Error estimations for seismic source inversions

Zacharie Duputel[1], Luis Rivera[1],
Yukitoshi Fukahata[2] and Hiroo Kanamori[3]

[1] IPGS, Université de Strasbourg, CNRS, France
[2] DPRI, Kyoto University, Kyoto, Japan
[3] Seismolab, Caltech, USA

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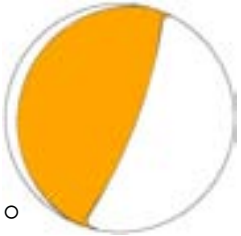
Introduction

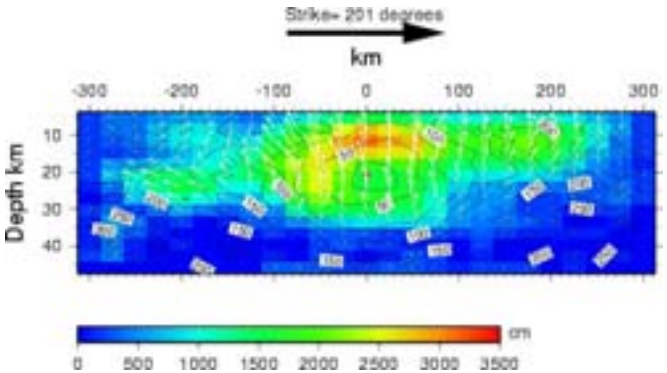
2011 Tohoku-oki earthquake (M_w=9.0): Examples of source inversion results and applications

Source Models:

M_w=9.0
ts=hd=68sec
latitude=37.92°
longitude=143.11°
depth=19.5km
$\phi_s = 196^\circ, \delta_s = 12^\circ, \lambda_s = 85^\circ$
$\phi_s = 21^\circ, \delta_s = 78^\circ, \lambda_s = 91^\circ$

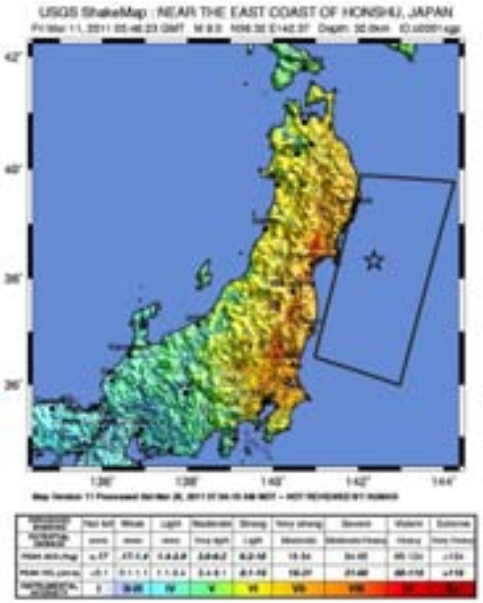


Wphase-solution

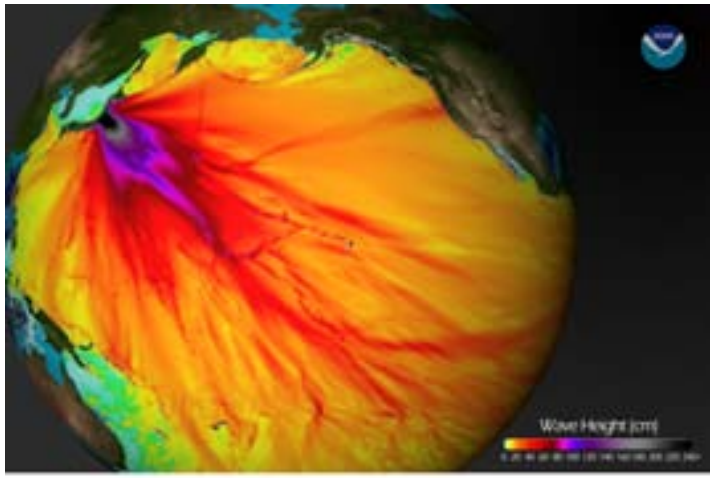


Slip-distribution (Caltech)

Applications:



Shake-map (USGS)



Tsunami-modeling (NOAA)

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W phase source inversion algorithm

- Fast source inversion of moderate to large earthquakes ($M_w \geq 6.5$)

- Data, **d**:
  - Long period seismograms

- Model, **m**:
  - Seismic moment tensor

Linear Theory:
$\mathbf{d} = \mathbf{G}\mathbf{m}$

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$\mathbf{C}_\mathrm{d}$, $\mathbf{C}_\mathrm{T}$, $\mathbf{C}_\mathrm{m}$ and all that

Notation

a simple case: linear gaussian case

$$\widetilde{\mathbf{m}} = \left(\mathbf{G}^t\mathbf{C}_\mathrm{D}^{-1}\mathbf{G} + \mathbf{C}_\mathrm{m}^{-1}\right)^{-1}\left(\mathbf{G}^t\mathbf{C}_\mathrm{D}^{-1}\mathbf{d}_\mathrm{obs} + \mathbf{C}_\mathrm{m}^{-1}\mathbf{m}_\mathrm{prior}\right)$$

