Calibration of the hydrogeological model of the Baltic Artesian Basin

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Abstract

The aim of the present study is to calibrate a hydrogeological mathematical model for the Baltic Artesian Basin (BAB). The calibration of the model is an essential part of the creation of a hydrogeological (groundwater flow) model inside a system with limited knowledge about the boundaries of geological layers and the material properties (hydraulic conductivities).

1. Mathematical Model of Baltic Artesian Basin

BAB is a multi-layered sedimentary basin and a complex hydrogeological system located in Northern part of Europe.

The finite element method is employed for the calculation of the steady state 3D groundwater flow with free surface. The model of geological structure consists of 42 layers based on 2D triangular base mesh (Figure 1). No-flow boundary conditions were applied on the rock bottom and the side boundaries of BAB, while simple hydrological model is applied on the surface. The level of the lakes, rivers and the sea is fixed as constant hydraulic head. The infiltration through the top surface initially is assumed as the distribution from the regional climate models and is adjusted during the automatic calibration process. Averaged long-term water extraction was applied at the water supply wells.

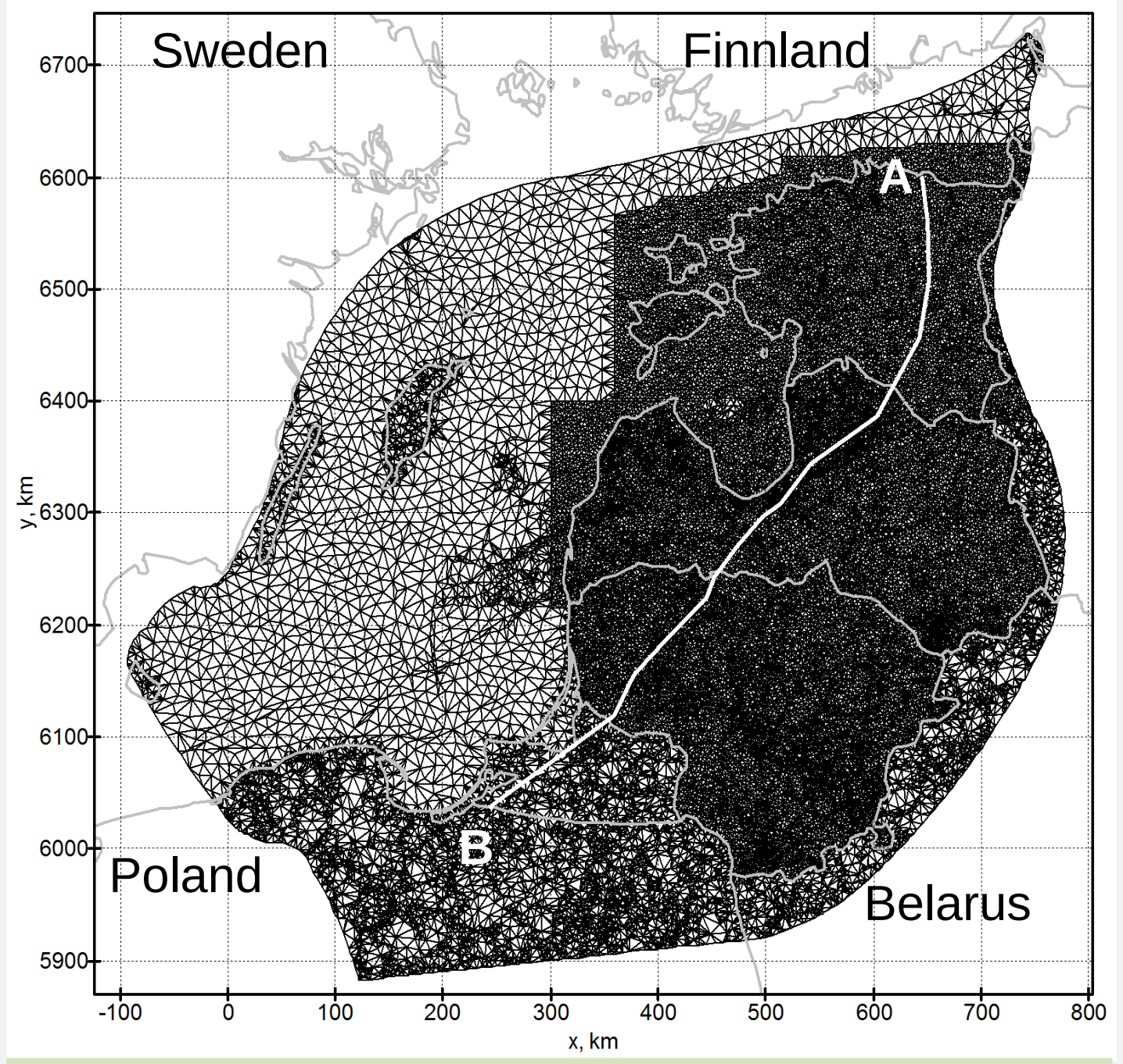

Figure 1: Triangular 2D base mesh of BAB. Grey lines – borders of the countries. White line – line AB

2. Calibration

The mathematical model for the BAB is calibrated on the statistically weighted borehole water level measurements applying automatic parameter optimization method L-BFGS-B for the hydraulic conductivities of each layer.

Both water level measurements in monitoring wells and level measurements in boreholes during the installation are used for calibration. As the available data is not uniformly distributed over the covered area, spatial weight coefficient is assigned to each borehole in order not to overestimate the clusters of boreholes (Figure 2).

