GPR is an invaluable tool for civil and geotechnical engineering applications. One of the most significant objectives of such applications is the detection of fractures, inclined interfaces, empty or filled cavities, and other unexpected features. These types of targets, especially fractures, are usually not good reflectors and are spatially localized. Their response is therefore a factor significantly affecting their detectability. Quite frequently, systemic or extraneous noise, or other significant structural characteristics swamp the data with information which blurs, or even masks reflections from such targets, rendering their recognition difficult.

This paper reports a method of extracting information (isolating) oriented and scale-dependent structural characteristics, based on oriented two-dimensional B-spline wavelet filters and Gabor wavelet filters.

**B-SPLINE WAVELETS**

Typically, the m-th order B-spline is defined recursively by convolution:

\[ \phi_m(x) = \sum_{n=0}^{\infty} c_n \phi_{m-1}(x-n) \]

Since B-splines are scaling functions, they can be associated with wavelets φm formed by a linear summation of functions ϕm.

The location of the peak with respect to the frequency (F) and wavenumber (k) is determined by the length (scale) of the wavelet.

Example of tuning a filter on a given temporal scale (reference frequency):

- Assume a B-scan of 512 to 512 samples, a 400 MHz antenna, 0.097 ns sampling rate and 0.025 m trace spacing.
- Let a Quadratic B-spline mother wavelet with a 21-point span.
- Filter locks on k = 4.55 m^{-1}.

**2-D B-SPLINE WAVELET FILTERS**

When the data form a matrix or image, it can be decomposed into a series of images, each of which contains information at a specific location, of features at a single scale.

The existence of two independent variables (time/space) allows each component of the 4k spectrum wavelength to be coupled with a particular orientation.

Scale and orientation can be varied to construct a matrix filter tunable to any type of data:

1. Generate a one-dimensional wavelet of the desired length.
2. Sidewise array a number of identical one-dimensional wavelets to create a 2-D matrix with the desired span.
3. Taper the span (edge-parallel) direction with a smoothing window (e.g. Gaussian, Hamming).
4. This is a departure from strict wavelet construction because the smoothing window is not a wavelet.
5. Rotate the matrix filter to the desired orientation φ.

**DIFFERENT FILTERS EXTRACT DIFFERENT INFORMATION**

The Cubic B-spline Derivative allows extractions of a broad spectral band around the turning frequency, while the Linear, Quadratic and Cubic B-splines pass progressively narrower frequency bands.

- Linear: progressively lower frequency localization and temporal scale extraction.
- The frequency response mimics the shape of the radar source wavelet – this type of filter is better adapted for GPR data, than the standard frequency domain filters.
- The Ricker (or Mezonat Hat) wavelet, one of the commonly implemented models of the radar source pulse, can also be approximated by derivatives of Cardinal B-splines.

**APPLICATION**

- Data collected with a GSSI SIR-2000 system and 400 MHz antenna.
- Raw B-scan is noisy:
  - Bandwidth: 20-1024
  - Sampling rate is 0.1957 ns.
  - Trace spacing is 0.025 m.
- Observe up-dipping reflector between 45-60 ns and 6.78 m, which may extend laterally.
- Treat with a 13-21 Linear B-spline/ Hamming window filter rotated to phi=160°, so that k = 4.55 m⁻¹.

**SCALE EXTRACTION AND DENOISING**

For complete data analysis at a given temporal or spatial scale:

1. Apply the filter rotated to different angles under adaptive control so as to remain locked on a given frequency or wavenumber. If D is the data matrix and G is the filter matrix, this will yield a series of orientation-dependent outputs.

\[ D \ast (\theta) = \sum_{\phi} G(\phi) \cdot \mathcal{F}^{-1} \left[ \mathcal{F}[D(\theta)] \mathcal{F}^{-1}[G(\phi)] \right] \]

2. Stack the orientation-dependent outputs in the weighted least-squares sense:

\[ D(\theta) = \mathcal{F}^{-1} \left[ \mathcal{F}[D(\phi)] \mathcal{F}^{-1}[G(\theta)] \right] \]

This is a departure from strict wavelet construction because the smoothing window is not a wavelet. The weights implemented herein are of the form

\[ w(\theta) = \frac{\mathcal{F}[D(\phi)] \mathcal{F}^{-1}[G(\theta)]}{\sum_{\phi} \mathcal{F}[D(\phi)] \mathcal{F}^{-1}[G(\phi)]} \]

**EXAMPLES**

Above: Section distributed with package of Lucy and Powers (2002). Data measured with GSSI SIR-2000/ 500 MHz antenna; size 1024-1024 sampling rate 0.099 ns, 0.194 m.

**REFERENCES**

- Data collected with a Måla system and 250 MHz antenna on a ridge (Mt Ktenias, altitude ~1600m, NE Peloponnesus, Greece), where high-lying wind-powered electricity generators were to be erected. The geology comprises dipping, thin-plated limestones with intercalations of argillaceous material, intensely fragmented and karstified. Fragmentation is due to faulting and (apparently) multi-phase jointing and karstification subsequently nucleated at faults and major joints.

- For complete data analysis at a given temporal or spatial scale:

**CONCLUSION**

- The weighting scheme guarantees that the output will not be disproportionally dominated by powerful spectral components.
- Makes possible to combine "partial" (orientation-dependent) scale analysis into a "whole" scale analysis adapted to local variations in the angle of dipoles.
- Smooths orientation-dependent noise features eluing the filter at a given temporal or spatial scale.

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