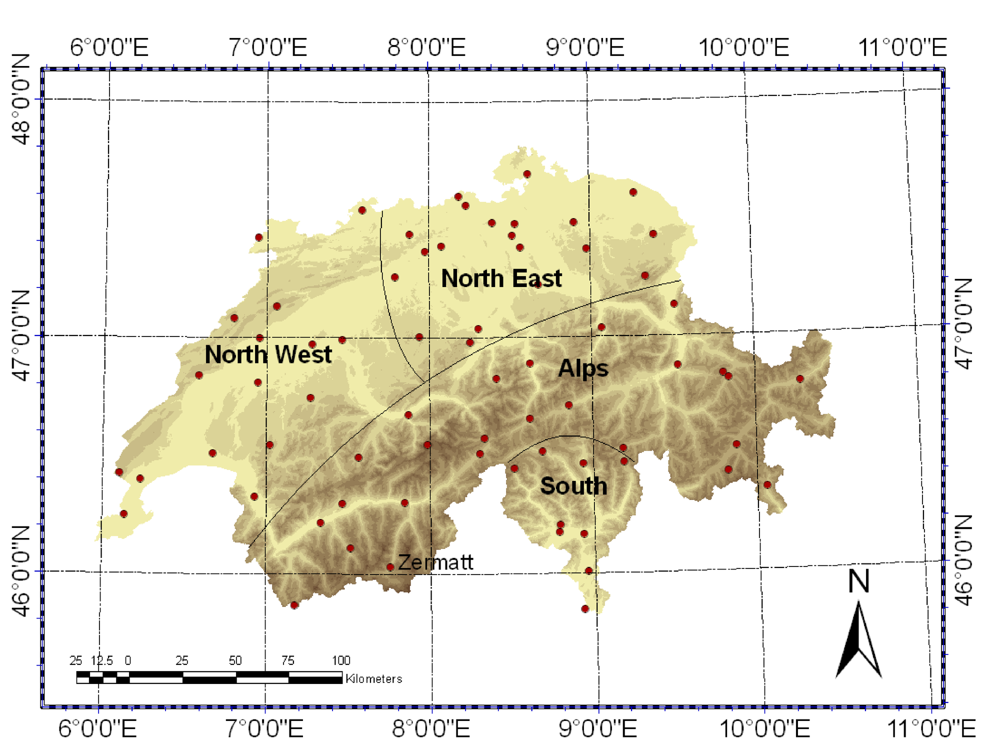
Abstract

In this study we investigate the applicability of the discrete random multiplicative cascade framework for the representation of temporal precipitation. Our effort focuses on the correlation structure of cascade weights across different climatic regimes and problems of inconsistency between the widely used class of multifractal random cascade models and observed temporal precipitation. This study based on the work of *Carsteanu and Foufoula* (*1996*) quantifies the dependence structure of observed precipitation in the complex Alpine terrain of Switzerland and proposes an intermittent RMC multifractal model that takes into account simple cases of temporal dependence structure in its weights.

Data

The data used in this study are 10 minute precipitation records at 69 meteorological stations of the SwissMetNet network. The stations are equipped with a heated tipping bucket rain gauge with precision 0.1mm per tip and cover a time period on average of 26 years. In addition, the data recorded from a high depth precision and time resolution weighing gauge will be used for comparison. The time resolution of this dataset is 1 minute with depth precision 0.001mm. Due to the complex Alpine climatology (Frei and Schär, 1998) the area of Switzerland was roughly divided into four sub regions.



Data Analysis Methods

In this study we are focusing only on the dependence structure of weights on each scale and not the potential cross dependence structure between scales (*Rupp et al.*, 2009). Moreover it will be assumed that the correlation structure is the same between all scales of interest.

Since the reconstruction of the weights or the breakdown coefficients is impossible in an intermittent time series, we adopt here the nonparametric notation of *Carsteanu and Foufoula-Georgiou* (1996) for the quantification of the dependence between weights.