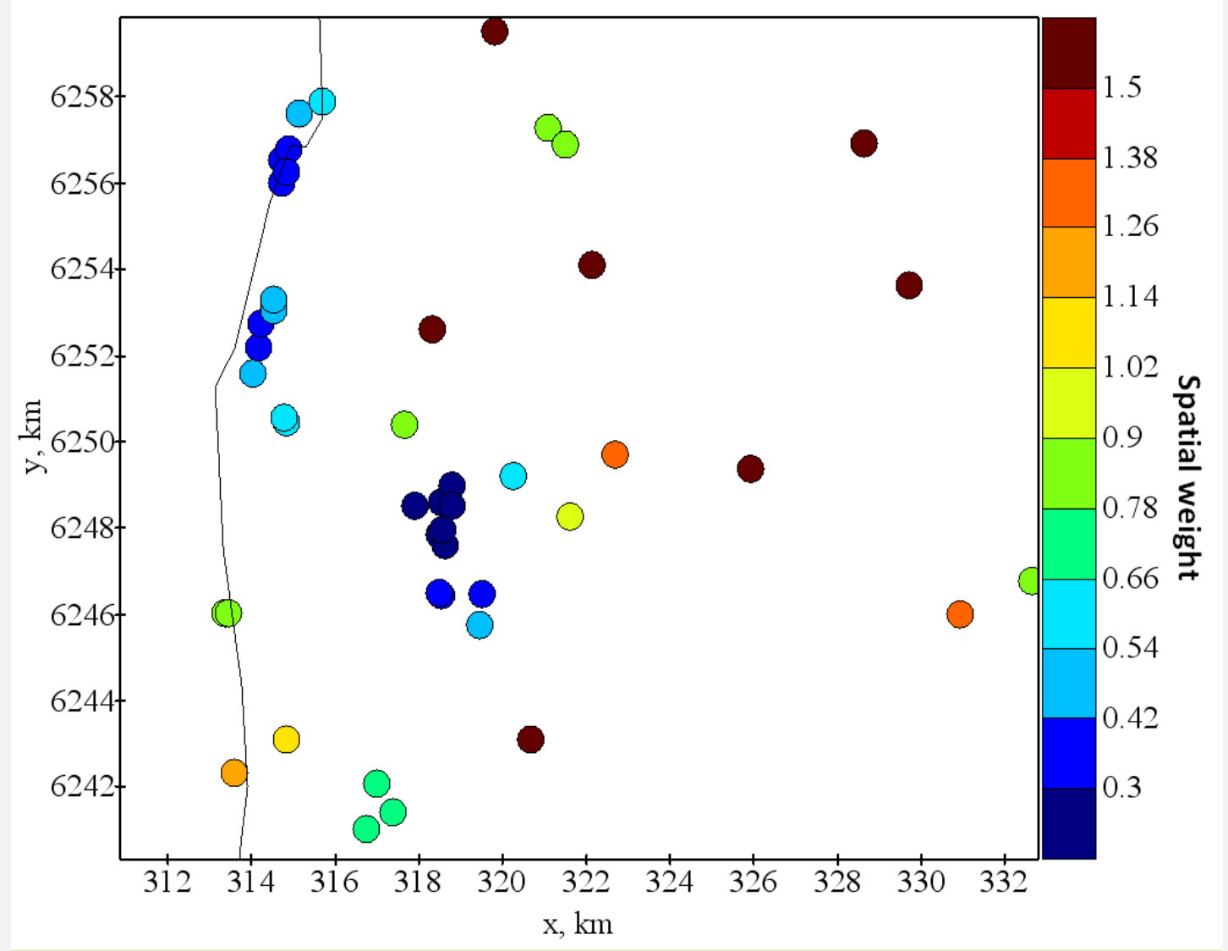

Figure 2: Spatial weight function in layer D3gj near city Liepāja.

The coefficients are: $c_{ri} = \sum_{i=1}^{N} \left(\frac{1}{\sum_{j=1}^{N} e^{-\frac{(r_i - r_j)^2}{\sigma^2}}} \right) / \sum_{j=1}^{N} e^{-\frac{(r_i - r_j)^2}{\sigma^2}}$, here r_i is the coordinate vector of the corresponding borehole, r_j is the coordinate vector of *j*-th borehole from *N* boreholes in hydrogeological layer, σ is the distance of influence (currently 1500 m).