- No prior information $\mathbf{m}_\mathrm{prior}$ ($\mathbf{C}_\mathrm{m}^{-1} = \mathbf{0}$),
- Two sources of error, data covariance: $\mathbf{C}_\mathrm{D} = \mathbf{C}_\mathrm{d} + \mathbf{C}_\mathrm{T}$
  - Observational error $\rightarrow$ covariance $\mathbf{C}_\mathrm{d}$
  - Modeling error $\rightarrow$ mismodeling covariance $\mathbf{C}_\mathrm{T}$
- Solution:

$$\widetilde{\mathbf{m}} = \left(\mathbf{G}^t\mathbf{C}_\mathrm{D}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^t\mathbf{C}_\mathrm{D}^{-1}\mathbf{d}_\mathrm{obs}$$

$$\widetilde{\mathbf{C}}_\mathrm{m} = \left(\mathbf{G}^t\mathbf{C}_\mathrm{D}^{-1}\mathbf{G}\right)^{-1}$$

$$\cancel{\mathbf{C}_\mathrm{D} = \sigma^2\mathbf{I}_\mathrm{d}} \rightarrow \mathbf{C}_\mathrm{D} = ?$$

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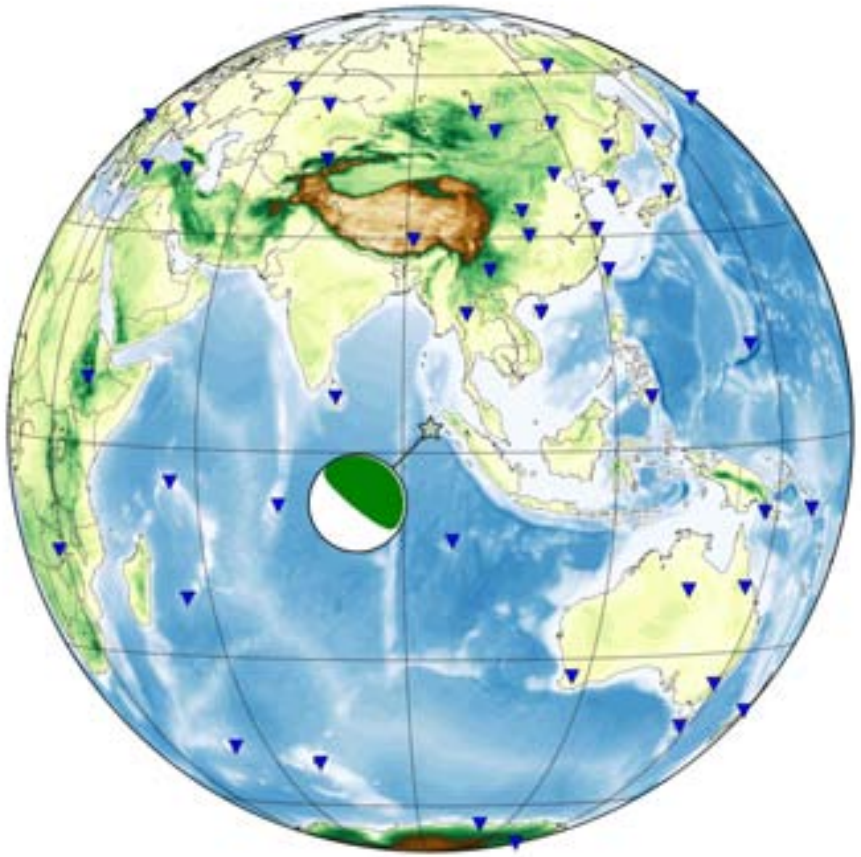
Observational error
Dealing with noisy traces: W phase standard approach

Source-network geometry



Real noise + synthetic signals (M_W=6.0, 7.0, 8.0)

