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Effect of the potential due to lunisolar deformations on the Earth precession

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Introduction

- In 2006, the IAU General Assembly endorsed Resolution B1 adopting a new model of precession that entered in force on January 1st, 2009

- It is model *P03* by *Capitaine et al* (1) and includes both the precession of the ecliptic and the precession of the equator

- The latter is given by

$$\psi_A = 5038.481507''\, t - 1.0790069''\, t^2 - 0.00114045\, t^3 + 0.000132851''\, t^4 - 0.0000000951''\, t^5$$
$$\omega_A = \epsilon_0 - 0.025754''\, t + 0.0512623''\, t^2 - 0.00772503''\, t^3 - 0.000000467''\, t^4 + 0.0000003337''\, t^5$$

$t \longrightarrow$ in *cy*

(1) Capitaine, N., Wallace, P. T., and Chapront, J., 2003**c**, "Expressions for IAU 2000 precession quantities," *Astron. Astrophys.*, **412**(2), pp. 567–586,

- Main term of precession in longitude depends linearly on time t in all models
- It is made up of:
  - First order effects of lunar and solar attraction on the Earth's equatorial bulge (proportional to dynamical ellipticity H and cosinus of the obliquity)
  - Several higher order effects on the precession rate
- Observed precession is the key to determine H and the scale of nutation theories

Relevance of accurate modelling of various effects on precession

- Sources of error in main term of precession are
  - Observational errors or processing of observations
  - Missmodelled effects
  - Unmodelled effects
- Errors in the ***t*** coefficient of $\dot{\psi}_A$ results in changes in the value of ellipticity ***H*,** therefore re-scale nutations

$$\frac{\delta H}{H} = -\frac{error}{\dot{\psi}_A}$$

Well known effects on precession

Various components of precession rate can be found in *Williams 1994* or *Capitaine et al 2003*

TABLE 4. Time and obliquity dependence of precession and obliquity rates ("/century) which are needed to calculate the evolution of precession and obliquity with time.

Source	Rate in "/century	ε Dependence
	Precession	
Luni-solar, direct planetary torque	$P_0 \cos \varepsilon_0 - 0.003395\ t - 6\text{x}10^{-6}\ t^2$	cos ε
Geodesic precession	$--1.919362 + 2.7\text{x}10^{-6}\ t$	1
Second order (M_3)	-0.03310	$6 \cos^2 \varepsilon - 1$
Second order	--0.01368	$3 \cos^2 \varepsilon - 1$
J_4 precession	+0.00260	$\cos \varepsilon\ (4-7 \sin^2 \varepsilon)$
J_2 tilt	-0.2630	$\cos 2\varepsilon\ / \sin \varepsilon$
Planetary tilt and direct torque	-0.00643+0.001074 t	$\cos 2\varepsilon\ / \sin \varepsilon$
Tides on lunar orbit	-0.000102 t	$\cos^2 \varepsilon$
Tides on spin and moments	-0.000133 t	$\cos^3 \varepsilon$
J_2 rate	-0.0140 t	Cos &
	Obliquity	
Planetary tilt and direct torque	$-0.0268-0.000044\ t + 3\text{x}10^{-6}\ t^2$	Cos ε
Tides	+0.0024	sin ε cos ε

Taken from

Williams, J. G., 1994, "Contributions to the Earth's obliquity rate, precession, and nutation," *Astron. J.*, **108**(2), pp. 711–724, doi:10.1086/117108.

Less widely known effects on precession

- **Effects of second order** in the previous tables (in the perturbation theory sense) **are for a rigid earth**.
- It was found that ***FCN*** **resonance increases rigid body second order contribution in a noticeable amount** (1, 2). Computations were carried out for two models:
  - rigid mantle + liquid core (Poincaré)
  - elastic mantle + liquid core

Contribution to precession (mas/cy units)				
Model	dp_0	dp_{10}	dp_{11}	TOTAL
Rigid	−45.3641	−.3979	−.2976	−46.0596
Poincaré	−45.3641	−22.5908	−1.2625	−69.2174
Poincaré + Elastic Mantle	−45.3641	−12.5195	−.7433	−58.6269

- *Williams* value for the rigid Earth (1994) is similar: *- 46.8 mas/cy*
- **Rigid effect** is about ***-9 ppm*** of the precession rate.
- **Non-rigidity adds -13.2 mas/cy,** about *1/3* more: ***-2.6 ppm***

1. *Ferrandiz et al AJ 2004.*
2. *Ferrandiz et al 2005* (Influence of the core on the Earth precession, Proc. of the workshop organized in honour of Prof. Jacques Henrard at the occasion of his retirement / / Presses universitaires de Namur, 2007).