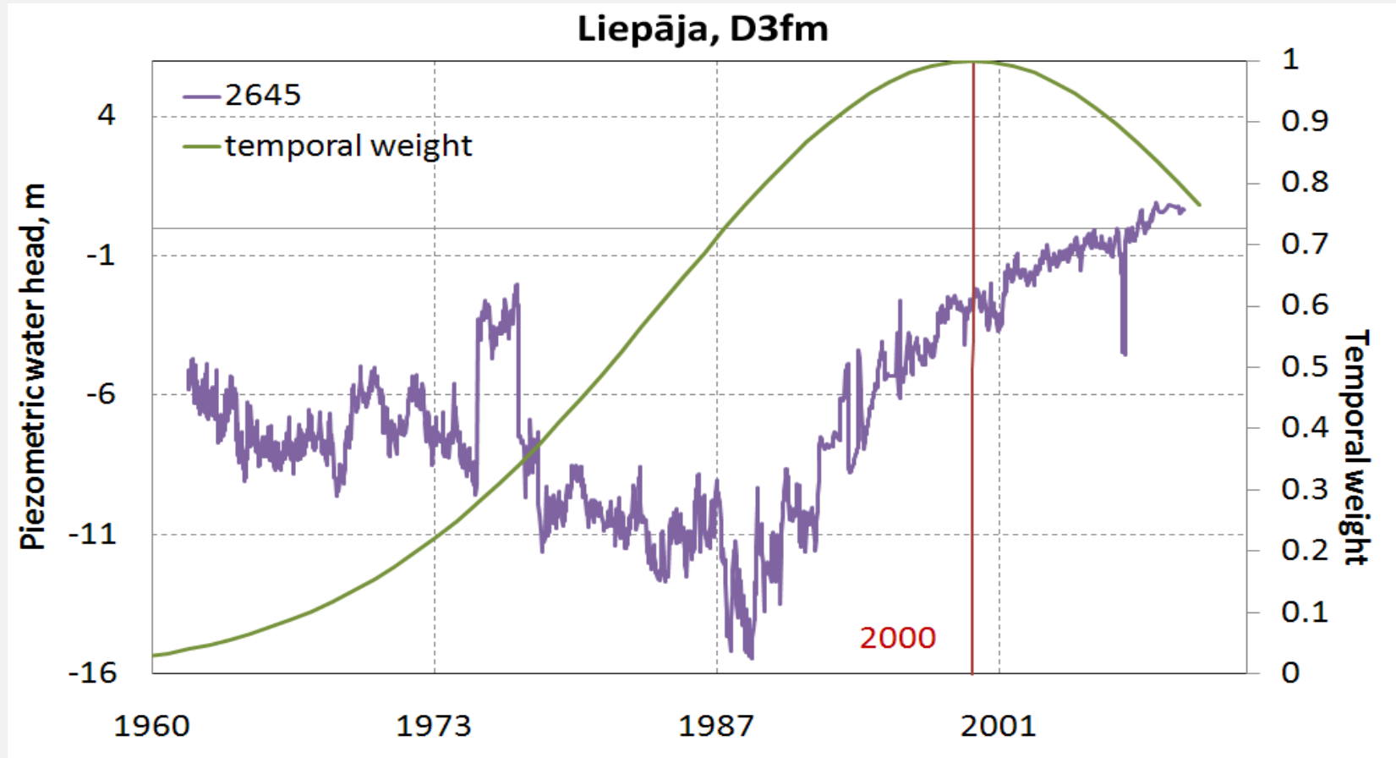

Figure 3: Temporal weight function for observations of the monitoring well #2645.

The monitoring data show distinct time dependence of water level in aquifers intensively used for groundwater abstraction. The data taken exactly in year 2000 are insufficient, therefore the observations from surrounding years are also taken into account but with smaller weighting coefficients (Figure 3).

The coefficients are: $c_{ti} = \frac{e^{-\frac{(t_i - t_0)^2}{2\tau^2}}}{\sum_{i=1}^{N} e^{-\frac{(t_i - t_0)^2}{2\tau^2}}}$ here t_0 = 2000 is the year for calibration, t_i is the year of i-th observation and τ is the time of influence.

The objective function Z_j of layer *j* is the weighted sum of squared differences between observed and modeled piezometric heads:

$$Z_j = \frac{1}{\sum_{i=1}^{N} c_{ti} c_{ri}} \sum_{i=1}^{N} c_{ti}\, c_{ri}\, (h_{obs} - h_{mod})^2$$

here h_{obs} is the observed head, h_{mod} is the modeled head and *N* is the number of the observations in the layer *j*. The overall objective function *Z*, which is minimized by the optimization method L-BFGS-B, is the sum of Z_j, equal importance of each layer is supposed. The parameters of the calibration are the horizontal and vertical hydraulic conductivities of the layers. The initial values of conductivities are taken from the available field pumping test measurements or based on the lithology of individual hydrogeological layers.

3. Results

The minimization of objective function typically converges in several hundreds of iterations (Figure 4) and the mean squared difference in one layer is 7 m (Figure 5). In Fig. 5 the objective functions for two different initial conditions converging to the similar result are seen. The ratio between the horizontal and vertical conductivity is kept fixed in each optimization run. The correlation between the modeled and observed data is shown in Figure 6.

