GOCE long-wavelength gravity field recovery from high-low satellite-to-satellite-tracking using the acceleration approach

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Introduction

The restricted sensitivity of the GOCE (Gravity field and steady-state Ocean Circulation Explorer) gradiometer instrument requires satellite gravity gradiometry to be supplemented by orbit analysis in order to resolve long-wavelength features of the geopotential. In this context, the energy conservation method has been adopted within official ESA products. On the other hand, various investigations showed the energy conservation principle to be a sub-optimal choice. For this reason, we propose to apply the acceleration approach, which proved to be an efficient tool in former missions. However, the application of this method to GOCE-SST data, given with a 1s-sampling, showed that serious problems arise due to strong noise amplification of high frequency noise. In order to mitigate this problem, tailored processing strategies with regard to low-pass filtering, variance-covariance information handling, and robust parameter estimation have been adopted. By comparison of our GIWF (Geodetic Institute (GI), Institut für Weltraumforschung, (IWF)) solutions and the official GOCE models with a gravity field solution derived from GRACE (Gravity Recovery And Climate Experiment), we conclude that the acceleration approach is better suited as the energy conservation method. Comparisons with solutions from alternative methods (i.e. Celestial Mechanics Approach, CMA) show a similar performance of the acceleration approach.

Methods

Observation equation

The acceleration approach is based on Newton's equation of motion.

$$\frac{d^2\mathbf{x}}{dt^2} = \mathbf{a}(t) = \nabla V(t)$$

According to the equivalence principle, the satellite's acceleration $d^2\mathbf{x}/dt^2$ in the Earth's gravitational field is equal to the gravitational attraction $\mathbf{a}(t)$ on the satellite, which is the gradient of the geopotential $V(t)$.

Numerical differentiation

The satellite acceleration $d^2\mathbf{x}/dt^2$ is determined from the orbit $\mathbf{x}(t)$ by numerical double differentiation.

- 8th order polynomial fit to n = 9 points performs best
- acceleration computed at the central point of mask → moving differentiation filter (Fig. 1)

Down-sampling

Numerical differentiation has the property of amplifying high-frequency noise. This holds especially for the GOCE hl-SST data, which is provided with a Δt = 1 s sampling.

- hl-SST is sensitive up to degrees L ≈ 80 – 120. The CHAMP sampling rate of Δt = 30 s with a Nyquist frequency of 2 Δt = 1 min (orbit period of about 90 min!) fits optimally to this.
- however the GOCE sampling of Δt = 1 s corresponds to gravity field features up to L = 2700

A brute force method to mitigate the impact of high-frequency signal/noise is "down-sampling".

- Implementation of "*Extended Differentiation Filter*" EDF(Δt_{ex}): the sampling points are chosen Δt_{ex} apart, whereas the differentiation filter moves along the original 1 s-sampled orbit (Fig. 1)
- acts as low-pass filter; the filtering is controlled by the time interval of neighbouring points

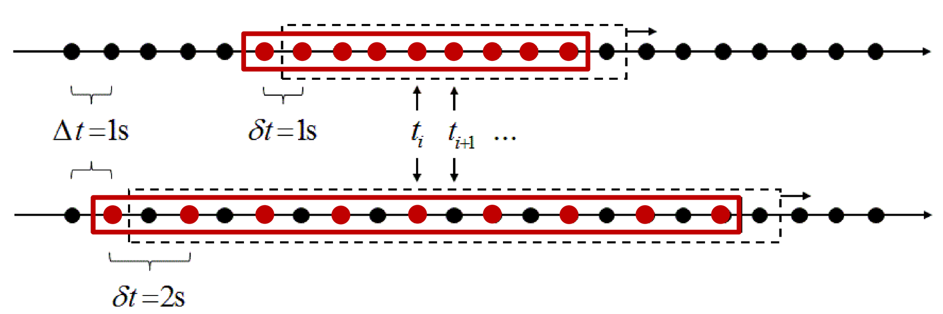

Figure 1: EDF-Differentiation filter design (here: 9-point-filter and Δt = 1 s). Upper panel: EDF(1s); lower panel: EDF(2s)