Purpose of presentation: Revisiting effects due to solid tides raised by luni-solar attraction

- Attraction of sun and moon causes a mass redistribution and the gravitational potential is changed by an additional term (usually expressed in terms of Love numbers k)

- The effect of the zonal tidal additional potential was computed by:

  - *Lambert & Capitaine, Effects of zonal deformations and the Earth's rotation rate variations on precession-nutation A&A 428, 255–260 (2004)*

  - *Bourda and Capitaine, Precession, nutation, and space geodetic determination of the Earth's variable gravity field A&A 428, 691–702 (2004)*

Summary of the approach in those papers

- The classic relation between H and J_2 parameters is used

$$H = \left(C - \frac{A+B}{2} \right) / C = \frac{MR_c^2}{C} J_2 = -\frac{MR_c^2}{C} C_{20} = -\sqrt{5}\,\frac{MR_c^2}{C} \overline{C}_{20}$$

as well as the invariance of the trace of the inertia tensor.

- Periodic components of J_2 affect nutations, constant terms influences the precession rate and linear time variation produces an acceleration of the precession in longitude.
- Solid earth tidal deformations are modelled as usual as an additional potential

$$U_{soltid} = GM \frac{R_c^2}{r^3} \overline{P}_{20}(\sin\phi) \overline{C}_{20_{Moon+Sun}}(t)$$

where $$\overline{C}_{20_{Moon+Sun}}(t) = \frac{k_{20} R_c^3}{5M} \sum_{p=moon}^{sun} \left(\frac{m_p}{r_p^3} \overline{P}_{20}(\sin\phi_p) \right)$$

Observed variations of J_2 are analyzed too

Present approach: We compute the effect of the full additional tidal potential (zonal + tesseral + sectorial)

- The main part of the potential of a rigid earth acting upon a external body (Moon, Sun) can be written as

$$V_0 = \frac{Gm^*}{r^{*3}}\left[\frac{2C_0 - A_0 - B_0}{2} P_2(\sin\delta) + \frac{A_0 - B_0}{4} P_2^2(\sin\delta)\cos 2\alpha\right]$$

- Tidal deformation causes an additional potential V_t usually described in terms of the Love number k_2

$$V_t = \frac{Gm^* m' R_\oplus^5}{r^{*3} r'^3} k_2 P_2(\cos S') \quad \text{with} \quad \cos S' = \frac{r^* \cdot r'}{r^* r'}$$

Brief account of derivations

- Using the notation by *Getino & Ferrandiz, 1995* we get the equivalent expansion in terms of D_t , proportional to k_2

$$V_t = \frac{Gm^*}{r^{*3}} D_t [\frac{2t_{33} - t_{11} - t_{22}}{2} P_2(\sin\delta) + \quad + (t_{13}\cos\alpha + t_{23}\sin\alpha) P_2^1(\sin\delta) +$$

$$+ \left(\frac{t_{11} - t_{22}}{4} \cos 2\alpha + \frac{1}{2} t_{12} \sin 2\alpha \right) P_2^2(\sin\delta)]$$

with

$$\tilde{\Pi} = D_t \left(\frac{a^*}{r^*} \right)^3 \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{12} & t_{22} & t_{23} \\ t_{13} & t_{23} & t_{33} \end{pmatrix}$$

$$t_{11} = 2P_2(\sin\tilde{\delta}) - P_2^2(\sin\tilde{\delta})\cos 2\tilde{\alpha}$$

$$t_{22} = 2P_2(\sin\tilde{\delta}) - P_2^2(\sin\tilde{\delta})\cos 2\tilde{\alpha}$$

$$t_{33} = -4P_2(\sin\tilde{\delta})$$

$$t_{12} = -P_2^2(\sin\tilde{\delta})\sin 2\tilde{\alpha}$$

$$t_{22} = -2P_2^1(\sin\tilde{\delta})\cos\tilde{\alpha}$$

$$t_{33} = -2P_2^1(\sin\tilde{\delta})\sin\tilde{\alpha}$$

- The kinetic energy, in canonical Andoyer variables for a two layer earth, reads as

$$
\begin{aligned}
T &= \frac{1}{2A_m}M^2 + \left(\frac{1}{2C_m} - \frac{1}{2A_m}\right)N^2 \\
&+ \frac{1}{2A_m}M_c^2 + \left(\frac{1}{2C_m} - \frac{1}{2A_m}\right)N_c^2 \\
&+ \frac{1}{2A_c}M_c^2 + \left(\frac{1}{2C_c} + \frac{1}{2A_c}\right)N_c^2 - \frac{1}{C_m}NN_c \\
&+ \frac{1}{A_m}\sqrt{M^2 - N^2}\sqrt{M_c^2 - N_c^2}\cos(\nu + \nu_c)
\end{aligned}
$$

- The Hamiltonian is $H = T + V_0 + V_t + additional\_terms$
- Precession angle is $\psi_A = -\lambda$, its derivative is $\dot{\psi}_A = -\dfrac{\partial H}{\partial \Lambda}$
- The effect on precession of term V_t is found to be $\delta p_0 = -\dfrac{\partial V_{t\sec}}{\partial \Lambda}$

where V_{tsec} is the secular part of V_t

- Using Kinoshita-like expansions, it is found that V_t can be cast as the sum of three components corresponding to zonal, tesseral and sectorial harmonics (denoted by subscripts 0, 1, 2 resp.)