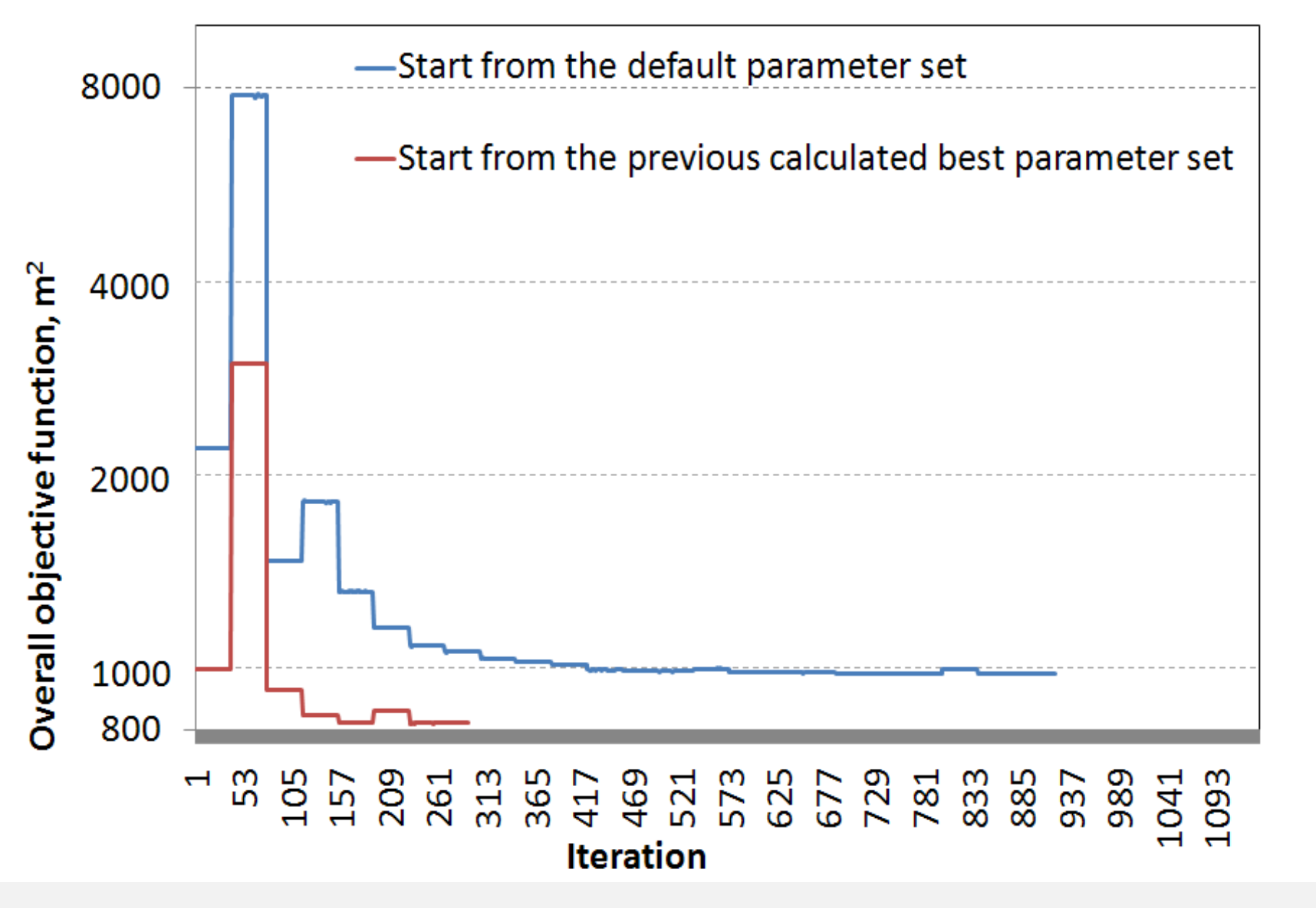

Figure 4: Optimization procedure of the objective function.