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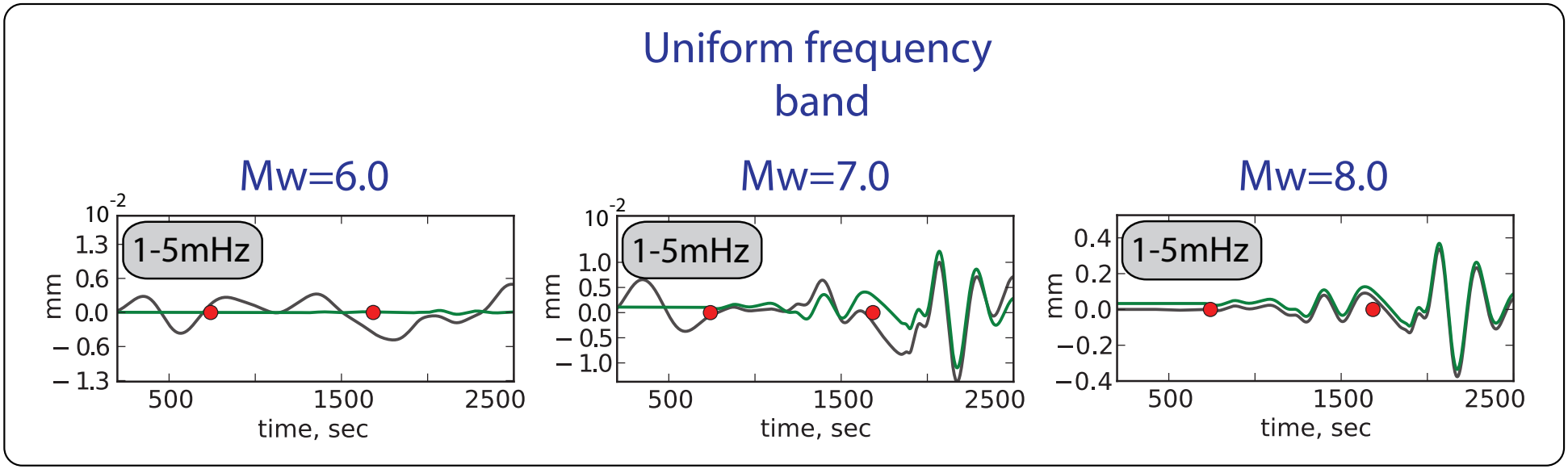
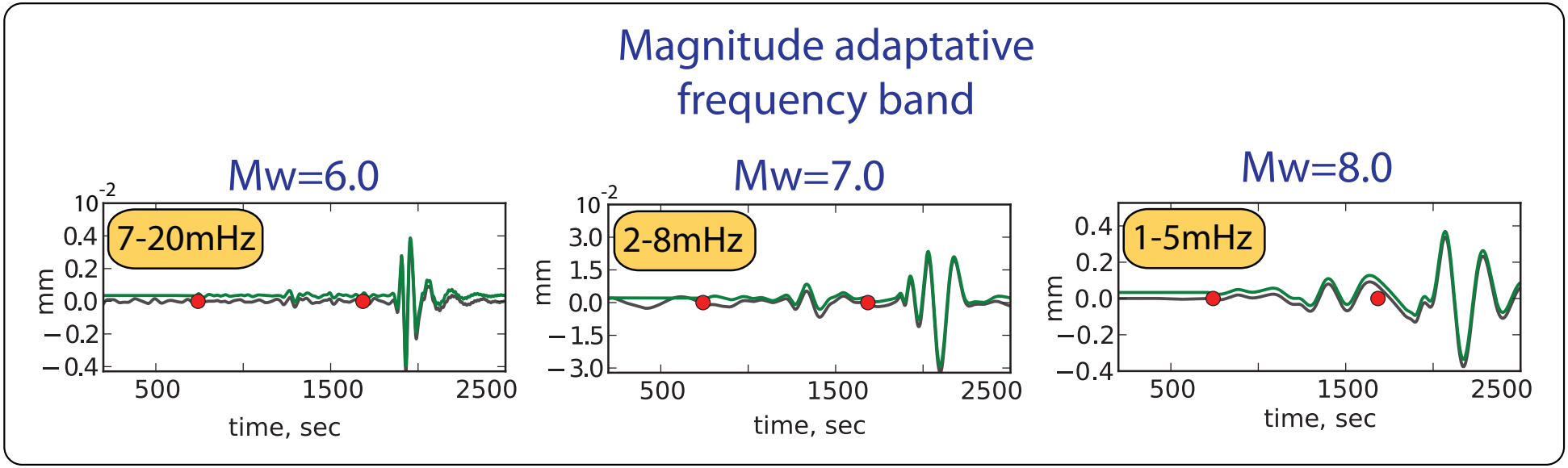
Dealing with noisy traces: W phase standard approach

Example for CRZF (G, LHZ, Crozet Island)

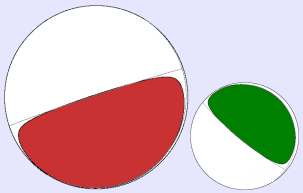




Dealing with noisy traces: W phase standard approach

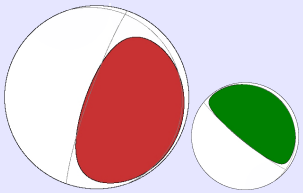
- Uniform frequency band: 1-5mHz
- Raw data set (no data screening)

$M_w = 6.0$



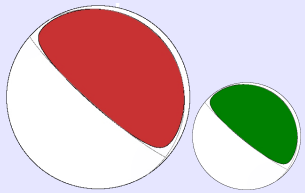
74 channels, 1-5mHz
$M_{w-wcmt} = 7.0$

$M_w = 7.0$



74 channels, 1-5mHz
$M_{w-wcmt} = 7.0$

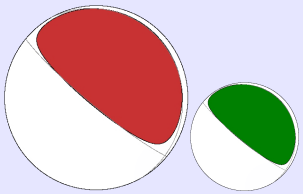
$M_w = 8.0$



74 channels, 1-5mHz
$M_{w-wcmt} = 8.0$

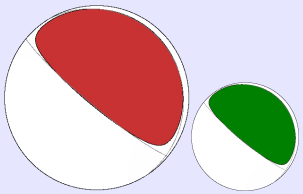
- Magnitude adaptive frequency band (preliminary M_w needed)
- Data screening (pre-event noise level, misfit)

$M_w = 6.0$



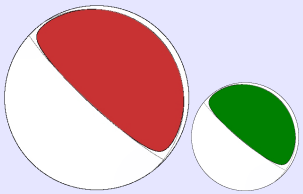
36 channels, 7-20mHz
$M_{w-wcmt} = 6.0$

$M_w = 7.0$



47 channels, 2-8mHz
$M_{w-wcmt} = 7.0$

$M_w = 8.0$



62 channels, 1-5mHz
$M_{w-wcmt} = 8.0$

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Dealing with noisy traces: W phase standard approach

- Uniform frequency band: 1-5mHz
- Raw data set (no data screening)

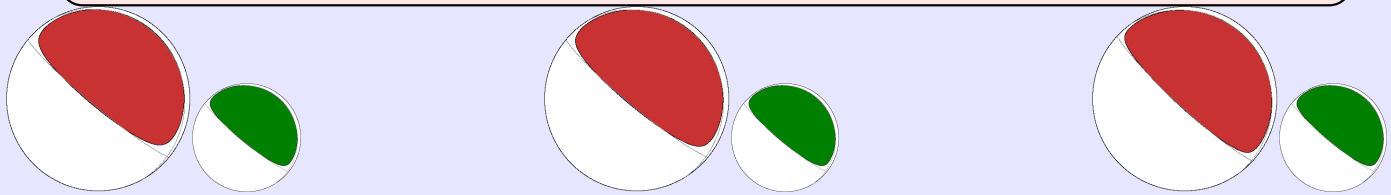
$M_w = 6.0$ $M_w = 7.0$ $M_w = 8.0$

74 cha mHz
M_w)

- Magn needed)
- Data

An alternative:

- No preliminary magnitude,
- Uniform frequency band: 1-5mHz,
- Complete data set (no data screening),
- Including data error analysis.



