Seismological Modeling of Traveltime Curves with Global Velocity Field Parameterized by Trigonometric Series

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INTRODUCTION

Motivation: implementation of **Inversion** Procedure that can be able to give us an Image of Seismological **Earth Interior Structures** using **Traveltimes** as **Data**;

Means: an adequate Expression to **Quantify** Earth Interior;

Possibility: to represent Seismic Velocity Fields by a **Parameterization** such as **Trigonometric Series**, where theirs **Coefficients** are thought as **Parameters to be Estimated** by some possible **Inversion Procedure**;

Question: is it **Trigonometric Series** an appropriated way to **Parameterize Earth Interior P** and **S Velocity Fields?**

Strategy: to observe a Numerical Reproduction of some Events of the **Jeffreys-Bullen Traveltime Curves** using **Earth Interior Velocities Fields Parameterized by Trigonometric Series.**

THEORY

Mathematical Aspects

Trigonometric Series

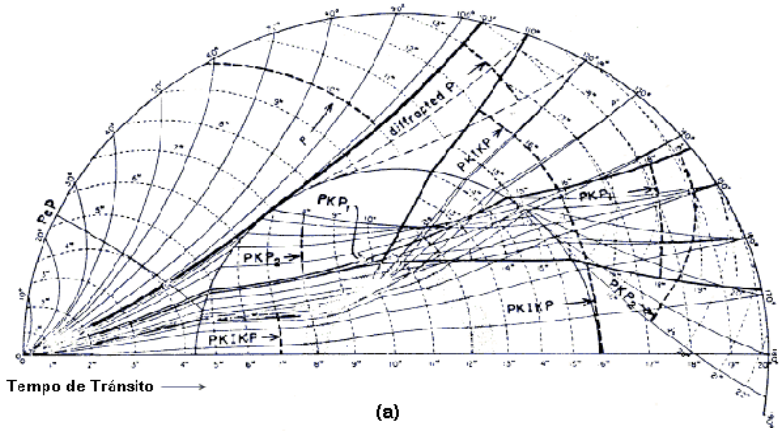
$$f(x) \cong a_0 + \sum_{n=1}^{N} \left[a_n . cos(ncx) + b_n . sin(ncx) \right]$$

THEORY

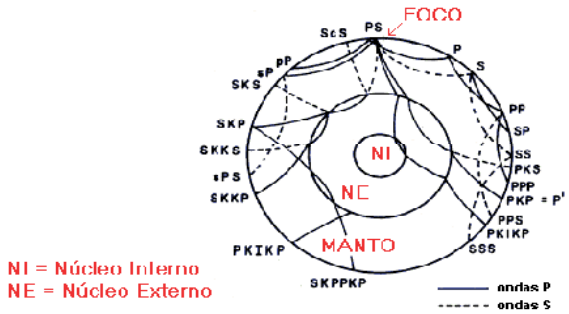
Seismological Aspects

(a) Direct P Wavefront Representation and their Rays Trajectories produced by a possible Earthquake (Gutenberg, 1959)

(b) Ray Trajectories for some Events illustrating used Nomenclature (adapted from Sheriff, 1999)



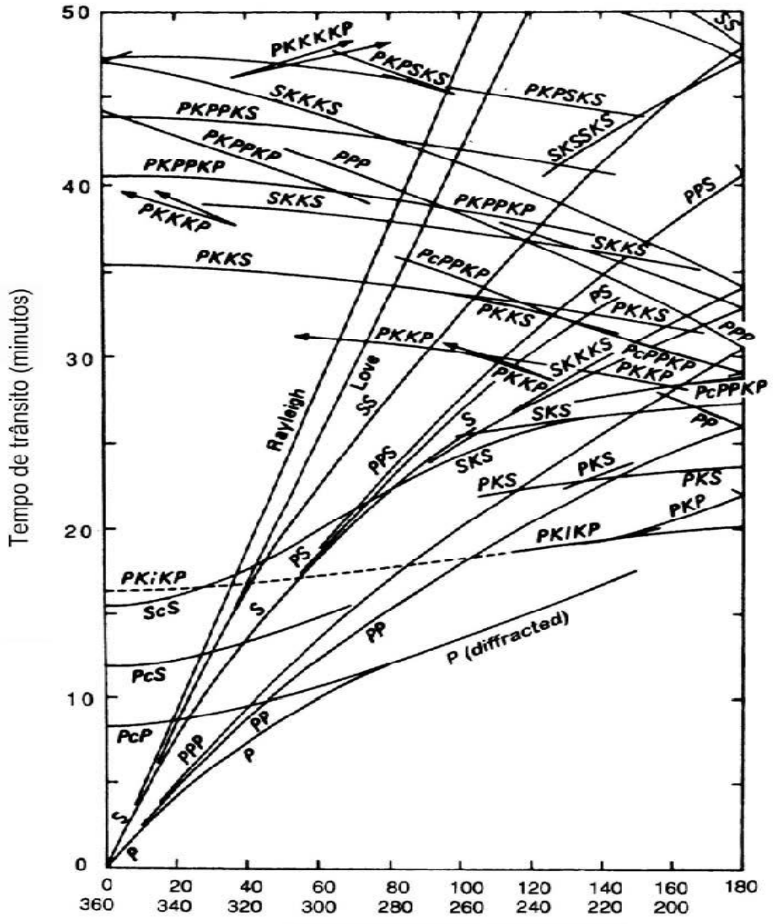
(a)



(b)