Outlier detection

Outlier detection by comparison of satellite accelerations with a propagated reference model (EGM96)

- conservative threshold selection
- only elimination of gross errors

Robust estimation

Robust estimation does account for the differing quality of the observations by means of iterative reweighted least squares adjustment

- reweighting $w_i^{(j+1)}$ according to the residuals $\hat{v}_i^{(j)}$ of the preceding step (j) with the Huber estimator

$$w_i^{(j+1)} = \begin{cases} 1 & \text{if} \quad |\hat{v}_i^{(j)}| \le \varepsilon \\ \varepsilon / \hat{v}_i^{(j)} & \text{if} \quad |\hat{v}_i^{(j)}| > \varepsilon \end{cases} \qquad \varepsilon \ldots \text{threshold}$$

- convergence typically after 2 iteration steps

Results

The results shown here were computed from the 61-day period 1st Nov – 31th Dec 2009, which corresponds to a full GOCE repeat cycle of kinematic orbits (Bock et al., 2011). These are delivered within the SST_PSO_2 Level-2 product (c.f. EGG-C, 2010).

Down-sampling

The following results are obtained for different EDF-designs (no decorrelation applied), see Fig. 2:

- for EDF(1s) with absence of any filtering the noise amplification heavily distorts the solution
- the EDF(30s) is superior to the other designs over almost all degrees
- the EDF(10s) shows inferior performance in the long wavelengths up to degree l ≈ 55
- EDF(45s) shows good results for the low degrees l < 35; above effects of over-smoothing appear
- the misfit of "true" and formal errors show the deficiencies of the (missing) stochastic model

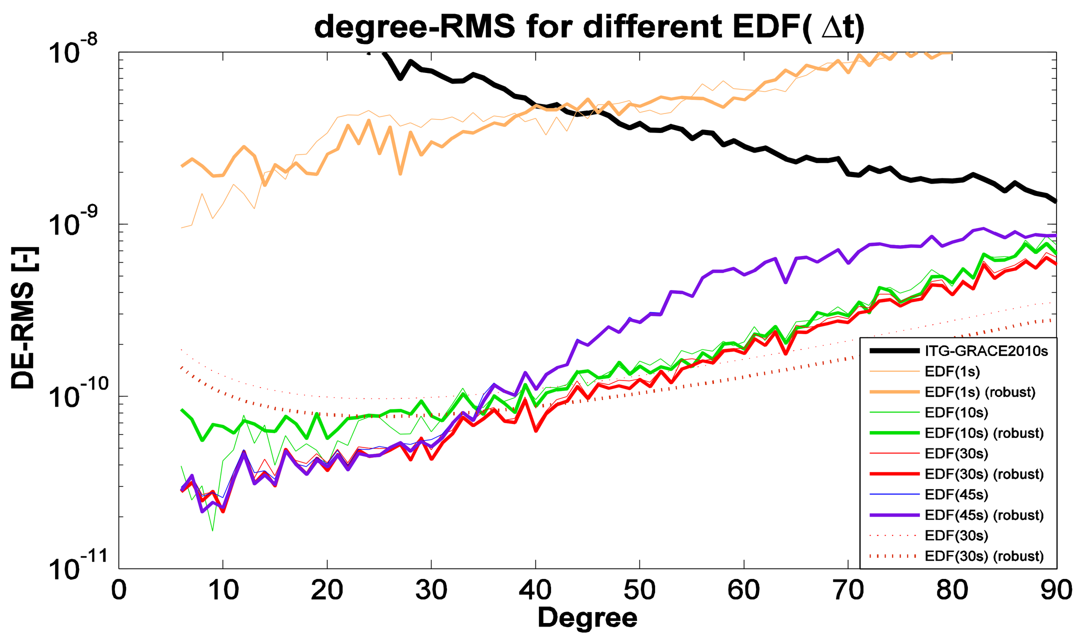

Figure 2: GOCE real data results for various EDF designs (no decorrelation applied); period: 1st Nov – 31th Dec 2009). Black graph: ITG-Grace2010s signal; solid graphs: degree-RMS; dashed graphs: formal errors; thin/bold graphs: without/with robust estimation