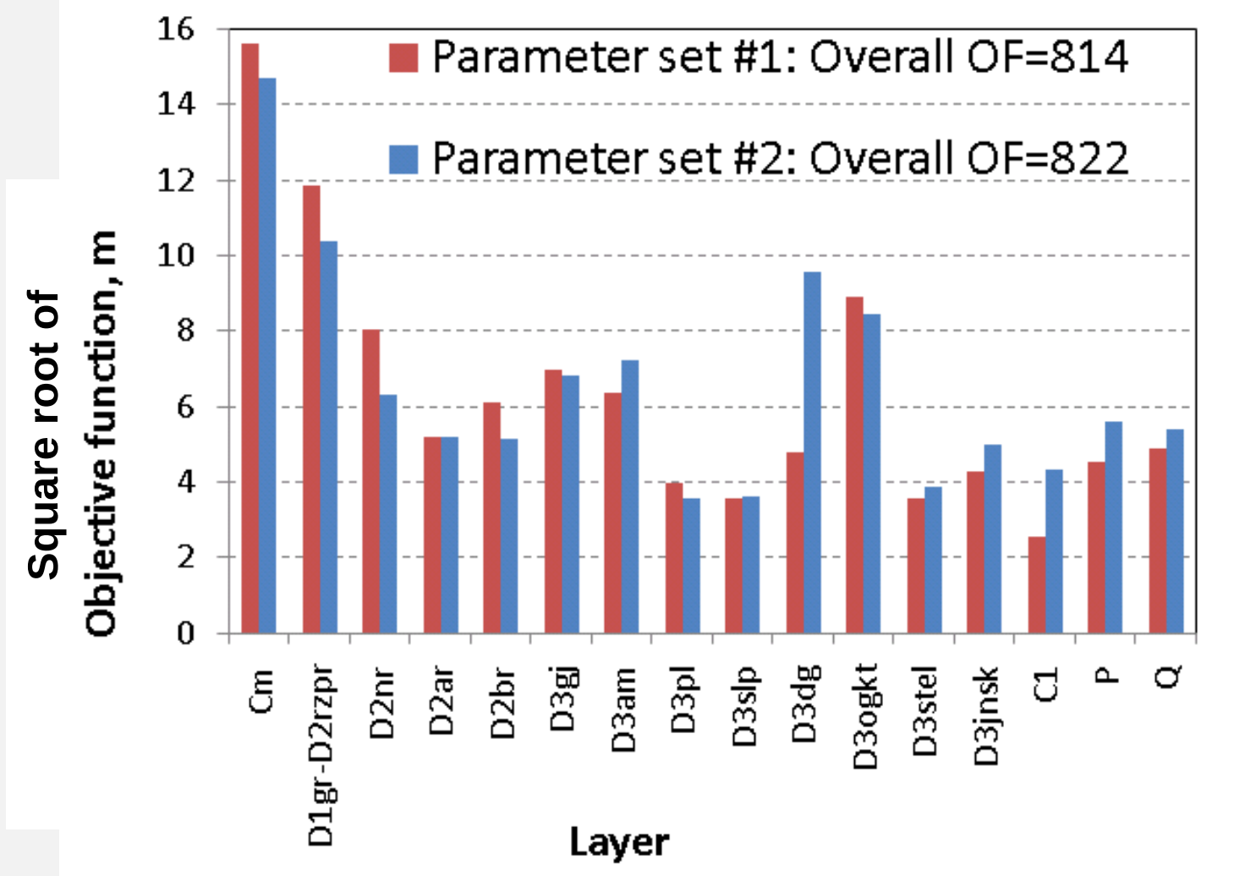

Figure 5: The square root of the components of objective function in each layer.