36 channels, 7-20mHz
$M_{w-wcmt} = 6.0$

47 channels, 2-8mHz
$M_{w-wcmt} = 7.0$

62 channels, 1-5mHz
$M_{w-wcmt} = 8.0$

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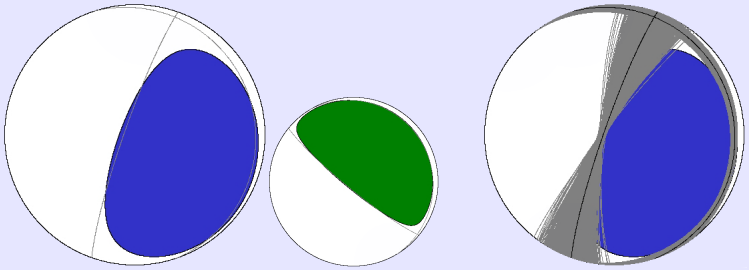
Data covariance matrix $\mathbf{C}_d$: Inversion for $M_w = 7.0$

1. Uniform observational error:

   $M_{w-wcmt} = 7.0$

   1-5mHz

   74 channels

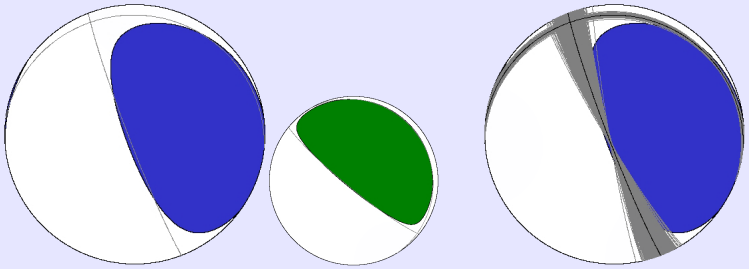


2. Observational error per-trace:

   $M_{w-wcmt} = 7.0$

   1-5mHz

   74 channels

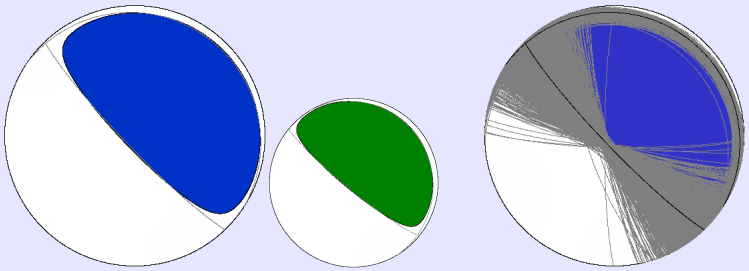


3. Data covariance terms:

   $M_{w-wcmt} = 7.0$

   1-5mHz

   74 channels



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$$\mathbf{C}_D = \mathbf{C}_d + \mathbf{C}_T$$

Observational error $\rightarrow \mathbf{C}_d$ ✔

Modeling error $\rightarrow \mathbf{C}_T$

- We often know that our modeling is not perfect e.g.:
  - Earth model
  - mislocation (lat, lon, depth)
  - finiteness
  - assumed fault plane geometry (ϕ, δ, L, etc.)
  - etc.
- How to introduce this uncertainty into the solution of the problem?
- One possibility is to allow for a non-deterministic theory incorporating such fuzzyness.

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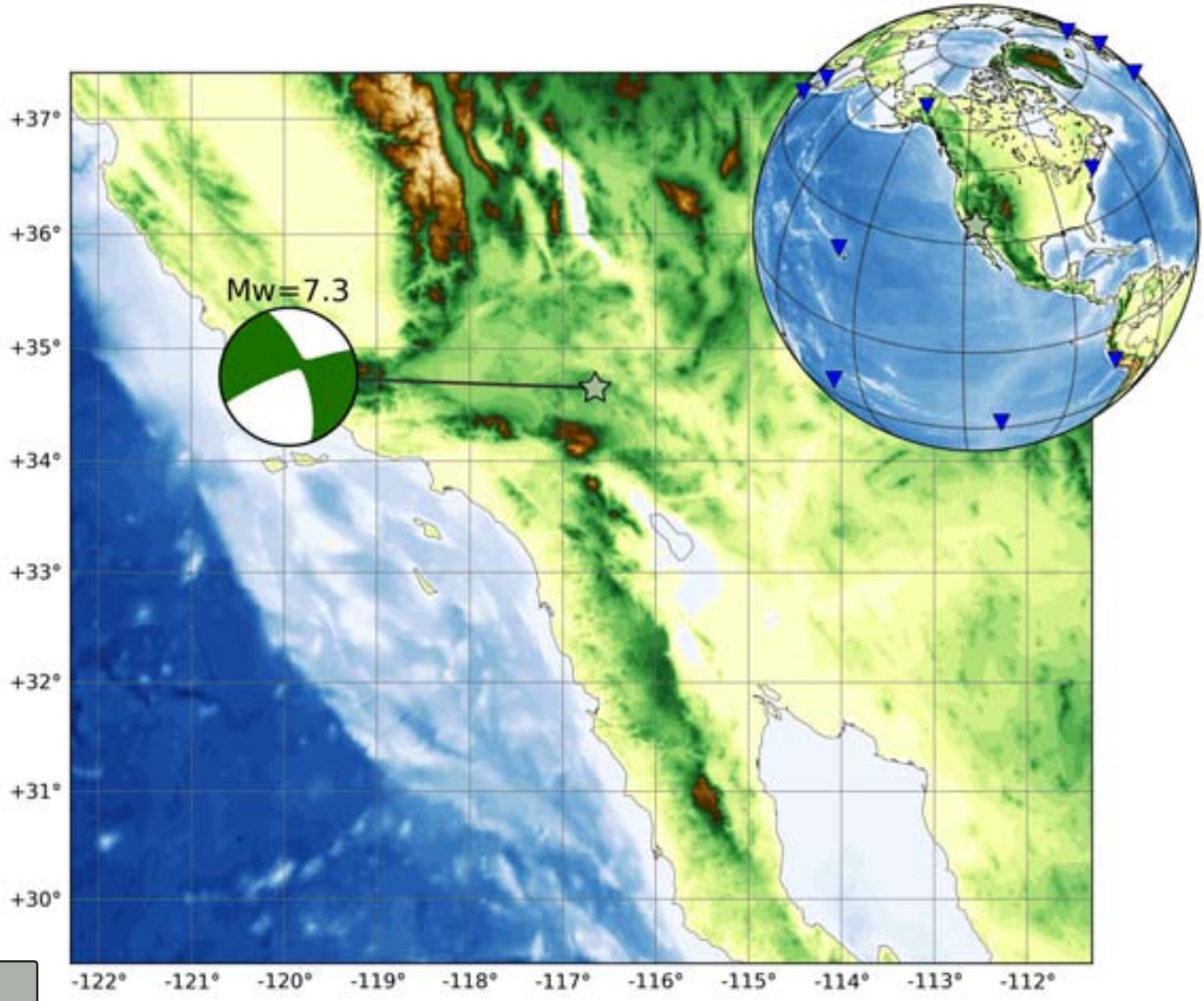
Mismodeling: C_T, NS mislocation (65km), Synth Landers