Let r_i be the recorded rainfall depth at time i and let as define a_i as

$$a_i := (r_i - r_{i+1}) \cdot (r_{i+1} - r_{i+2})$$

Then we define the quantity oscillation coefficient as

$$C_\updownarrow := \frac{\#(a<0)}{\#(a \neq 0)}$$

This quantity is the fraction of the triplet occurrences (up-down-up or down-up-down) in the sample are is dependent on the dependence structure of the weights.

Stochastic model

In order to quantify the connection between the values of the oscillation coefficient and the dependence structure of the cascade weights, we build a simple variant of the canonical intermittent beta lognormal multiplicative cascade model (Over et al.,1994).

The model in principal is defined as a mass subdivision across scales according to a multiplicative manner. The mass in a scale n is defined as

$$\mu_n(\Delta_n{}^i) = R_0 b^{-n} \prod_{j=1}^{n} Y_i$$

for $i = 1, 2, \ldots, b^n$.

In our case we define the weights Y_i as the product of two different and independent stochastic processes X_i and Z_i.

$$Y_i = c \cdot X_i \cdot Z_i$$

where X_i is a first order two state Markov chain model with transition probability matrix

$$(p_{ij}) = \begin{bmatrix} 1-\alpha\gamma & \alpha\gamma \\ \alpha(1-\gamma) & 1-\alpha(1-\gamma) \end{bmatrix}$$

and lag one correlation coefficient

$$\rho_X = 1 - \alpha$$

and Z_i is a first order unit mean autoregressive process AR(1) with marginal lognormal distribution defined as

$$Z_i = \exp(Z_i{}^*)$$

where $Z_i{}^*$ a first order autoregressive process with correlation parameter φ.

$$Z_i{}^* = \mu(z^*) + \varphi[Z_{i-1}{}^* - \mu(z^*)] + \varepsilon_i$$

Finally c is the intermittency parameter related to β of the model of Over et al. (1994) with the relationship

$$c = b^{-\beta} = {}^1/_\gamma$$

Relation between model parameters and $C_\updownarrow$

Since an analytical expression between the model parameters and the oscillation coefficient would be extremely cumbersome, we derive the above relationships and their confidence intervals through an extensive Monte Carlo simulation. The initial assumption was that the dependence structure is a result of only one of the two independent processes defined above in a sense to distinguish between in-storm processes and intermittency. The parameter space of the simulation is

$$\varphi = 0 \cap \{\sigma(z^*) \in [0.05, 0.5] \cup \beta \in [0.05, 0.5] \cup \rho_X \in [-0.9, 0.9]\}$$