Individual weighting of components

Due to the necessity to simultaneously solve for epoch-wise clock corrections and positions the radial component of the kinematic orbits is degraded. Thus a natural frame for analysis would be the orbit frame or the LNOF (Local North oriented Frame), where the radial component can be down-weighted directly (Fig. 3).

- least squares residuals reveal a reduced accuracy of the radial component by a factor of 2.5 – 3.
- down-weighting of the radial component significantly improves the quality of the solution
- result is insensitive to small variations in the variance factors

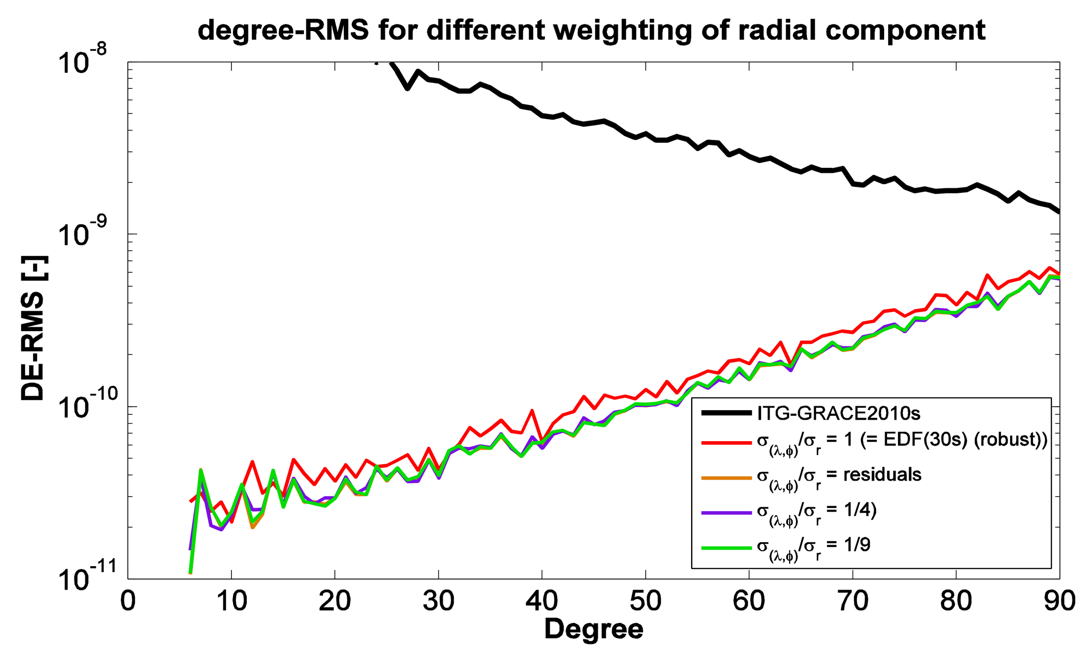

Figure 3: GOCE real data results for EDF(30s) (no decorrelation applied) with different down-weighting factors of the radial component (analysis in the LNOF). Red graph: identical to Fig. 2; orange graph: weighting according to RMS of residuals; blue/green graphs: down-scaling by factors 1/2 and 1/3.

Empirical covariance function

The influence of temporal correlations between the (pseudo-)observations on the gravity solutions (from EDF(30s)) is investigated (Fig. 4). Therefore empirical covariance-functions for data weighting are estimated from residuals (in the LNOF) by means of auto-covariance functions.