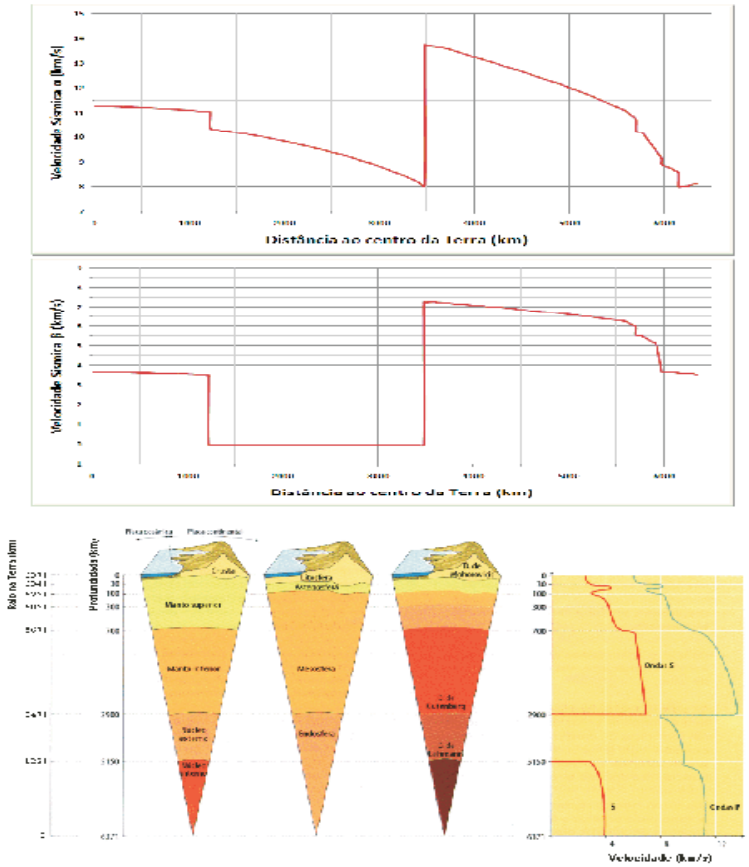
THEORY

Epicentral Angle Jeffreys-Bullen Traveltime Curves, built using Earthquake Data (Fowler, 2001)



THEORY

Graphs of Seismic Velocities for P and S Waves as a Function of the Centre Earth Distance and the most important Discontinuities in Earth Interior (adapted from http://domingos.home.sapo.pt/estruterra_4.html)



THEORY

General Aspects of Ray Tracing

Ray Equation : $\frac{d\vec{X}(\tau)}{d\tau} = \vec{P}(\tau)$ and $\frac{d\vec{P}(\tau)}{d\tau} = \frac{1}{2}\vec{\nabla}.\left(\frac{1}{V^2}\right)$,

where :

$\vec{X}(\tau) = (x(\tau), z(\tau))$ is a general point on the ray trajectory;

$\vec{P}(\tau) = (P_1(\tau), P_2(\tau))$ is the slowness vector tangent to the ray trajectory;

$V = V(x, z)$ is a velocity field;

$\tau = \int_0^t V^2 dt$ is a ray parameter; and

the equation $\left\|\vec{P}\right\|_2 = \sqrt{P_1^2 + P_2^2} = \frac{1}{V(x,z)}$ is satisfied.

APPLICATION

Numerical Version for Ray Tracing Equations

Position on the Ray Trajectory

$$\vec{X}(\tau+\delta\tau)=\vec{X}(\tau)+\frac{d\vec{X}(\tau)}{d\tau}.\delta\tau=\vec{X}(\tau)+\vec{P}(\tau).\delta\tau$$

Slowness Vector Tangent to the Ray Trajectory

$$\vec{P}(\tau+\delta\tau)=\vec{P}(\tau)+\frac{d\vec{P}(\tau)}{d\tau}.\delta\tau=\vec{P}(\tau)+\frac{1}{2}\vec{\nabla}.\left(\frac{1}{V^2(x(\tau),z(\tau))}\right).\delta\tau$$

Traveltime Calculation on the Ray Trajectory

$$T(x_{N+1},z_{N+1})=\sum_{i=0}^{N}\frac{1}{V_i}.\sqrt{(x_{i+1}-x_i)^2+(z_{i+1}-z_i)^2}$$

APPLICATION

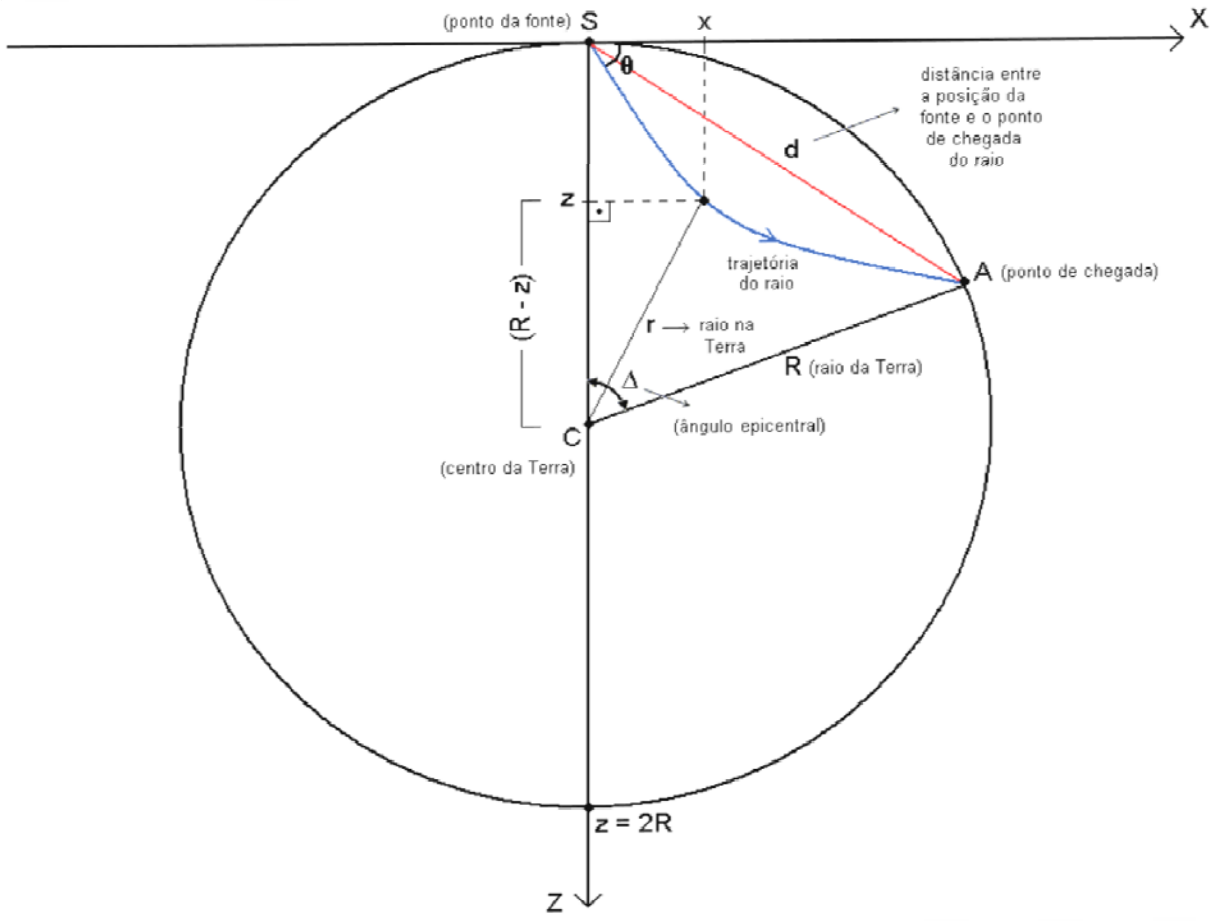
Trigonometric Series, where *R* is the Earth Radius and C_k is obtained by Adjustment to the PREM Data using the Least Squares Method