$$V_t = V_{t,0} + V_{t,1} + V_{t,2}$$

$$V_{t,0} = -54 \sum_{I,J=moon,sun} \frac{gm_I}{a_J^3} \tilde{D}_{t,J} \left[\sum_j \tilde{B}_{j,J} \cos \tilde{\Theta}_{j,J} \right] \cdot \left[\sum_i B_{i,J} \cos \Theta_{i,J} \right]$$

$$V_{t,1} = -18 \sum_{I,J=moon,sun} \frac{gm_I}{a_I^3} \tilde{D}_{t,J} \sum_{\substack{i,j \\ \tau,\tilde{\tau}=\pm 1}} C_i(\tau) \tilde{C}_j(\tilde{\tau}) \cos(\mu + \nu - \tilde{\mu} - \tilde{\nu} - \tau \cdot \Theta_i + \tilde{\tau} \cdot \Theta_j)$$

$$V_{t,2} = -\frac{9}{2} \sum_{I,J=moon,sun} \frac{gm_I}{a_I^3} \tilde{D}_{t,J} \sum_{\substack{i,j \\ \tau,\tilde{\tau}=\pm 1}} D_i(\tau) \tilde{D}_j(\tau) \cos(2\mu + 2\nu - 2\tilde{\mu} - 2\tilde{\nu} - \tau \cdot \Theta_i + \tilde{\tau} \cdot \tilde{\Theta}_j)$$

- The zonal component $V_{t,0}$ has a secular part as well as J_2, but the other two components have a **secular part** too.
- Therefore, we have three contributions to the precession.

Results of computations

Contributions to precession of **zonal tidal potential** *(in mas/cy)*

	δp *(mas/cy)*	Fraction of precession, ppm	Equivalent variation of dynamical ellipticity H = 3273794.48 E-9
Argument $\Theta_i=0$	***43.890***	*8.7*	*-28.51 E-09*
Argument $\Theta_i \neq 0$	*-4.121*	*-0.8*	*2.67 E-09*
Total effect	*39.769*	*7.9*	*-25.83 E-09*

As noticed by Lambert & Capitaine (2004) such kind of contributions would be automatically included in the observed value of the precession

The zero tide value of $\Delta\hat{C}_{20}$ equivalent to dp = 43.89 mas/cy (linked to the only strictly secular increment of Stokes coefficients) is -4.217269603*10^(-9).
The IERS reference value is -4.1736*10^(-9)

Contributions to precession of **all** components of the tidal potential *(mas/cy units)*

	Zonal (2,0)	Tesseral (2,1)	Sectorial (2,2)	Total
Argument $\Theta_i=0$	*43.890*	*-39.337*	*-4.553*	*0*
Argument $\Theta_i\neq0$	*-4.121*	*-27.572*	*31.693*	*0*
Sums	*39.769*	*-66.909*	*27.140*	***0***

- **The contributions of the three harmonic components of the tidal potential are cancelled out.**
- Regarding nutations, that kind of possibility was considered as matter of further research by Mathews (2003) and Lambert & Capitaine (2004)
- There is no net effect on the precession in longitude at this order of approximation

Consequences

- **The "observed" value of the dynamical ellipticity H** relevant for computing the precession of the equator **is not a zero tide value**, **unlike the observed value of** J_2, relevant for satellite motions.

- The permanent tide due to lunisolar attraction must not be used to correct the value of H, since its effect on the precession is cancelled with the non-zonal harmonics. That should be kept in mind when using relationships as

$$H = -\sqrt{5}\,\frac{MR_c^2}{C}\,\overline{C}_{20}$$

- Besides, there are no associated corrections of nutations.

Case of frequency dependent Love numbers

- It is known that Love numbers k_n vary with the frequency of tides due to anelasticity, specially in the diurnal band because of RFCN resonance.

- There are different sets of such Love numbers available

  - Mathews et al (JGR, 1995), adopted in IERS Conventions
  - Dehant et al (JGR, 2002)…

- The former Hamiltonian approach can be adapted to this case. Formulations are longer and will not be presented

Contributions to precession with frequency dependent Love numbers (mas/cy units)

	Zonal (2,0)	Tesseral (2,1)	Sectorial (2,2)	Total
Argument $\Theta_i=0$	*50.865*	*-39.337*	*-5.298*	*6.230*
Argument $\Theta_i\neq0$	*-4.776*	*-32.367*	*36.878*	*-0.265*
Sums	*46.089*	*-71.704*	*31.580*	***5.965***

- IERS recommended model was used in computations
- **Assuming frequency dependent Love numbers, the effects of the three harmonic components of the tidal potential are not cancelled out.**
- **There is a net contribution to the precession in longitude about 1.2 ppm**
- Values of some nutations are affected at the µas level

Conclusions

- Assuming linear elasticity, lunisolar tidal yielding of the solid earth does not effect precession.
  The relevant value of the dynamical ellipcity is not a zero tide value

- Assuming non-linear elasticity and frequency dependent Love numbers, there is a small effect of about 1 ppm.

- The value of H should be corrected accordingly and some nutations amplitudes are affected