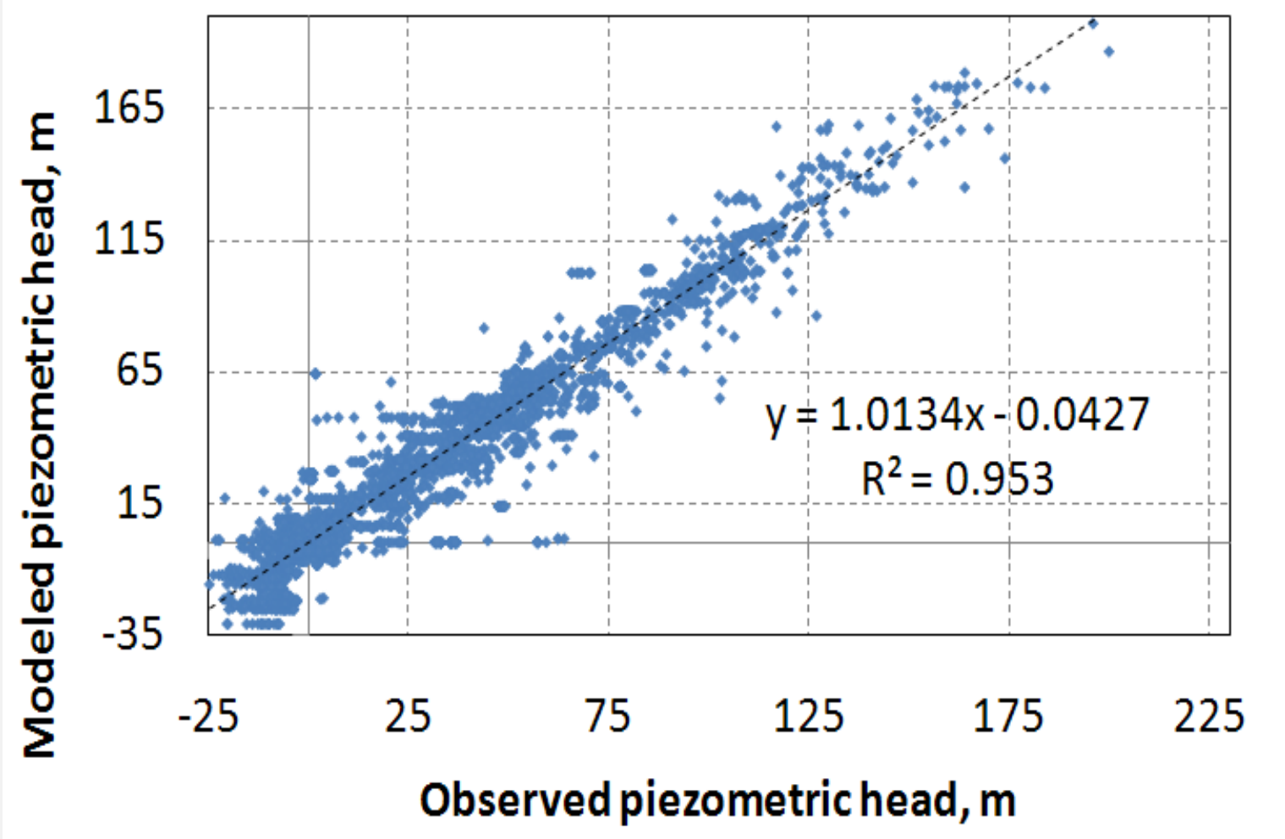

Figure 6: The comparision between observed and modeled data.

Figure 7 shows the conductivities after calibration in the cross-section along the line AB (white line in Figure 1).

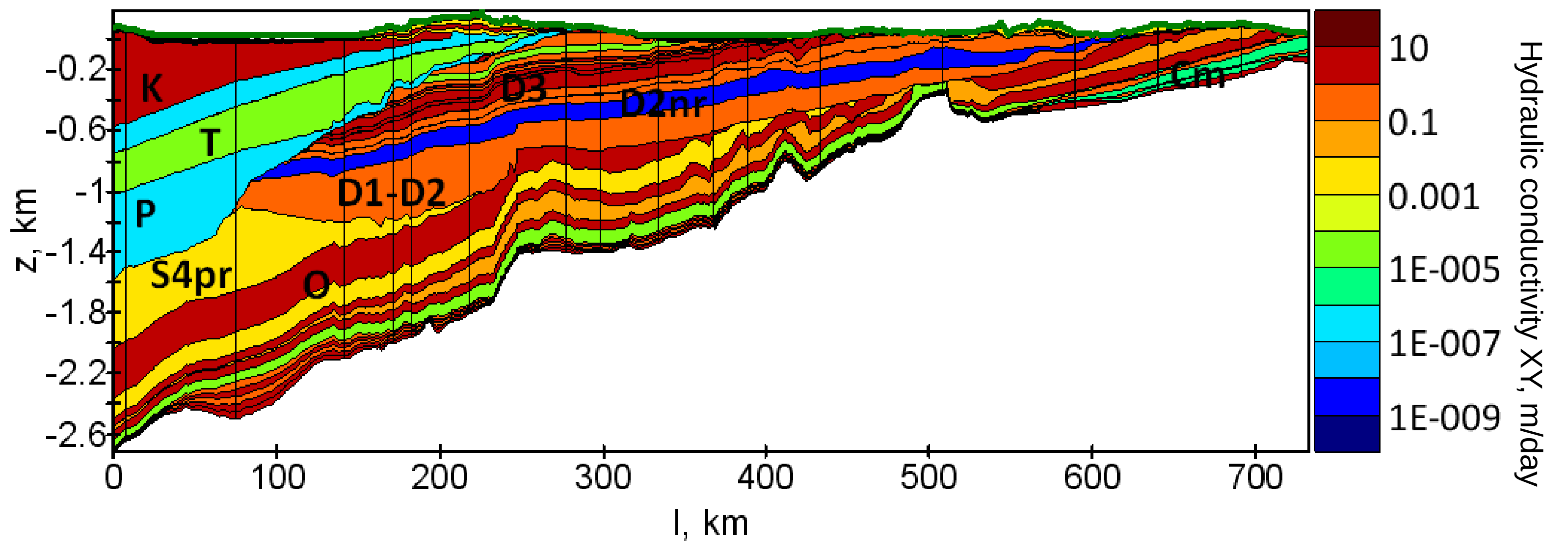

Figure 7: The horizontal hydraulic conductivities of the layers after calibration (cross-section along the line AB).

The allowed variation range (a multiplicative coefficient applied to the corresponding initial parameter value) of conductivities for all layers and infiltration rate is 0.01 to 100 times. Figure 8 shows the evolution of hydraulic conductivity of some aquifers corresponding to the optimisation procedure shown in Fig. 4 with blue line. However, hydraulic conductivities of some aquitards have reached their limits and the restarting of the calculation (red line) from the previous calculated (blue line) best coefficient set with changed limits results in further reduction of target function.