$$V(r) \cong C_0 + \sum_{n=1}^{N} \left[C_{2n-1}.cos\left(\frac{n\pi\, r}{R}\right) + C_{2n}.sin\left(\frac{n\pi\, r}{R}\right) \right],$$

where *r* means Distance to the Centre Earth.

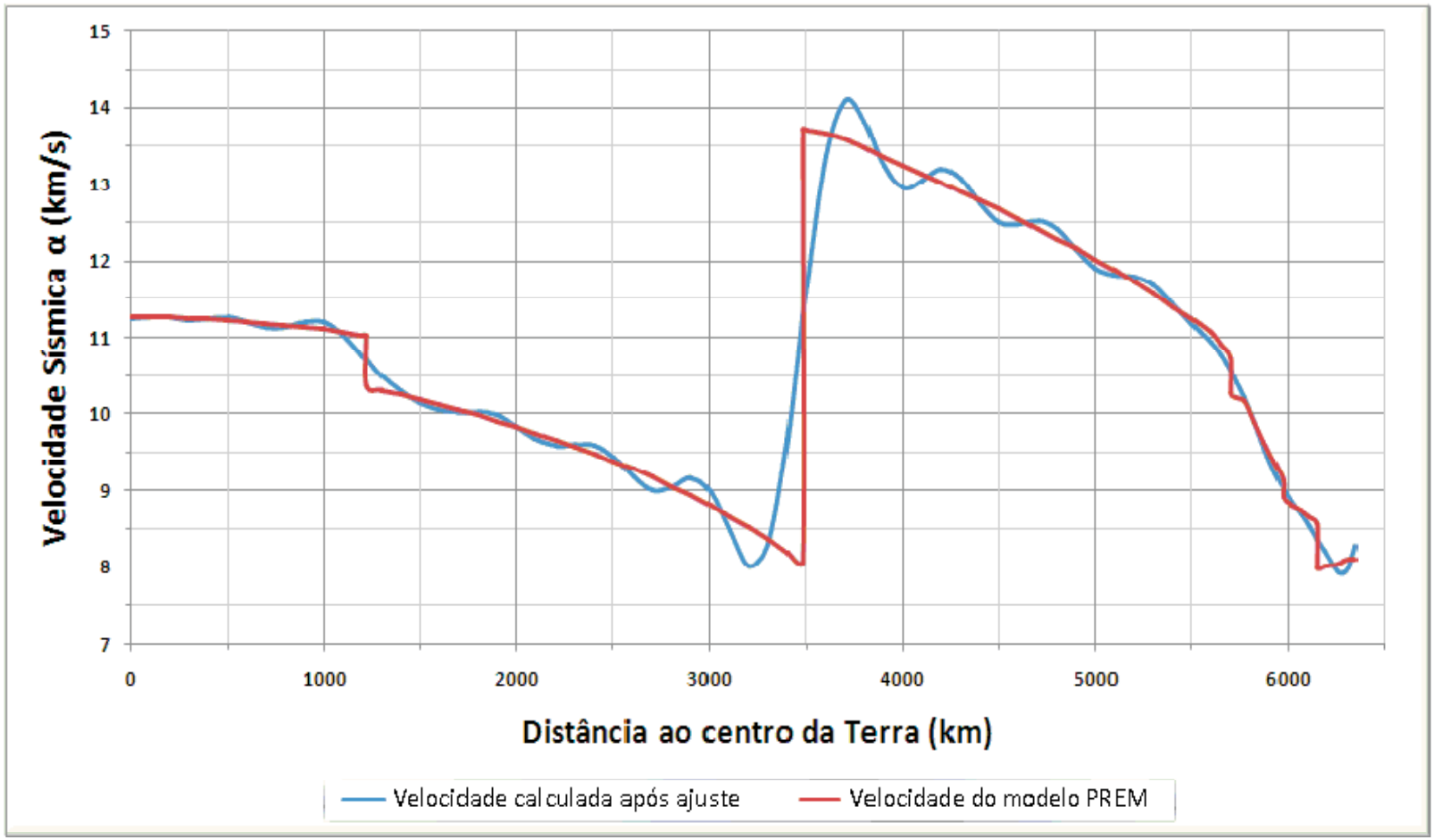
APPLICATION

Schematic Representation of Geometrical Elements of the Ray Tracing in Earth Interior