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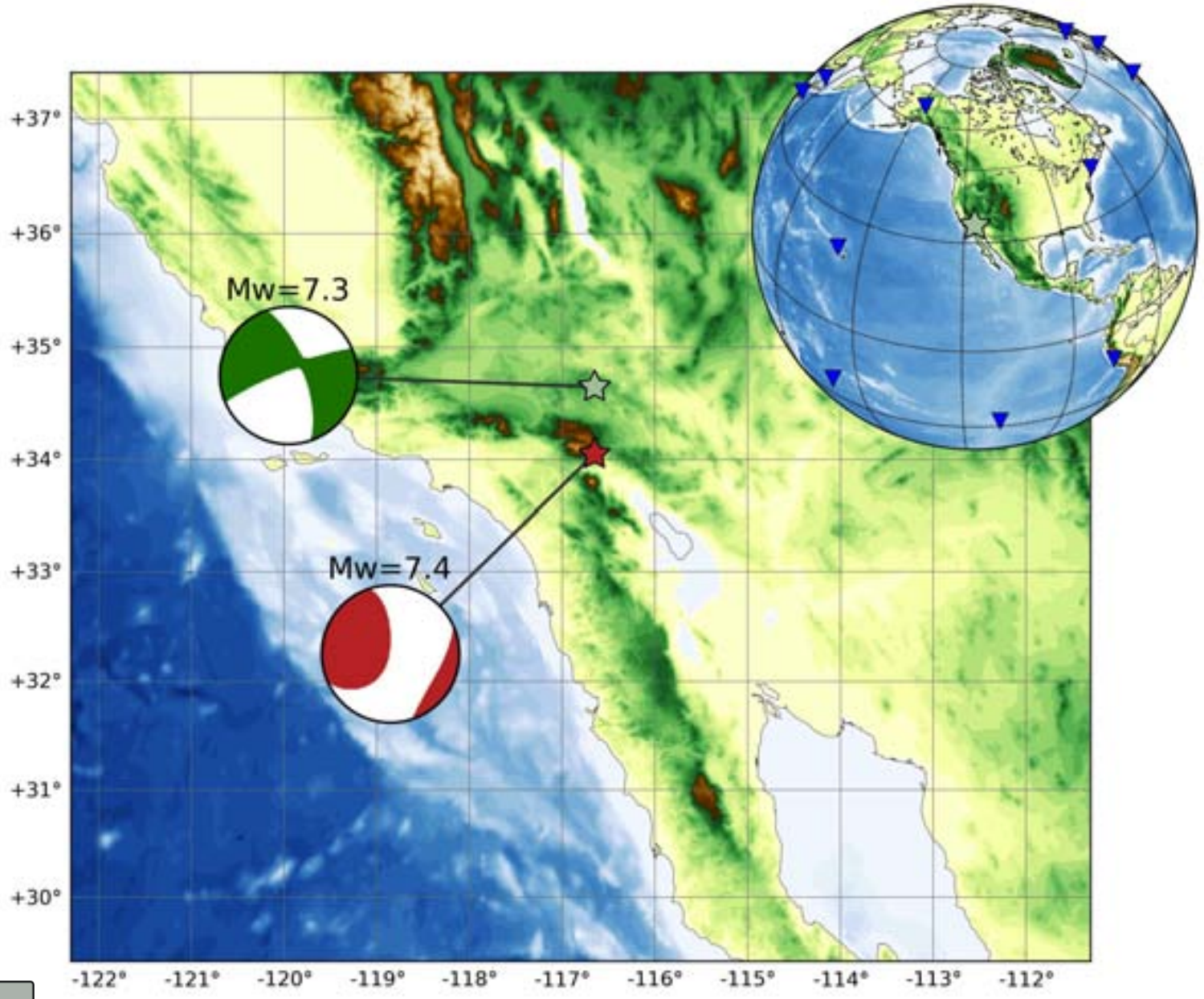
Mismodeling: C_T, NS mislocation (65km), Synth Landers



