A wetting and drying algorithm for non-hydrostatic models with combined pressure/free-surface

Simon W. Funke, C.C. Pain, S.C. Kramer and M.D. Piggott
Applied Modelling and Computation Group, Department of Earth Science and Engineering
Imperial College London

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1. Introduction

The recent tsunami event in Japan has underscored the importance of understanding free-surface flows and wetting and drying processes. In this work, a wetting and drying algorithm for free-surface problems is proposed, which, in contrast to most other methods, does not assume a hydrostatic pressure. Non-hydrostatic flow features are apparent in coastal regions (thus also in wetting and drying regions), but also play a significant role on a larger scale, see Figure 1.

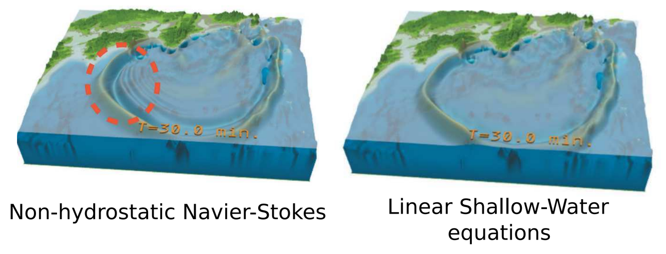


Figure 1: Furumura and Saito (2007) showed the importance of solving the Navier-Stokes equations for tsunami prediction.

2. Underlying equations

- Incompressible Navier Stokes equations with Boussinesq approximation:

$$\rho_0\left(\frac{\partial \vec{u}}{\partial t}+\vec{u}\cdot\nabla\vec{u}\right)-\nabla\cdot\mu\nabla\vec{u}+\nabla\tilde{p}=-\rho' g\vec{n}_z,$$

$$\nabla\cdot\vec{u}=0,$$

where $\vec{u}$ is the velocity, $\tilde{p}=p+\rho_0 gz$ the "piezometric" pressure, ρ_0 and ρ' the background and perturbation density, μ the viscosity and g and $\vec{n}_z$ the gravity magnitude and direction, respectively.

- Boundary conditions in an ocean-like domain:
  - Bottom/sides: *No-normal flow boundary condition*: $0=\vec{n}\cdot\vec{u}$.
  - Free-surface: For simplicity, the atmospheric pressure is assumed to be zero. Then the relationship between pressure on the surface and free-surface elevation η is given by:

$$\eta=\max\left(\frac{\tilde{p}}{\rho_0 g}, b+d_0\right). \quad (1)$$

Here b is the bathymetry function and d_0 enforces a positive water level in the domain. From that, the *combined kinematic free-surface boundary condition with wetting and drying* is derived (Funke et al. (2011)):

$$\frac{\vec{n}\cdot\vec{n}_z}{\rho_0 g}\frac{\partial}{\partial t}\max\left(\tilde{p},\rho_0 g(b+d_0)\right)=\vec{n}\cdot\vec{u}. \quad (2)$$

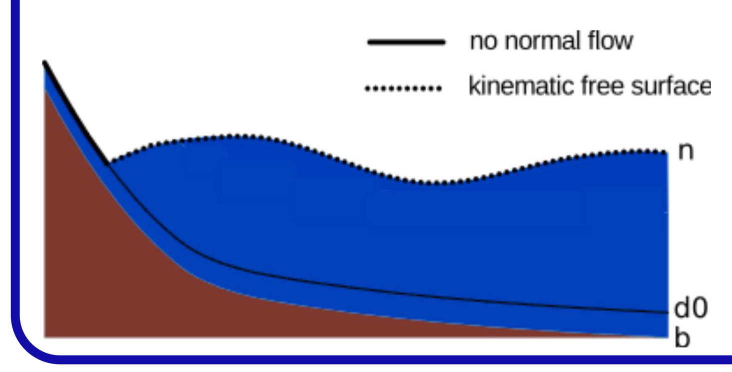
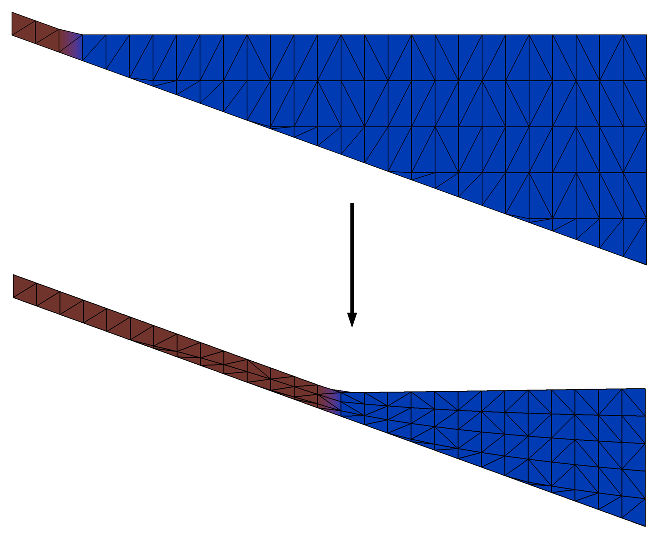


Figure 2: The free-surface boundary condition (2) reduces to the no-normal flow b.c. in dry regions and to the kinematic free-surface b.c. elsewhere.