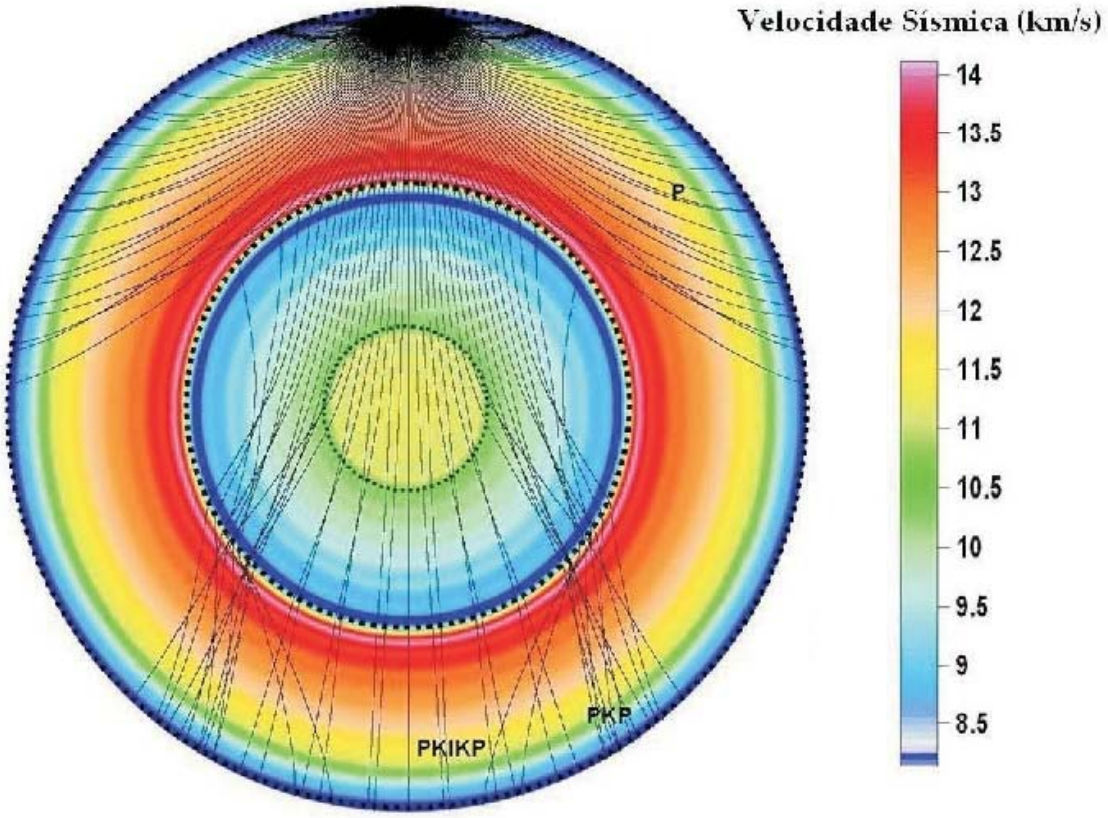
APPLICATION

Seismic P Velocity Curve Adjusted to the PREM Velocity Data by means of Least Square Method