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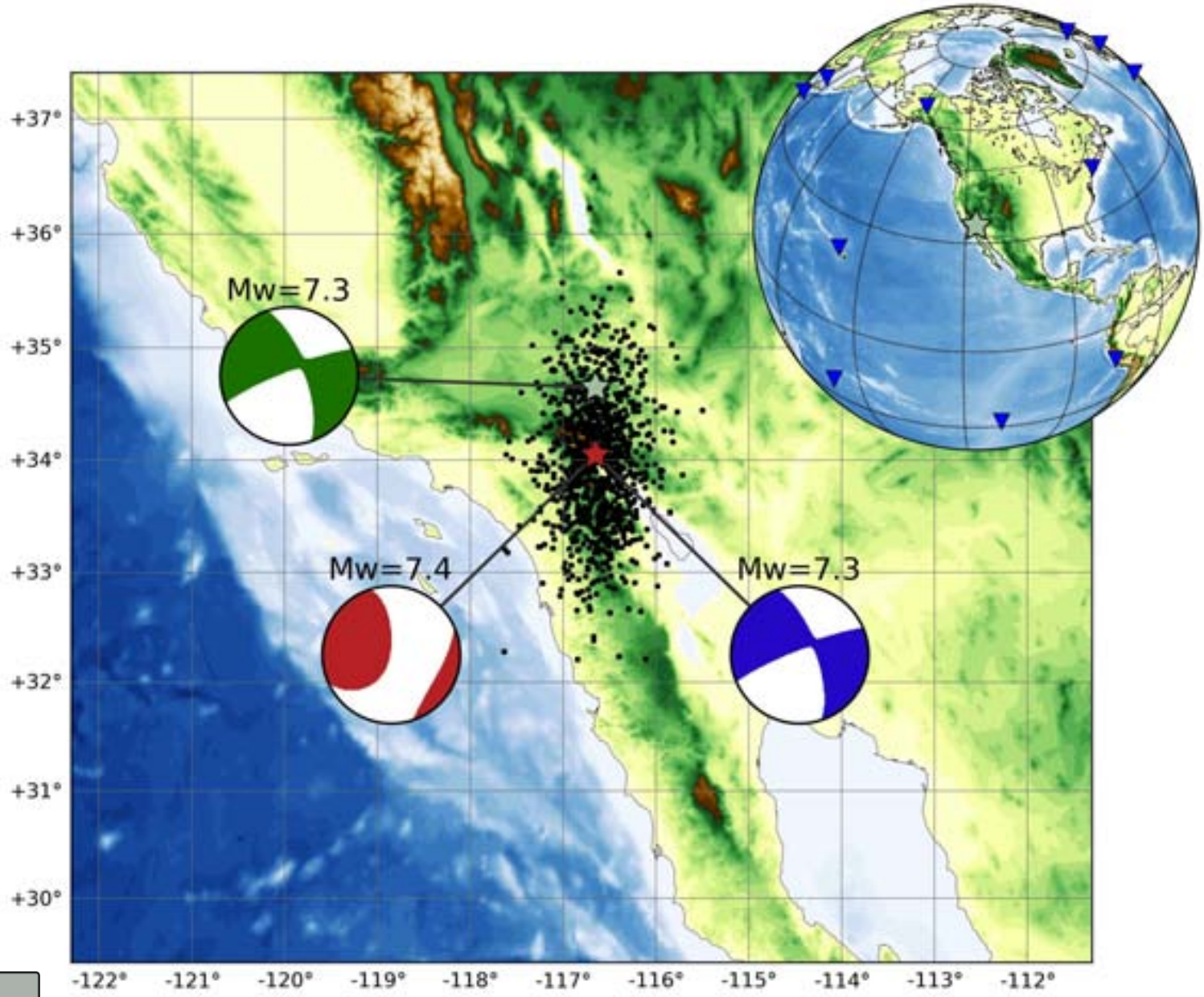
Mismodeling: $\mathbf{C_T}$, NS mislocation (65km), Synth Landers