3. Implementation

- Implemented in the finite-element code Fluidity-ICOM, see Ford et al. (2004).
- The implicit θ-method is used for the time discretisation.
- The resulting non-linear system is solved using an extended pressure-correction scheme (Kramer et al. (2010, in preparation)).
- At the end of each iteration the free-surface elevation is obtained from equation (1) and the mesh is deformed accordingly.
- The mesh deformation is performed only vertically, hence no remeshing is necessary:



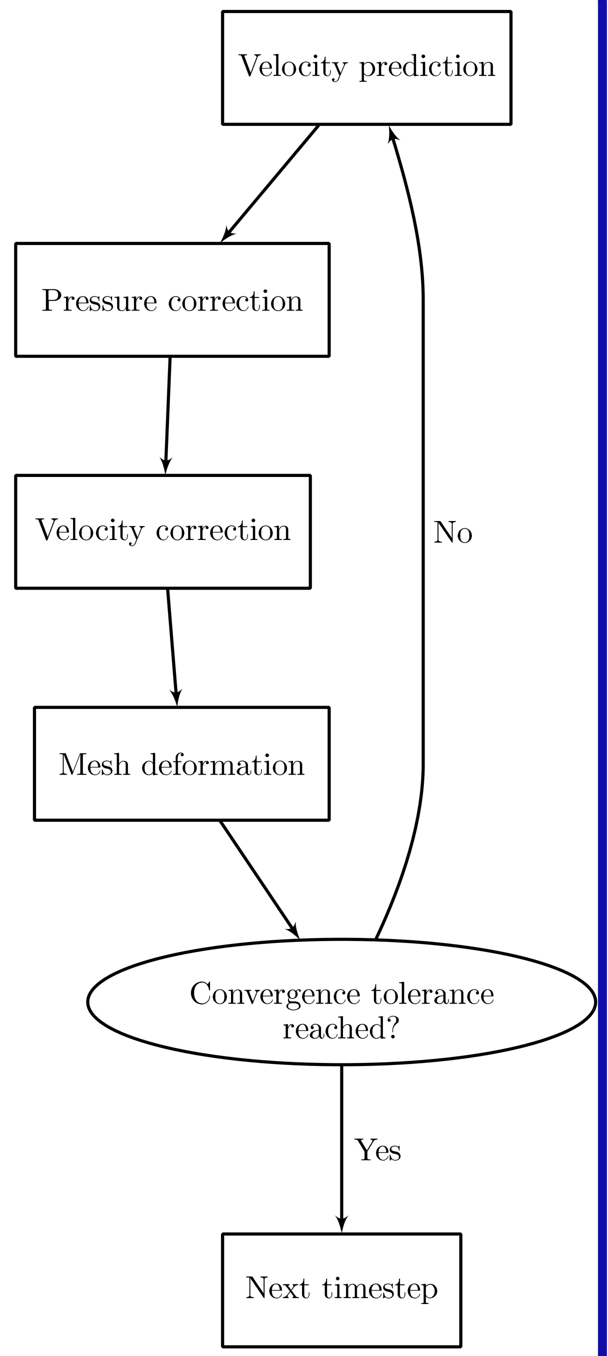


Figure 3: The extended pressure-correction scheme.

4. Conservation properties and validation

- Volume/mass conservation was proved (Funke et al. (2011)) for:
  - the continuous equations from Section 2.
  - the extended pressure-correction scheme from Section 3.
- Three-dimensional, non-viscous testcase in a closed basin, with analytical solution by Thacker (1981).
  - Highly accurate, see Figure 4. No visible numerical damping.
  - Volume is conserved up to machine precision ($< 10^{-14}$).