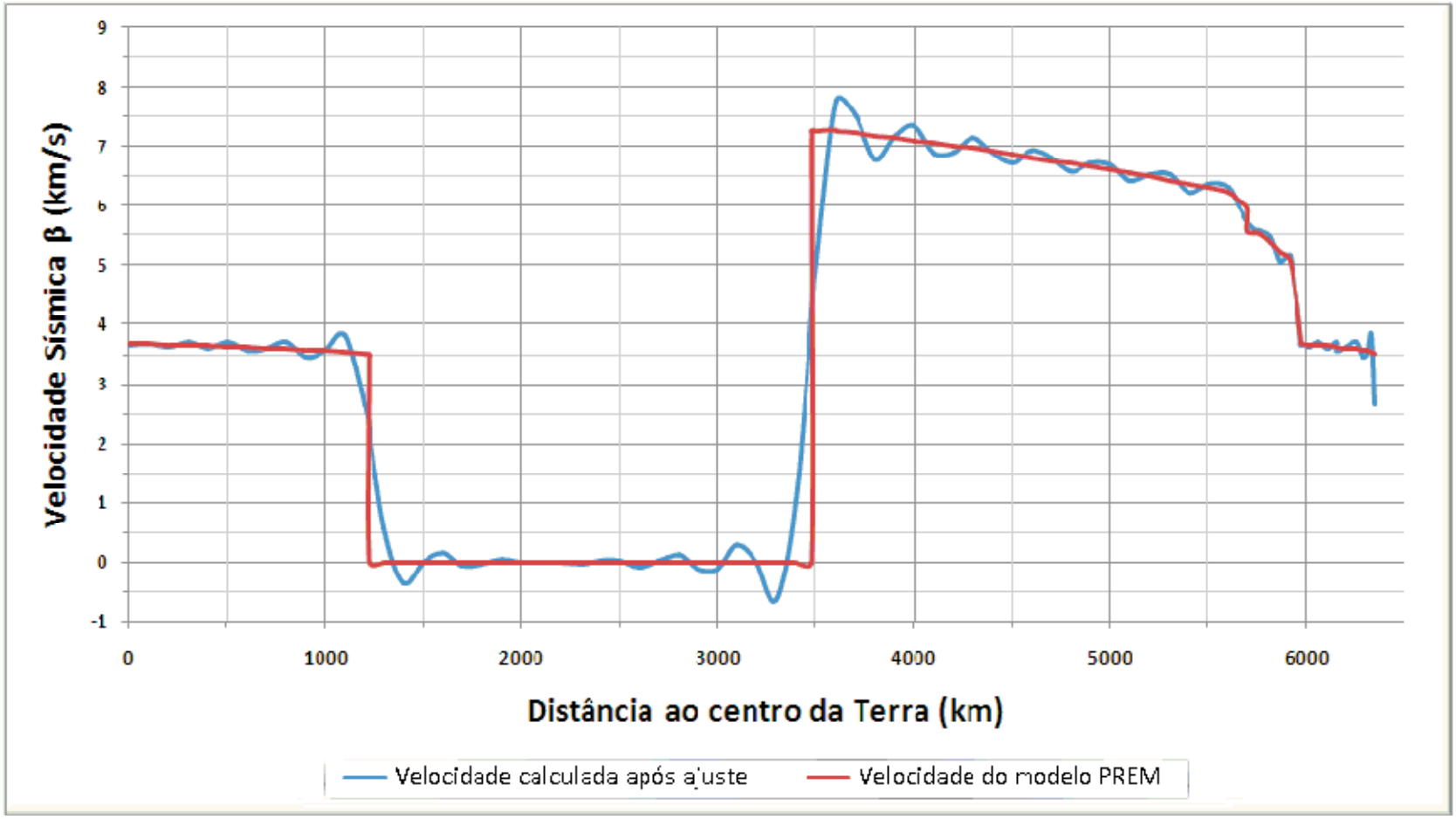
APPLICATION

Seismic Ray Trajectories of P Waves on the Velocity Field Parameterized by Trigonometric Series



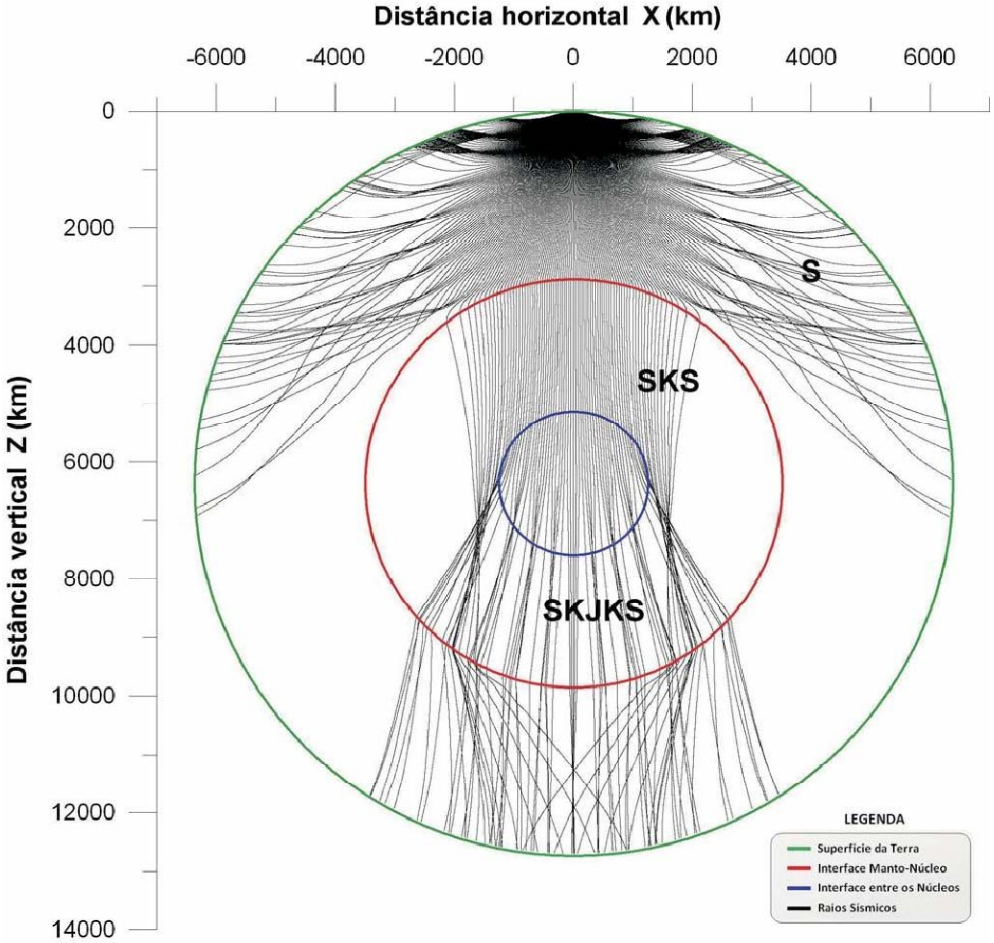
APPLICATION

Seismic S Velocity Curve Adjusted to the PREM Velocity Data by means of Least Square Method