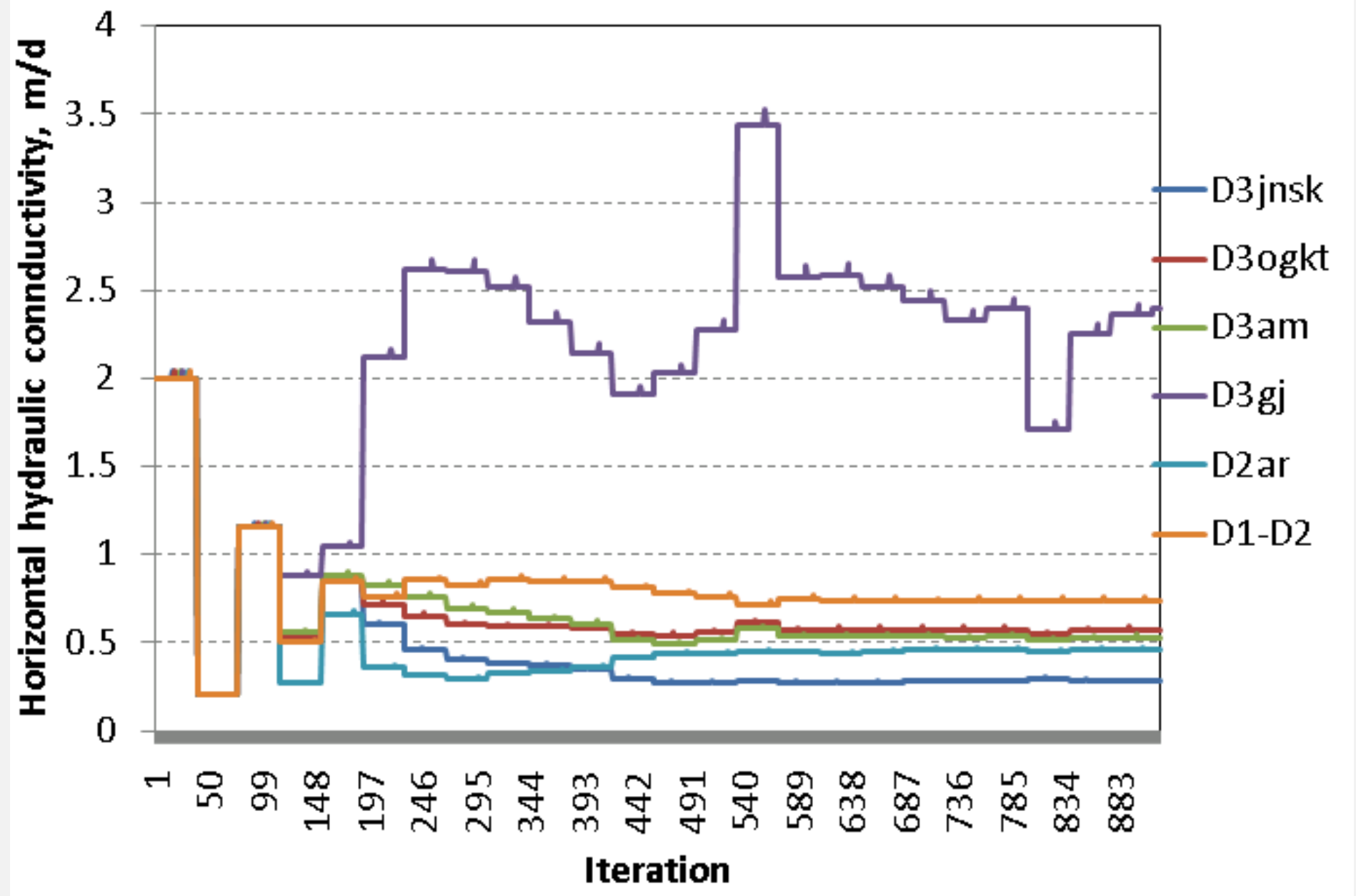

Figure 8: The evolution of the horizontal hydraulic conductivity during the optimization process.

4. Conclusions

The geometric model and the introduction of weight coefficients has allowed to reach a good correlation between modeled and observed data.

The mean squared difference in one layer is 7 m, which is considered as a satisfactory result for the current resolution of the BAB model mesh.

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