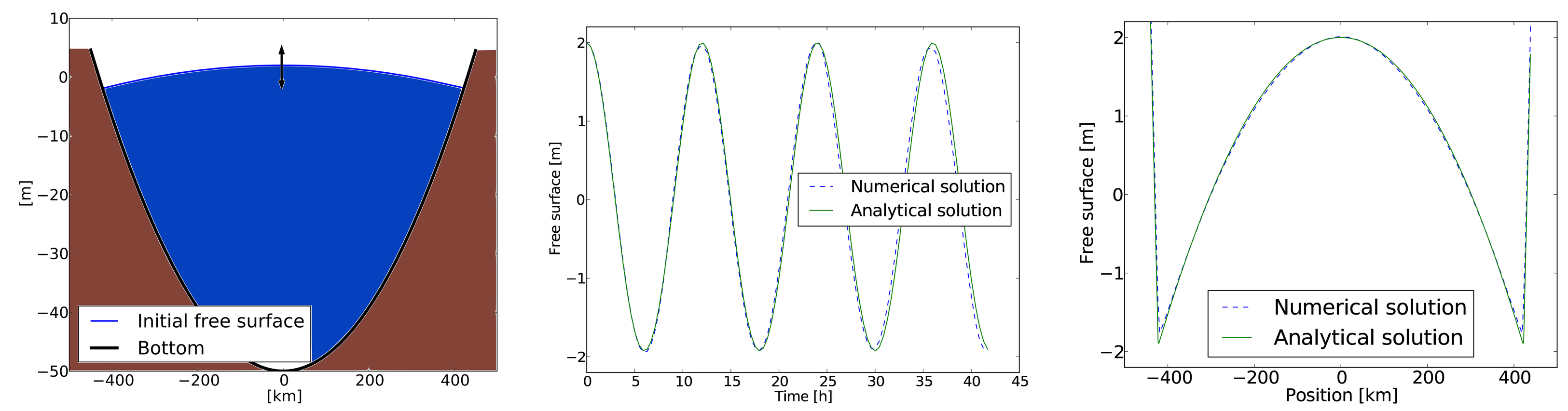
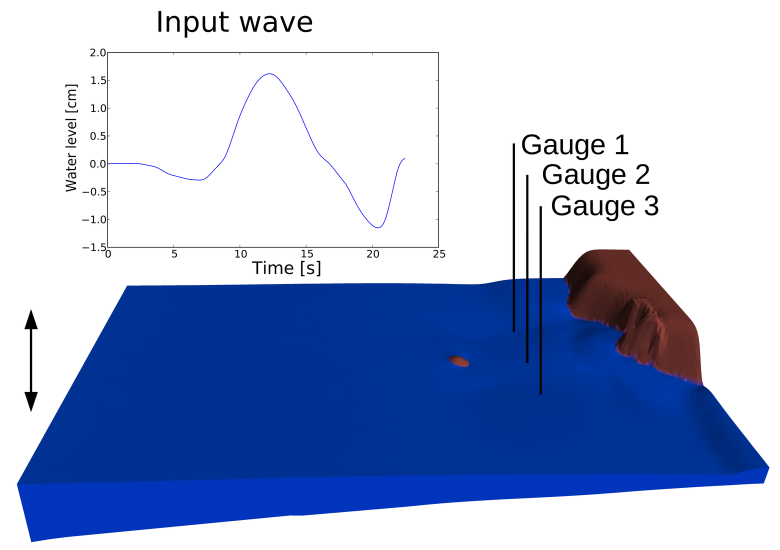
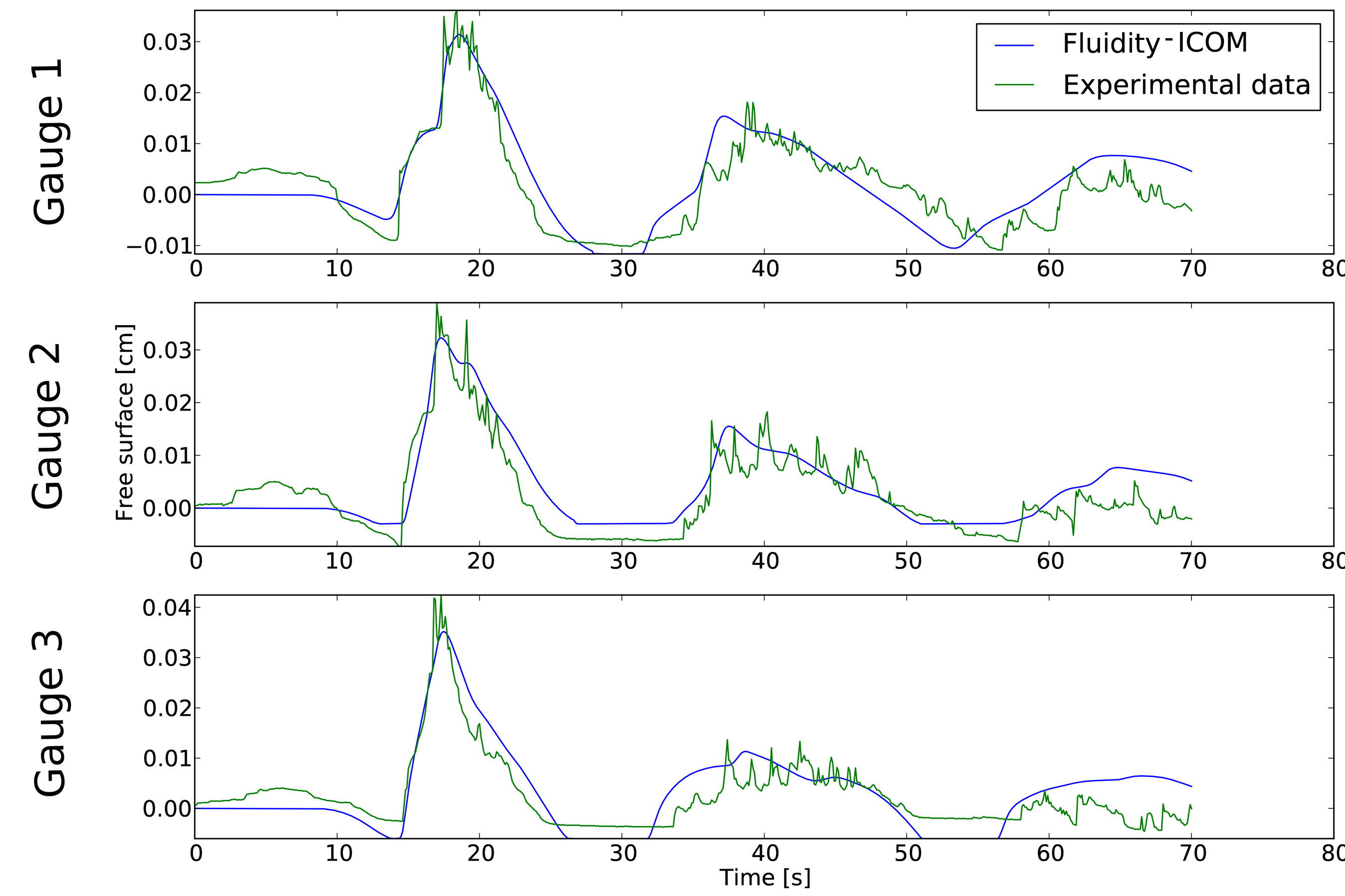