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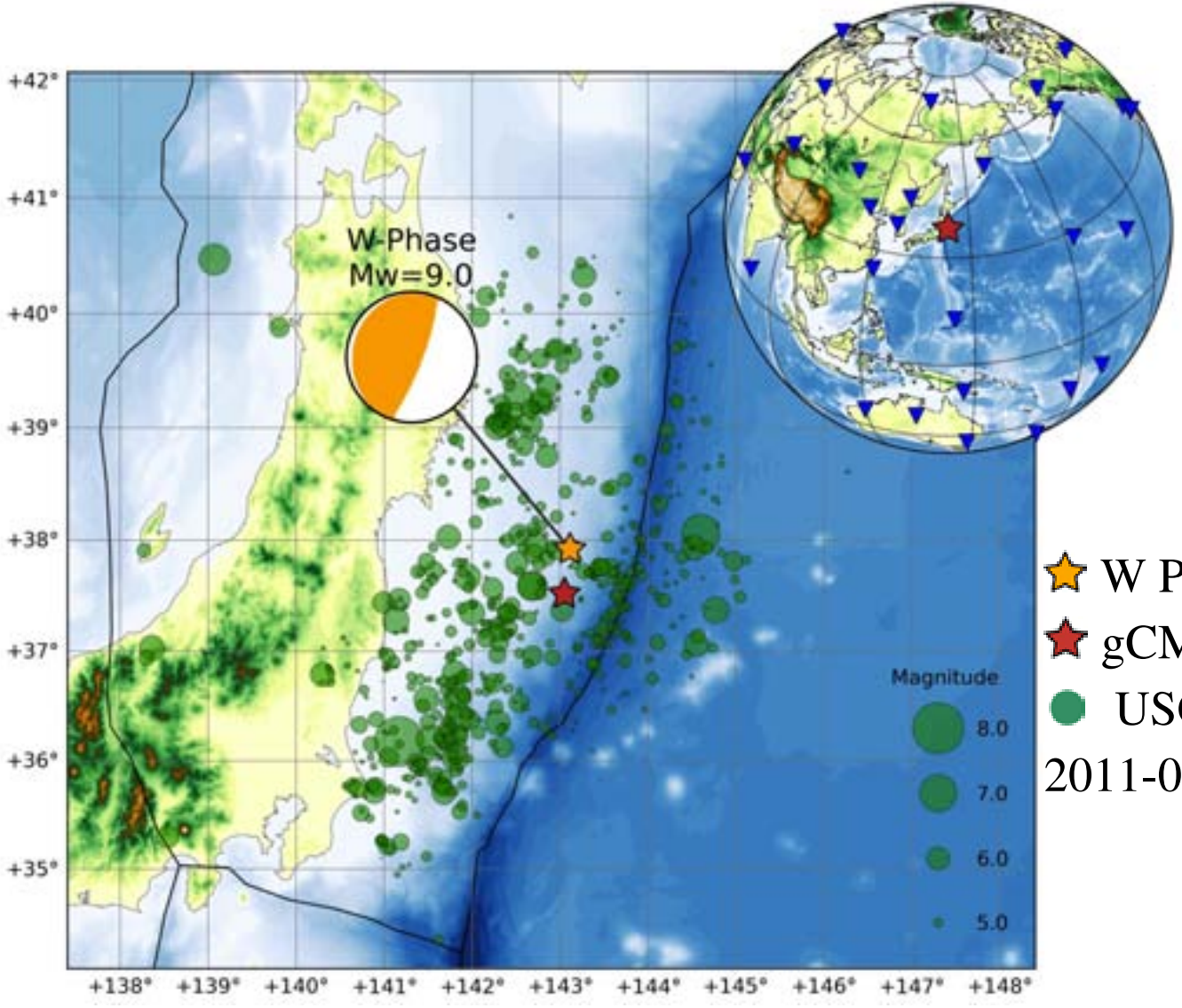
Black dots image the centroid location uncertainty ($\mathbf{C_x}$)

Real data: Tohoku-oki, 2011. Mw=9.0

W phase solution



W Phase centroid location

gCMT centroid location

USGS location of events from 2011-03-11 to 2011-03-20.

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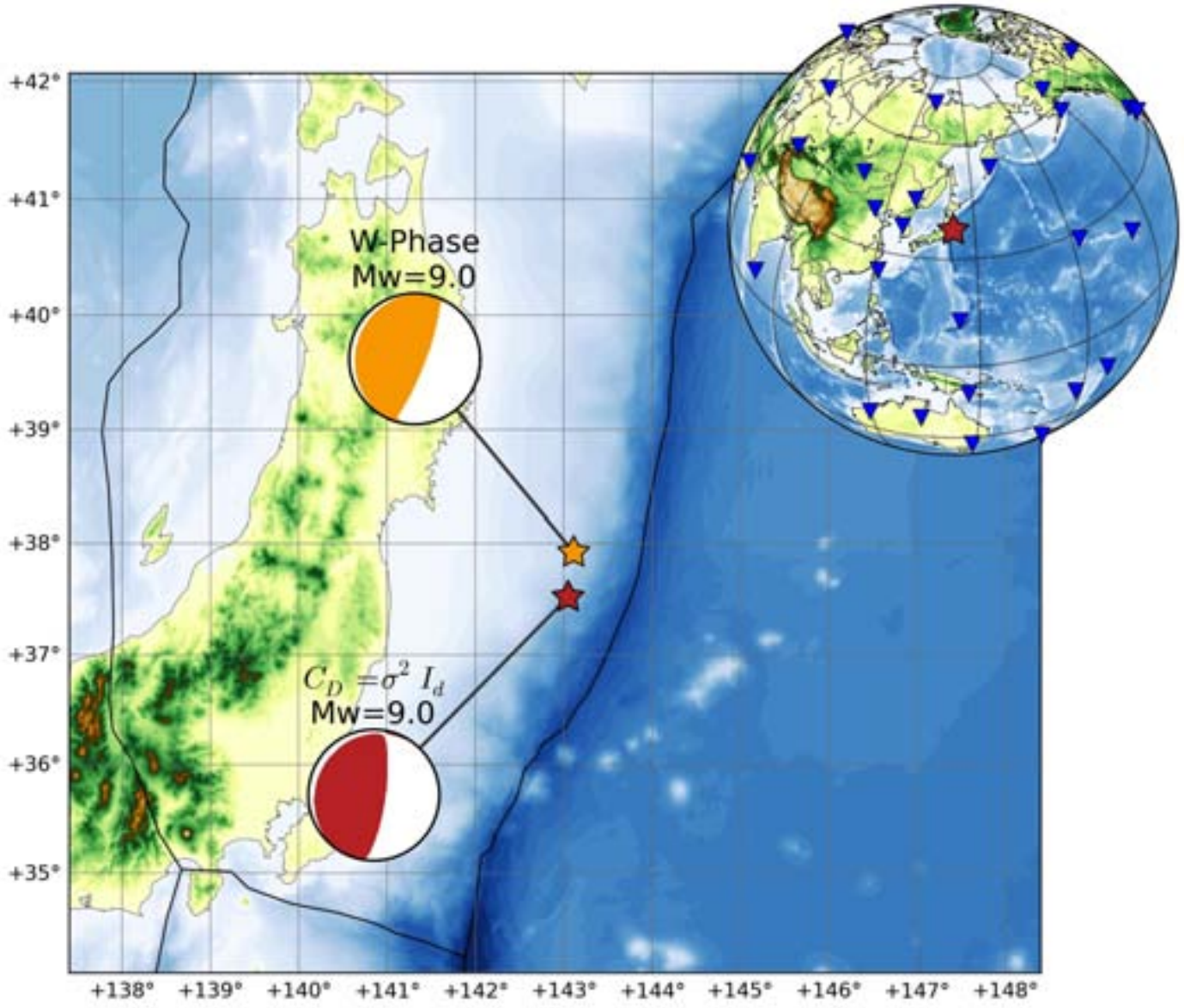
Real data: Tohoku-oki, 2011. Mw=9.0