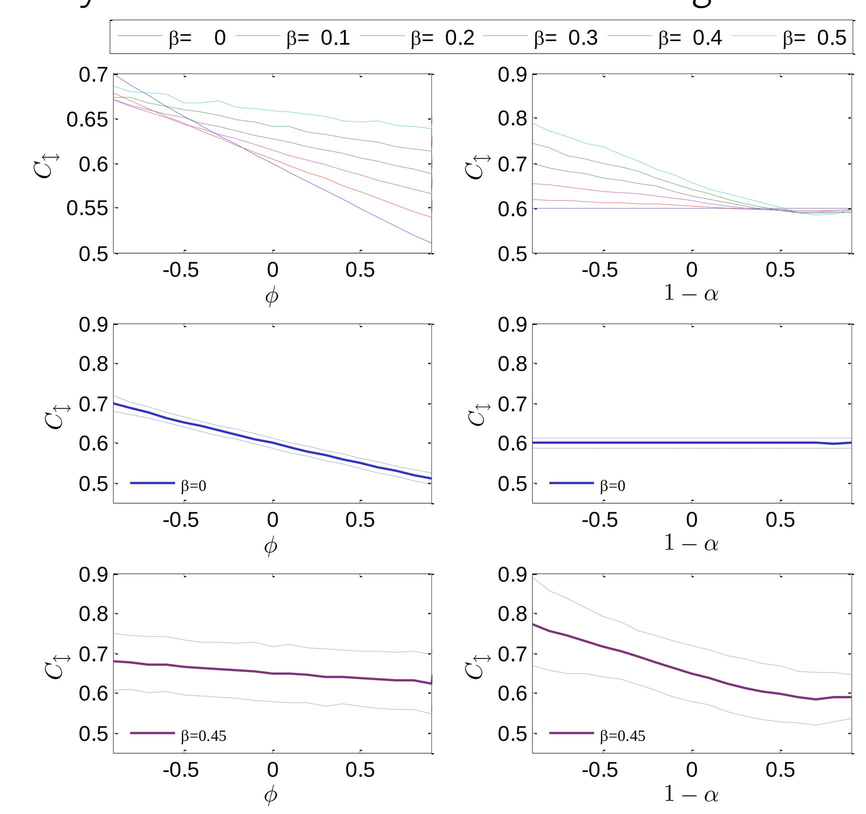
and

$$\rho_X = 0 \cap \{\sigma(z^*) \in [0.05, 0.5] \cup \beta \in [0.05, 0.5] \cup \varphi \in [-0.9, 0.9]\}$$

The quantity $C_\updownarrow$ was found to be independent of the marginal distribution of the continous part

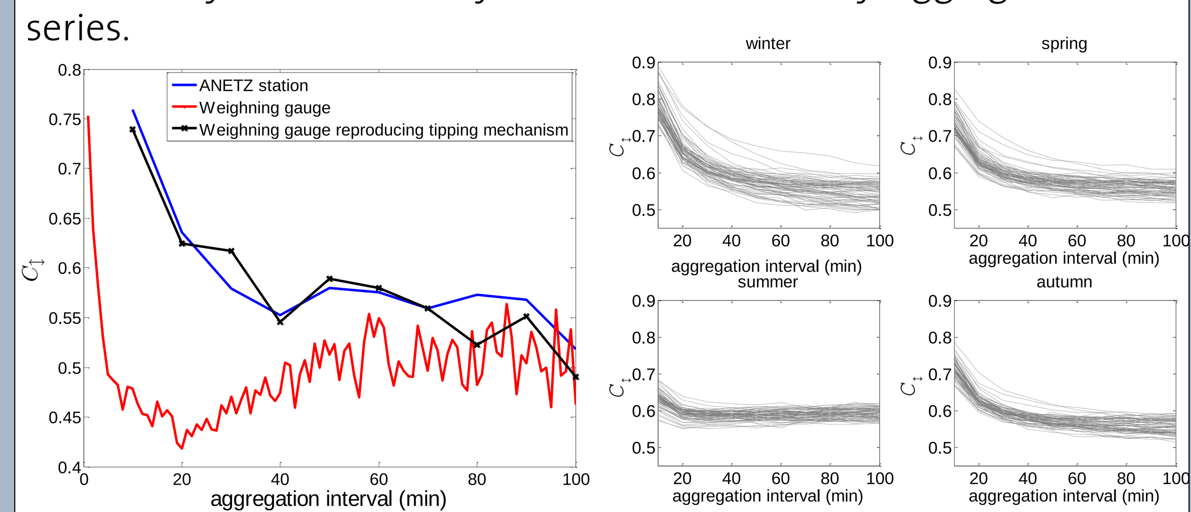
The cascade steps used in the study were 11, and the branching number was 2 in order to achieve the closest power 2 sample size as the seasonally hourly averaged time series.