- the estimated variance-matrices contain the relative weighting of the directions
- the consideration of correlations in EDF(30s) slightly increases the quality of the solution
- formal errors are similar to "true" errors; thus stochastic model is chosen properly

Further experiments for EDF(30s) showed that empirical covariance matrices outperformed analytical covariance matrices based on error propagation of the provided orbit covariance information. However, the degradation of the radial component is comparable. Application of empirical and analytical covariance-matrices directly for EDF(1s) lead to inferior results compared to EDF(30s), especially for the empirical case. This may serve as a hint for stochastic mismodelling of the orbit correlations and the high-frequency noise introduced by numerical differentiation.

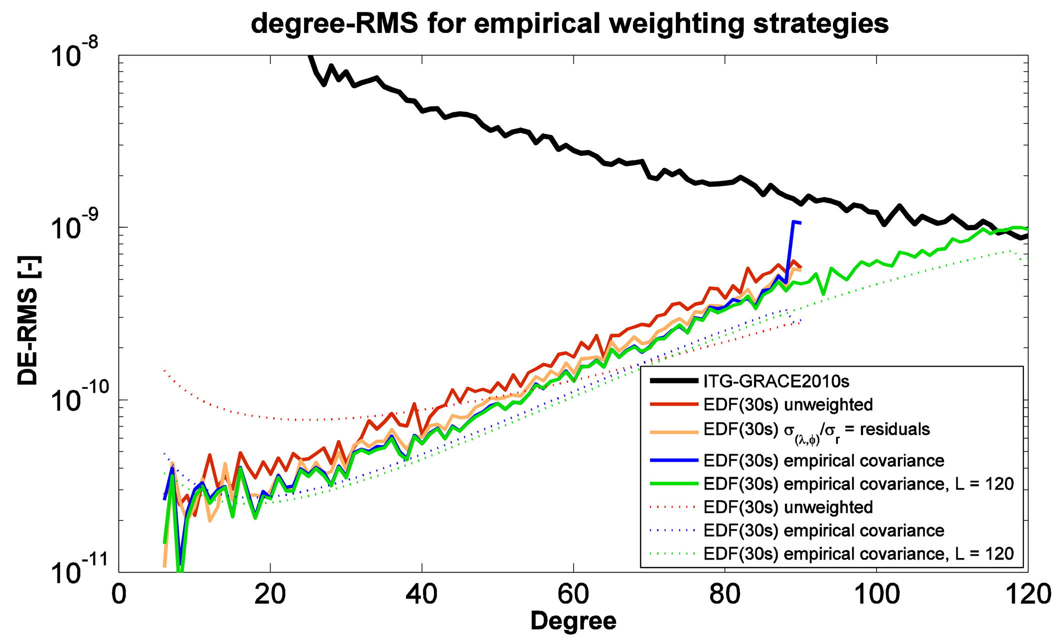

Figure 4: GOCE real data results for EDF(30s) with application of different weighting strategies (analysis in the LNOF); red/orange graph identical to Fig. 3. Red graph: no weighting; orange graph: down-weighting of radial component based on RMS of residuals; blue/green graphs (GIWF solutions): application of empirical covariance functions (estimation to SH degree 90/120); dashed graphs: formal errors

Comparison with external gravity field solutions

Figure 4 shows for a solution to L = 120 (same performance as L = 90 up to degree l ≈ 85) a signal-to noise ratio SNR > 1 up to degree l = 116. This solution is compared to external solutions:

- AIUB (Astronomical Institute of the University of Bern), Celestial Mechanics approach (Jäggi et al., 2011)
- INAS (Institute of Navigation and Satellite Geodesy, Graz University of Technology), energy balance approach; so far applied in the official GOCE-TIM gravity field series (Pail et al., 2011)

The results (Fig. 5) show a similar performance for AIUB and GIWF (AIUB slightly better for orders m > 5, GIWF better in the near-zonals) and a reduced accuracy of about a factor of sqrt(3) for the INAS solution, as assumed from former investigations of the energy balance approach.