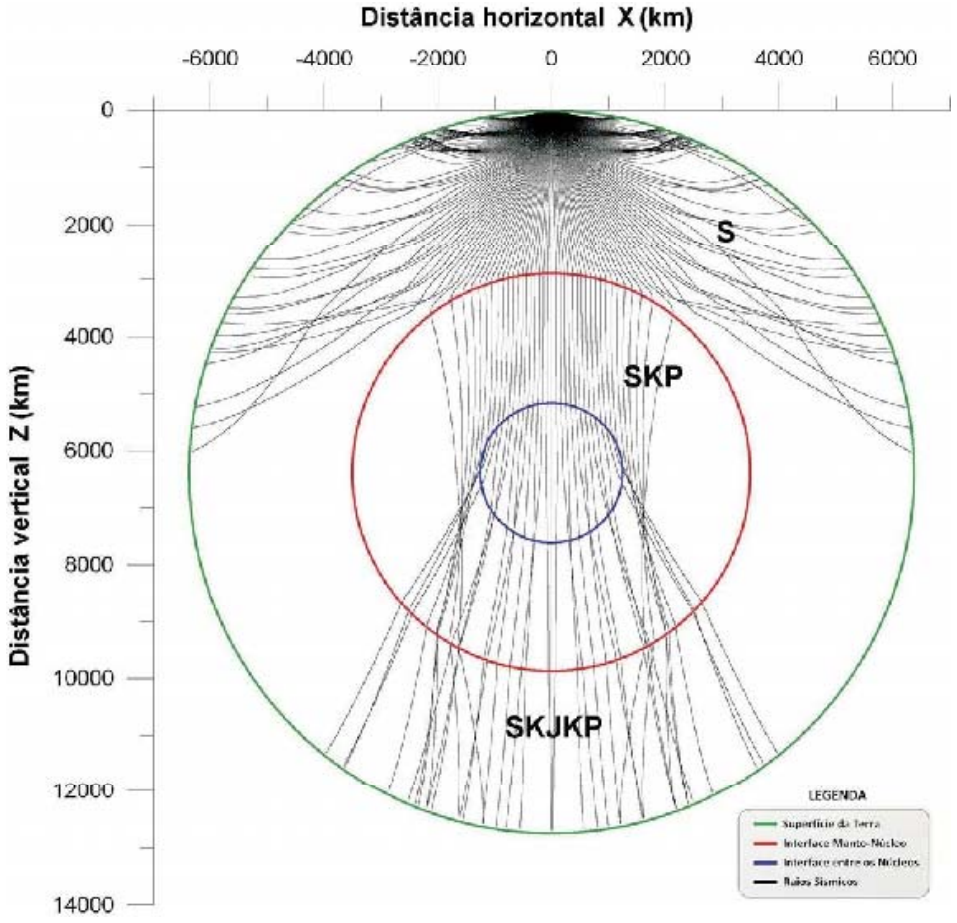
APPLICATION

Global Ray Field of Shear Wave obtained by Ray Tracing



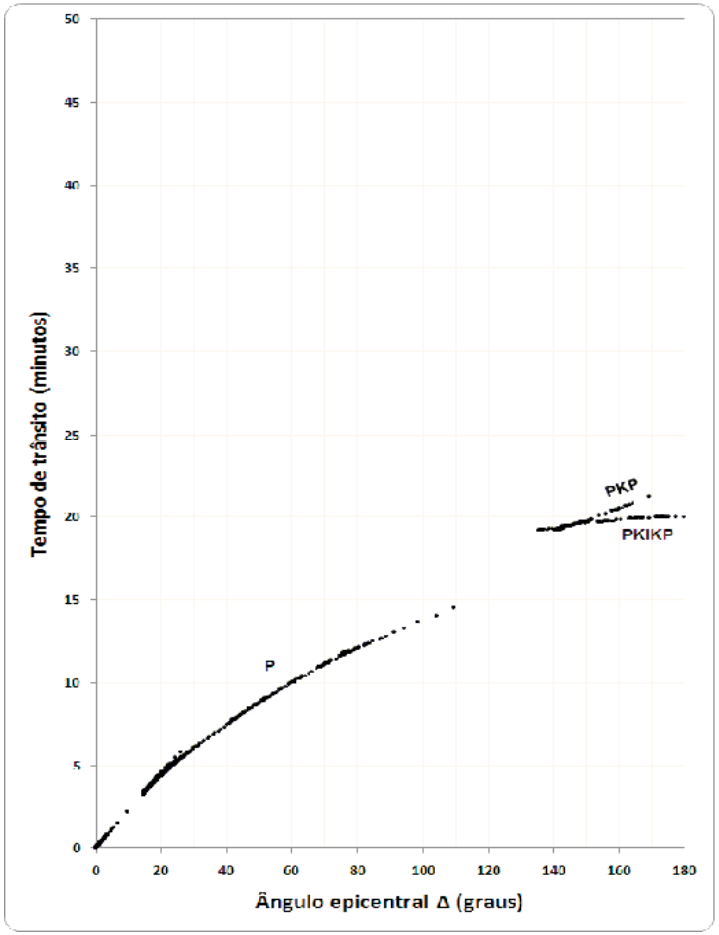
APPLICATION

Global Ray Field with some Converted Wave in the Interfaces



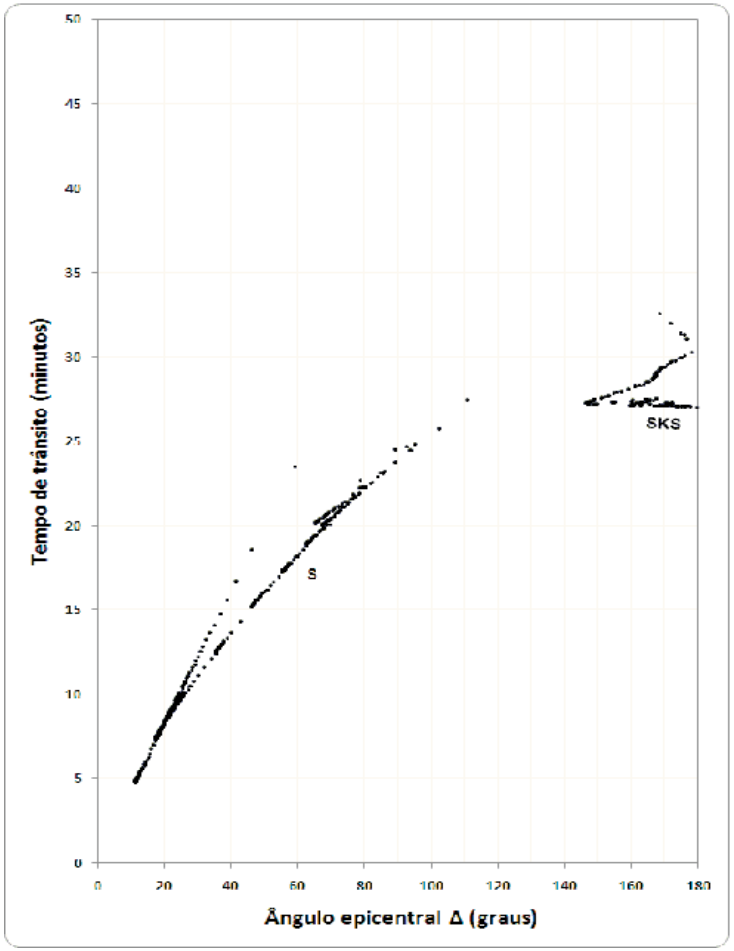
RESULTS

Traveltime of P Waves that cross the Earth obtained by Numerical Ray Tracing



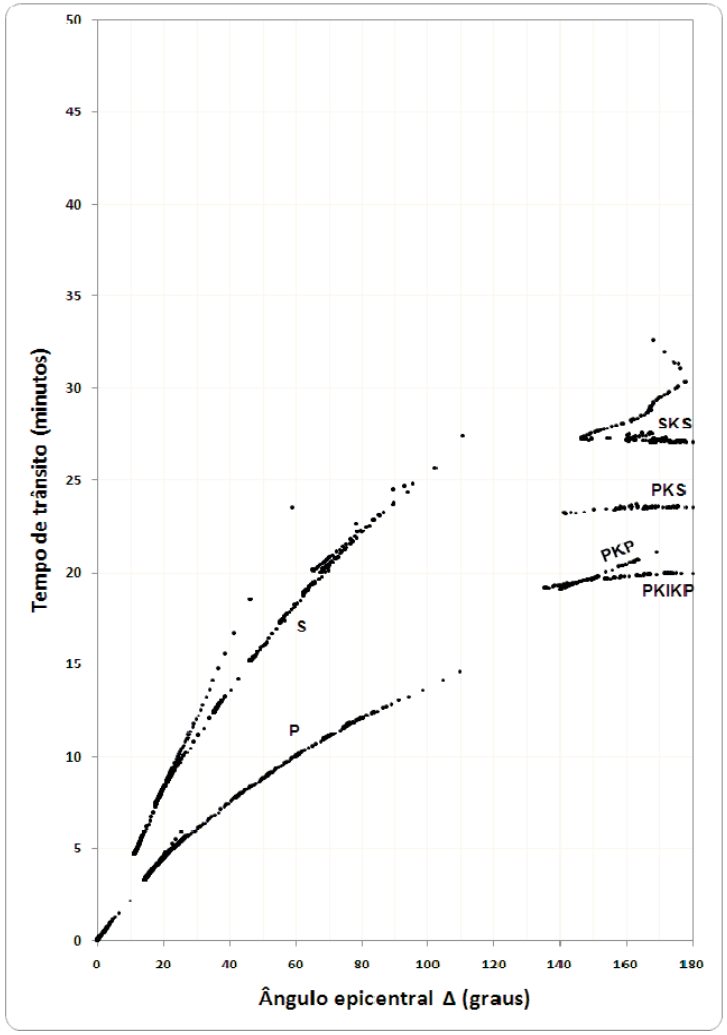
RESULTS

Shear Wave Traveltime in Earth Interior generates by Ray Tracing



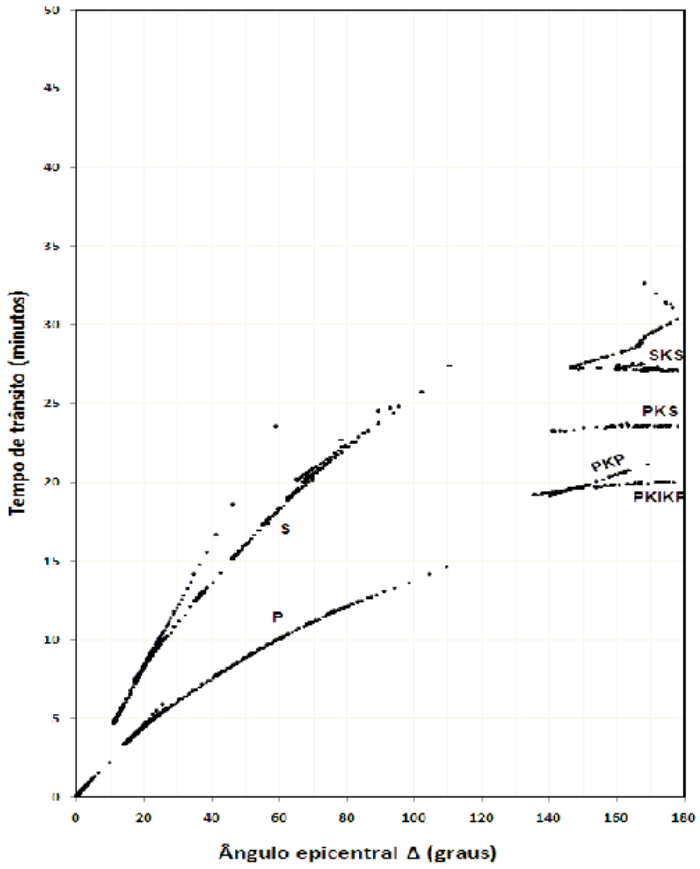
RESULTS

Gathering of all Events generated by this study using Ray Tracing