A summary of the simulation is shown in the figures below



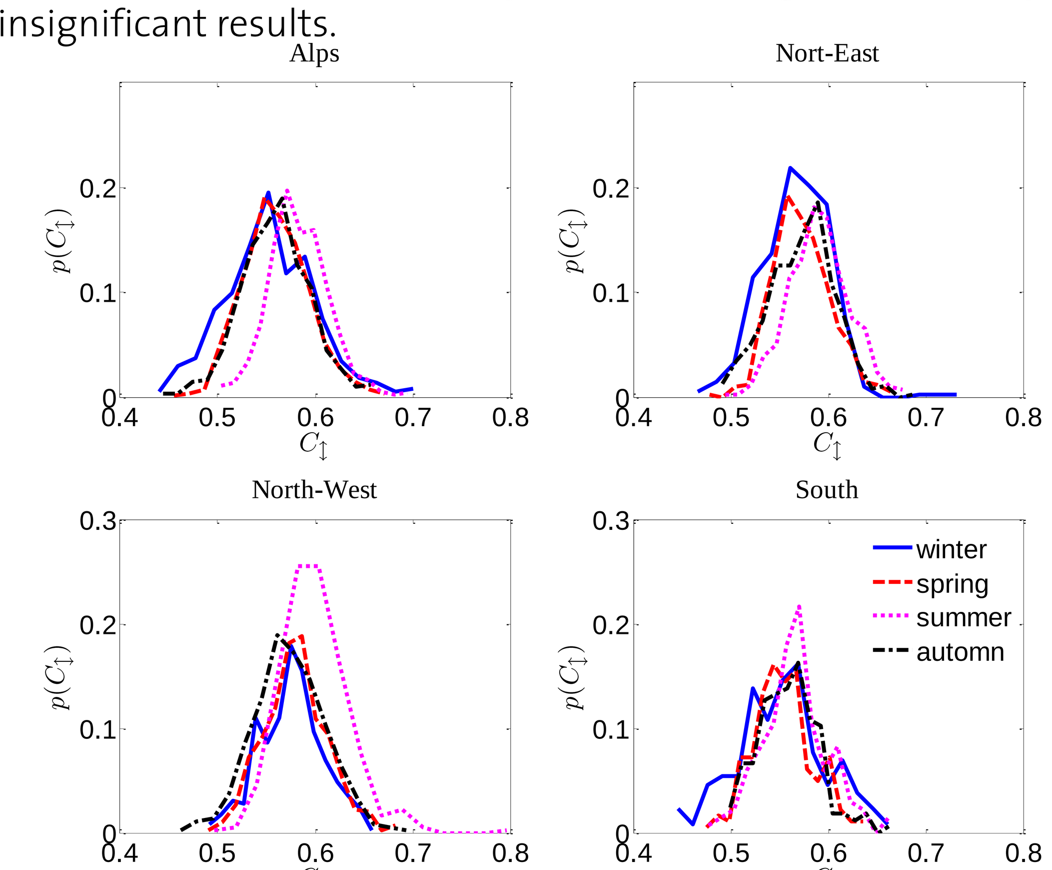
Low instrument precision effects

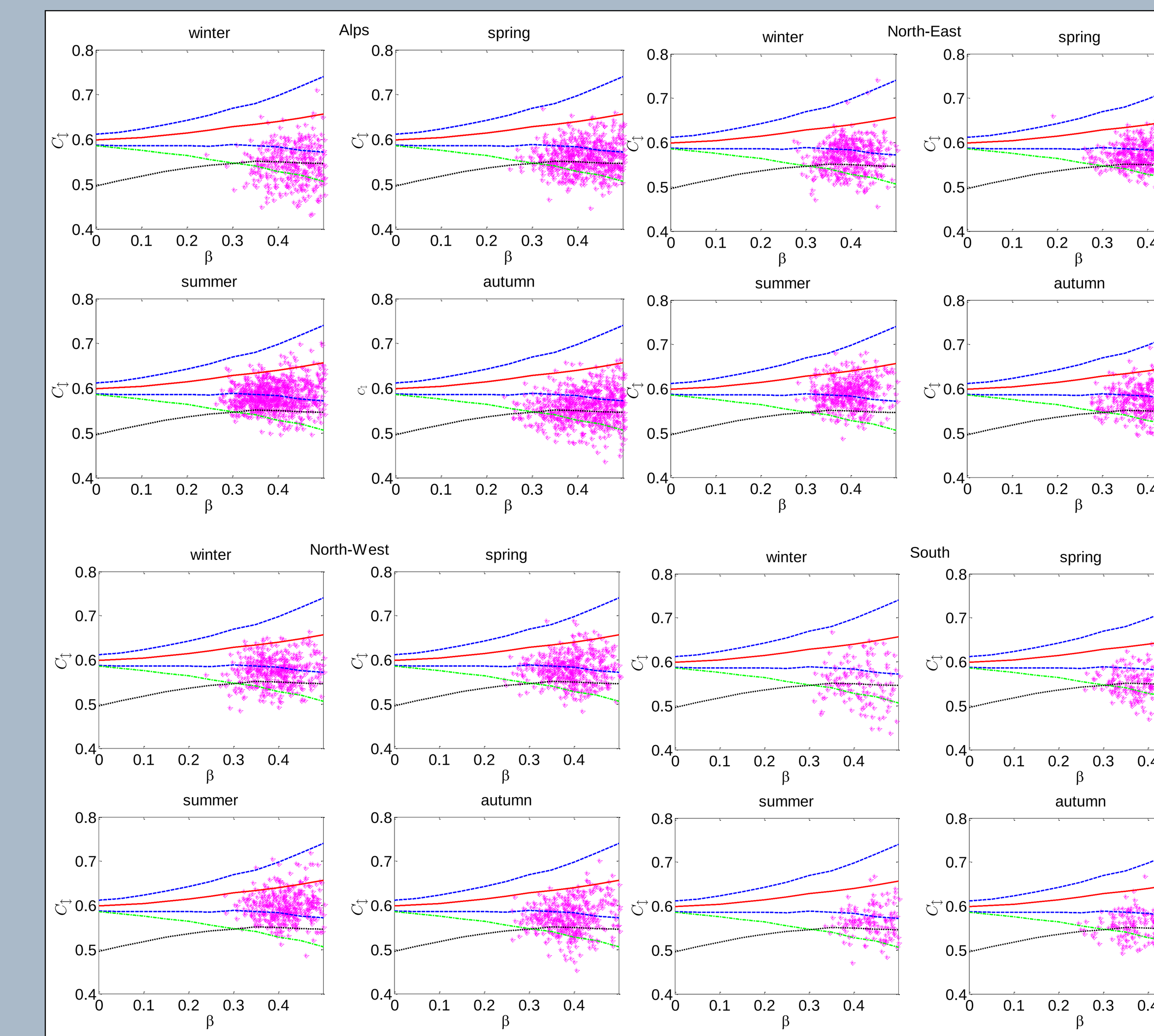
It can be theoretically proven (*Carsteanu and Foufoula-Georgiou*, 1996) that if the data follow a scaling law, the quantity $C_\updownarrow$ should be scale invariant. Based on the comparison between a low and a high precision precipitation gauge we conclude that even though precipitation time series are scaling approximately from scales of minutes up to a day, the low precision tipping bucket rain gauges lead to a wrong estimation for scales lower than one hour. This is the reason why our data analysis is based on hourly aggregated time series.



Data Analysis

The calculated oscillation coefficients for the four sub regions in a seasonal basis are shown below together with their histograms. Series with β parameter larger than 0.5 were omitted since they correspond to periods with very few data, that give statistically insignificant results.





Conclusions

- The widely used random multiplicative cascade framework with independent weights fails to describe adequately the observed behaviour of temporal precipitation at 69 Swiss stations.
- In a canonical cascade framework it appears that the weights are highly positively correlated.
- There exist a weak seasonal pattern of the correlation. Summer precipitation events caused mainly by atmospheric convection and orographic enhancement seem to be better described by the initial random multiplicative cascade scheme rather that winter stratiform events.
- No evident spatial correlation pattern seems to exist in our study domain.
- Simple Markovian correlation between cascade weights is also not able to describe the observed behaviour and this leads to the conclusion that more complex correlation patterns exist that are potentially influenced by cross scale dependencies. Also the case of cross dependence between the intermittent and continuous part of the cascade weights should be explored.
- Instrumental precision can also highly influence the correlation identification in high resolution data measured by tipping-bucket gauges.

Acknowledgments

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This study is submitted for review in Water Resources Research

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