$$\mathbf{C}_\mathrm{D} = \sigma^2 I_d$$



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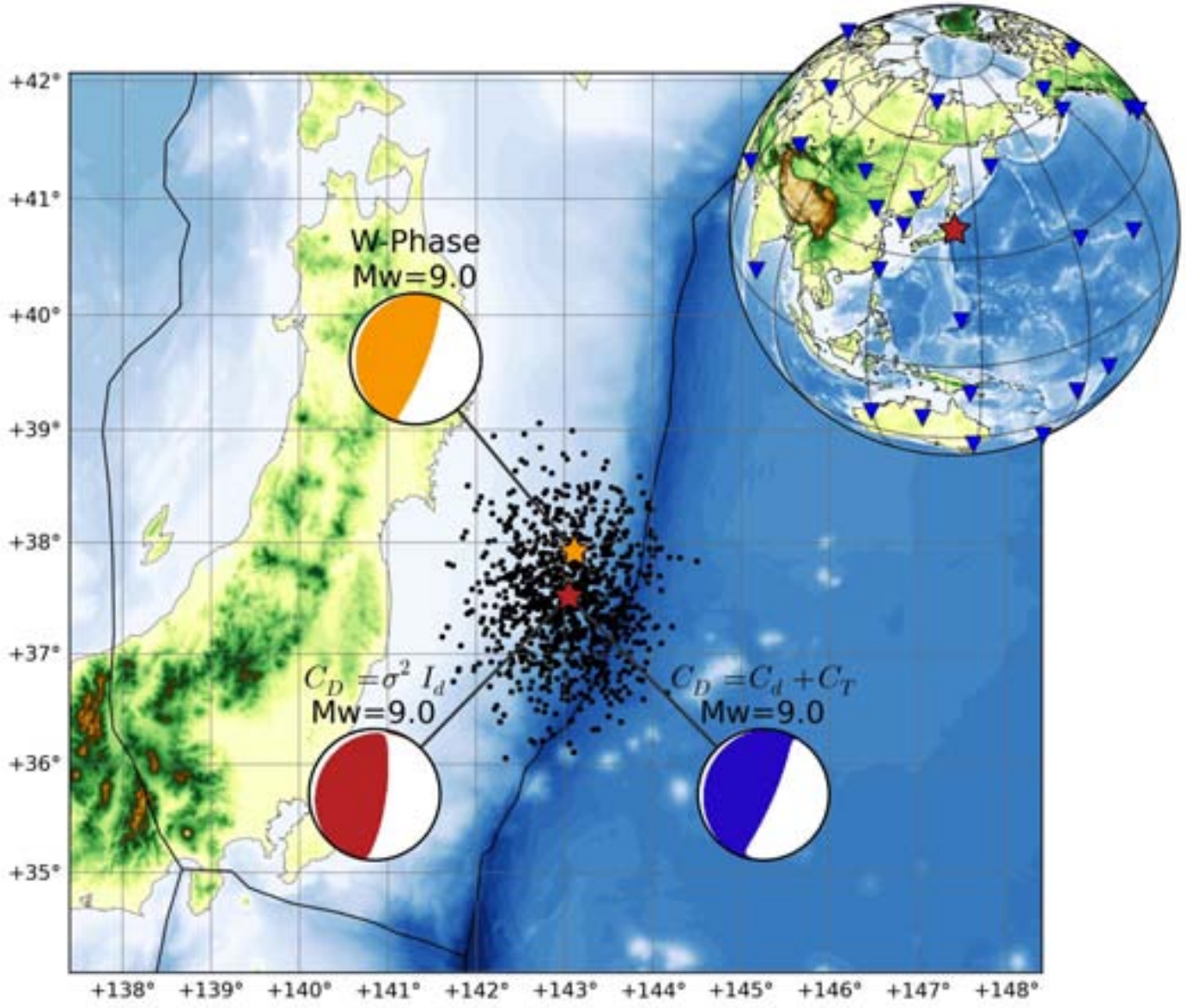
Real data: Tohoku-oki, 2011. Mw=9.0

$$\mathbf{C}_D = \mathbf{C}_d + \mathbf{C}_T$$



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Summary and Conclusion

- We advocate performing a more formal error analysis
  - To have more realistic estimate of errors
  - To improve the solution itself
- Three ingredients entering into C_D
  - Noise level measurements per-trace (non uniform diagonal).
  - Off-diagonal terms (e.g. oversampling).
  - Mismodeling which can significantly contribute to the solution. (full covariance).
- The approach presented here can be applied to much more general inverse problem.
- Practical problem; C_D can easily be a very large matrix. Say $10^4 \times 10^4$, or even larger (e.g. InSAR).

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Thanks for your attention