**Application of a normalized Nash-Sutcliffe efficiency to improve the accuracy of the Sobol' sensitivity analysis of a hydrological model**

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J. Nossent and W. Bauwens

Department of Hydrology and Hydraulic Engineering
Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium
(Phone: +32-2-649 88 77; e-mail: jnossent@vub.ac.be)

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**A. Introduction**
- Sensitivity analysis (SA)
  - Main practice in hydrological modelling
  - Identify influential and non-influential parameters
  - Insights on the model processes
- Sobol’ SA method [1]
  - Variance based method
  - For time series -> objective function is required -> scalar
  - Variance estimation assessed with Monte Carlo integrals
  - Accuracy of variance estimation may decrease when mean value of scalar inputs $\mu$ for SA is large [2]
- Nash-Sutcliffe efficiency (NSE) [2] yields more accurate results than e.g., the also commonly used Sum of Squared Residuals
- Sufficient for flow predictions (mean NSE is 0.73)
- Poor for water quality simulations (mean NSE up to ~46)

**B. Sobol’ sensitivity analysis**
- Very robust SA technique with almost ideal properties
- No assumptions about linearity, additivity, monotonicity
- Quantifies amount of variance that each parameter contributes to unconditional variance of model output
- Expressed with sensitivity indices:
  - First order index (main effect): $S_i = V_i / V$
  - Measure for variance contribution of individual parameter $i$ to total variance
  - Total index: $S_t = S_i + S_{i,j} + S_{i,j,k} + \ldots$
  - Sum of rank of effect of $i$ and all interactions with other parameters
- $f_i, V_i, V_{i,j}, \ldots$ numerically estimated with Monte Carlo integrals
- $E_i = V_i / (1 + 2V_i f_0)$ numerically
- A large number of random samples in the parameter hyperspace are required to evaluate model
- A sample size of 12000 is used, resulting in 33600 flow ($p$=26) model evaluations (with $p$=26, the number of parameters) for $S_i$ and $S_t$

**C. SWAT-model of the River Kleine Nete catchment [4]**
- SWAT (The Soil and Water Assessment Tool) [5]:
  - Small (580 km$^2$), lowland (av. 23 msl) catchment
  - 56% agriculture, 26% forests
  - Mainly sandy soils (95%)
- Physical basis, semi-distributed (HRUs), basin scale simulator for water quantity, water quality and sediment simulations
- The River Kleine Nete catchment
  - The model
    - 1997-2007
    - Precipitation: 700 to 1100mm/y
    - PET: 700mm/y, ET: 500mm/y
    - 5°C in winter & 14°C in summer
    - Short river reaches
    - 26 parameters

**D. The normalized Nash-Sutcliffe efficiency**
- The regular Nash-Sutcliffe efficiency:
  \[ NSE = 1 - \frac{\sum (\hat{y}_i - y_i)^2}{\sum (y_i - \bar{y})^2} \]
- with $y$ the simulated value on day $i$, $\bar{y}$ the observed value on day $i$ and $\bar{y}$ the average of the observations
- The normalized Nash-Sutcliffe efficiency:
  \[ NSE = 1 - \frac{\sum (\hat{y}_i - y_i)^2}{\sum (y_i - \bar{y})^2} \]

**E. Results**
- Mean objective function values ($\hat{y}$) for the NSE and NNSE for different variables
- Improved and more realistic sensitivity indices ($S_i$ and $S_t$) for NNSE for water quality variables
- Graphical representation of the evolution with increasing sample size of the total sensitivity index ($S_t$) for o-Po$_4$ with applying the regular NSE and the normalized NSE as an objective function

**F. Conclusions**
- A normalized Nash-Sutcliffe efficiency (NNSE) has been introduced with similar properties as the regular Nash-Sutcliffe efficiency (NSE) to evaluate model output, but with values between 0 and 1
- The NNSE values between 0 and 1 provide a more intuitive interpretation of the model evaluations
- The NNSE can be successfully applied for the Sobol’ sensitivity analysis of flow and water quality variables

**References**