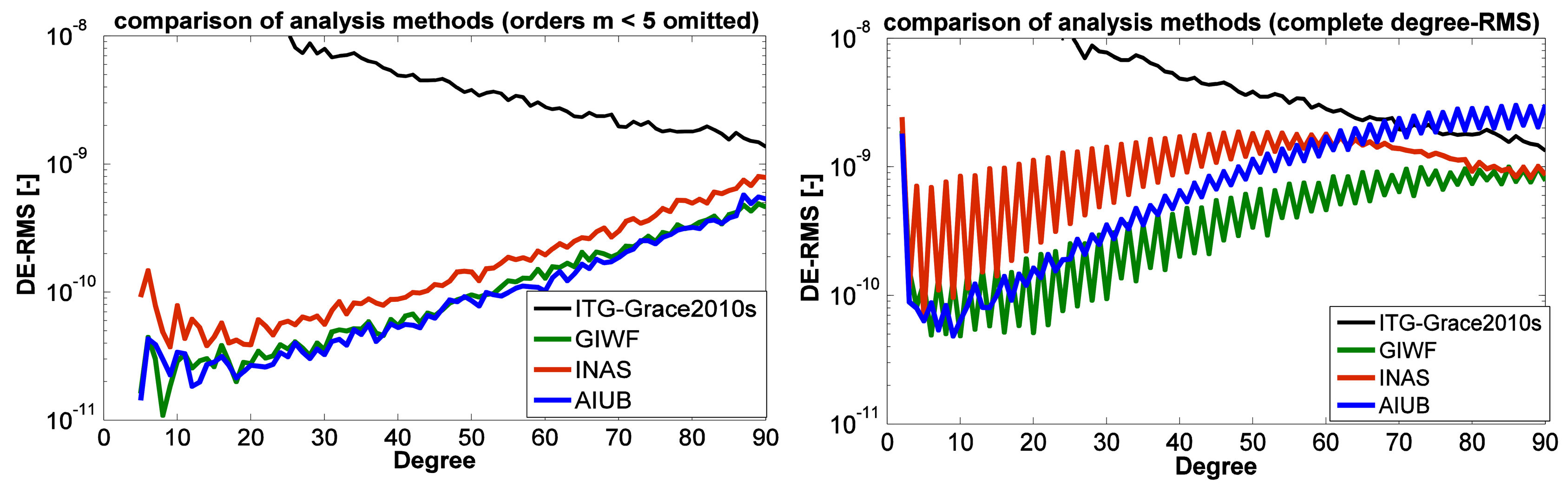

Figure 5: Comparison of real GOCE-SST results from different research groups applying different solution strategies. GIWF: L = 120, acceleration approach (green curve from Fig. 4); INAS: L = 100; AIUB: L = 120. Left: orders m < 5 omitted; right: complete degree-RMS

Concluding remarks

Based on the results we conclude that the acceleration approach is an appropriate method for GOCE long-wavelength determination from GOCE hl-SST (i.e. kinematic orbits).

- It shows a similar performance as alternative approaches and outperforms the currently applied energy balance method in official GOCE models. Compared to state-of-the-art GOCE-TIM models an improvement up to degree l = 25 seems possible.
- In contrast to the Δt = 10 s/30 s sampling of CHAMP/GRACE data the Δt = 1 s sampling of GOCE data involves certain problems, which necessitates tailored processing strategies.
- The best result was obtained by application of EDF(30s) lowpass-filter together with empirical covariance modelling in the LNOF and robust estimation.
- However, as formal errors and especially solutions applying directly the EDF(1s) show, the stochastic modelling of orbit error correlations and especially of high frequency noise introduced by numerical differentiation is quite imperfect.

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