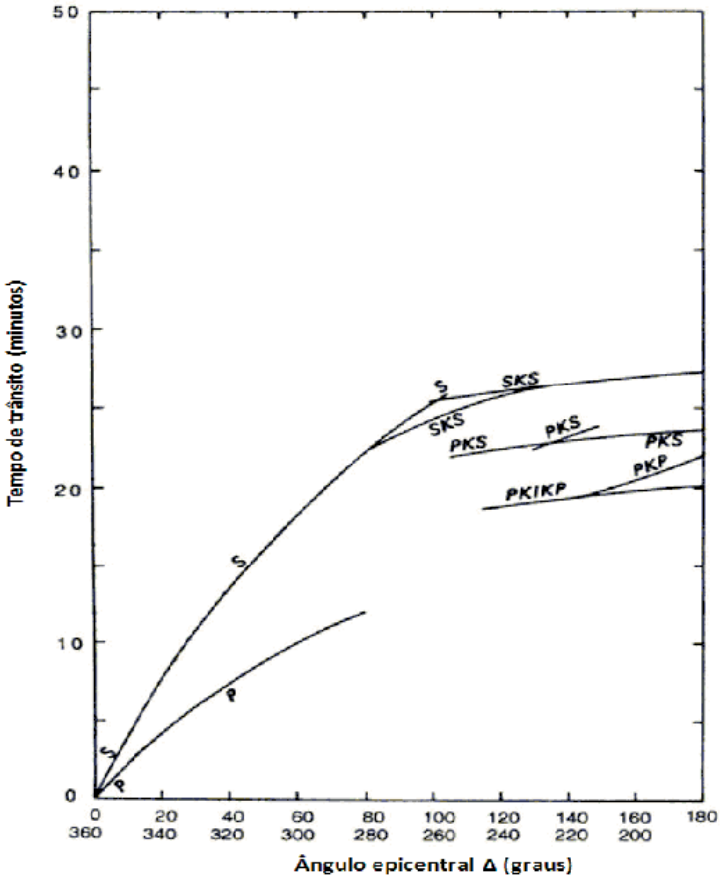


RESULTS

Comparison between the Modeled and Jeffreys-Bullen Traveltime Curves



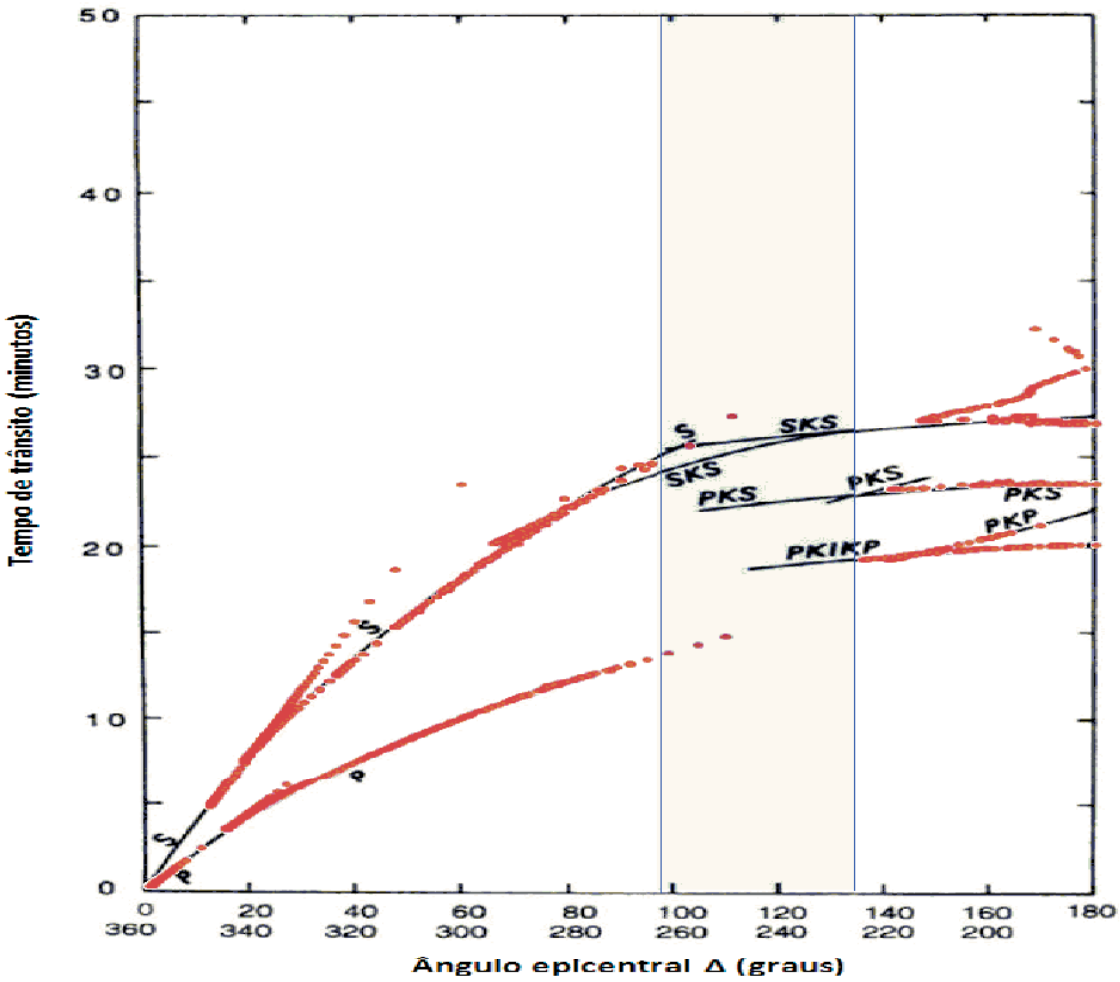
(a)



(b)

RESULTS

Superposition of Modeled Traveltime (red) on the Jeffreys-Bullen Curves



CONCLUSIONS

• Traveltime Jeffreys-Bullen Events (such as P, PKP, PKIKP, S, SKS, and PKS) were well Modeled;

• Trigonometric Series can be used satisfactorily as a mean to Parameterize Earth Interior P and S Velocity Field;

• Numerical Ray Tracing in Earth Interior produced reasonable Arrival Pattern on Earth Surface;

• Gibbs Effects and Velocity Field Discontinuities brought difficulties for Ray Tracing;

• Shadow Zones on Earth Surface were well Modeled;

• In the Transition Zone between Mantle and Outer Core results don´t have equivalence with their correspondent observed in Jeffreys-Bullen, this is specially dramatic for S waves;

• This work is consistent, because many similarities are observed between the calculated traveltime curves and those given by Jeffreys-Bullen.

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BRAZIL

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