Necessary Terms of Simulations Using Nonlinear Long Wave Equations

- **LINEARITY**
- **DISPERSION**
- **FRICTION**

**Linearity** may be sufficient in deep water. **Nonlinearity** is essential in shallow zone. The friction term becomes important in the shallow zone and at land.

The **dispersion** becomes important in long distance propagation and also when the wave amplifies in the shallow zones.

1. There are numerous numerical approaches for assessment of tsunamis from source to target locations.
2. Tsunami generation and propagation are solved by numerical modeling with a reasonable and acceptable error limit.
3. However, tsunami behavior in shallow region and at land are not solved clearly because of
   - breaking
   - dispersion due to the increase of $\eta$ at shallow region
   - Dispersion due to long distance propagation
   - shoaling

**NONLINEAR SHALLOW WATER EQUATIONS**

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$$

**Bottom Friction**

$$\rho \frac{\partial M}{\partial x} + \rho \frac{\partial N}{\partial y} - \rho \frac{\partial \eta}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\rho}{\tau} \frac{\partial \eta}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho}{\tau} \frac{\partial \eta}{\partial y} \right)$$

**Dispersion Potential**

$$\phi = \frac{1}{2} \left( \frac{\partial M}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial N}{\partial y} \right)^2$$

**NONLINEAR DISPERSIVE SHALLOW WATER EQUATIONS (BOUSSINESQ EQUATIONS)**

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$$

$$\frac{\partial M}{\partial t} + \frac{\partial M}{\partial x} \left( \frac{\partial M}{\partial x} \right) + \frac{\partial M}{\partial y} \left( \frac{\partial N}{\partial y} \right) - \frac{\partial}{\partial x} \left( \frac{\rho}{\tau} \frac{\partial \eta}{\partial x} \right) - \frac{\partial}{\partial y} \left( \frac{\rho}{\tau} \frac{\partial \eta}{\partial y} \right) = \frac{\partial \phi}{\partial x}$$

$$\frac{\partial N}{\partial t} + \frac{\partial M}{\partial x} \left( \frac{\partial N}{\partial x} \right) + \frac{\partial N}{\partial y} \left( \frac{\partial M}{\partial y} \right) - \frac{\partial}{\partial x} \left( \frac{\rho}{\tau} \frac{\partial \eta}{\partial x} \right) - \frac{\partial}{\partial y} \left( \frac{\rho}{\tau} \frac{\partial \eta}{\partial y} \right) = \frac{\partial \phi}{\partial y}$$

**IMPORTANT RATIOS**

- **Depth to wave length**
- **Amplitude to depth**

**SUMMARY:** IN WHICH STAGE OF TSUNAMI MODELING IS THE DISPERSION IMPORTANT?

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