Figure 4: From left to right: Initial setup, solution at center of the domain and after 24h.

5. Tsunami simulation

- Laboratory experiment of the Hokkaido-Nansei-Oki tsunami in 1993, Japan (Androsov et al. (2008)).
- Dimensions: $5.4m \times 3.4m$.
- Wetting and drying occurs both on the Okushiri island and the mainland.





6. Summary

The proposed wetting and drying method

- is stable and accurate.
- conserves volume/mass.
- uses a computationally cheap mesh deformation instead of remeshing.
- does not introduce any restrictions to the discretisation, in particular unstructured meshes and/or implicit timestepping methods can be used (and therefore the timestep size is not restricted by the CFL condition).

References

A. Androsov, S. Harig, J. Behrens, J. Schröter, and S. Danilov. Tsunami modelling on unstructured grids: verification and validation. In *Proceedings of the International Conference on Tsunami Warning (ICTW), Bali, Indonesia.*, volume 6, 2008.

R. Ford, C.C. Pain, M.D. Piggott, A.J.H. Goddard, C.R.E. De Oliveira, and A.P. Umpleby. A nonhydrostatic finite-element model for three-dimensional stratified oceanic flows. Part I: model formulation. *Monthly Weather Review*, 132(12):2816–2831, 2004.

S.W. Funke, C.C. Pain, Kramer S.C., and M.D. Piggott. A wetting and drying algorithm with a combined pressure/free-surface formulation for non-hydrostatic models. *Advances in Water Resources*, 2011. submitted.

T. Furumura and T. Saito. Three Dimensional Simulations of Tsunami Generation and Propagation. *Annual Report of the Earth Simulator Center April*, 2008, 2007.

S.C. Kramer, C.J. Cotter, and C.C. Pain. Solving the Poisson equation on small aspect ratio domains using unstructured meshes. *Ocean Modelling*, 35: 253–263, 2010. ISSN 1463-5003.

S.C. Kramer et al. A free-surface method for fully unstructured non-hydrostatic ocean models. in preparation.

W.C. Thacker. Some exact solutions to the nonlinear shallow-water wave equations. *Journal of Fluid Mechanics*, 107:499–508, 1981.