Geometry of the refractivity field for GNSS signals propagation

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Introduction

In order to improve the modelling of the propagation of GNSS electromagnetic signals through the neutral atmosphere and achieve millimetric accuracy at low elevation, the GRGS (Groupe de Recherche de Géodésie Spatiale) in collaboration with CLS (Collecte Localisation Satellite) has developed a new set of mapping functions called AMF (Adaptive Mapping Functions) for applications in geodesy (GNSS and DORIS orbits determination and stations positioning), altimetry or radar InSar. The idea is to use high resolution observational data assimilations produced by the ECMWF (European Center for Medium-range Weather Forecast) to model tropospheric delays from ray tracing for all elevations and azimuths. AMF are used to fit tropospheric ray-traced delays using a few numbers of coefficients for a given site at a given time.
The refractivity of the moist air is the key physical parameter which drive the propagation of GNSS signals through the troposphere. To rebuilt the geometry of refractivity field from ECMWF model-levels data, it is necessary to choose a model of gravity acceleration. In this poster, after a description of ECMWF model-levels data and an explanation about how to determine the geometry of the refractivity field from these data, we will study several usual models and their impact on the tropospheric delay.
The ray tracing algorithm consists of integrating the eikonal differential system using refractivity interpolated from the discretization by model levels. To do this integration in a simpler form, we can divide the atmosphere in layers and make 2D ray tracing using Snell's law at each levels. We will compare several models to approximate reference and Earth's surfaces and see how tropospheric delays change.

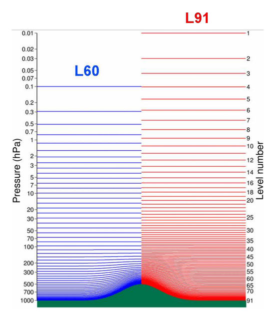
Ecmwf model-levels data

The current ECMWF model divides the atmosphere into 91 layers which are defined in terms of pressure instead of altitude. Theses layers are defined on half levels by :

$$p_{k+1}(\phi,\lambda,t) = A_{k+1/2} + B_{k+1/2}\, p_s(\phi,\lambda,t)$$

where $p_s(\phi,\lambda,t)$ is the surface pressure field (changing in time) and $A_{k+1/2}$ and $B_{k+1/2}$ are a set of fixed constant coefficients whose values effectively define the **vertical hybrid coordinates**. The horizontal discretization is defined by a spherical harmonics truncation which can be interpolated at a regular grid of geodetic latitude ϕ and geodetic longitude λ.

Vertical hybrid coordinates



The vertical resolution is higher in the troposphere and lower in the stratosphere. Levels follow the contours of the orography in the lower atmosphere and they are isobars above the level 30 because of the impact of orographic features on the atmospheric circulation decreases with altitude.

ECMWF data contains for one epoch :
– the **pressure** p_s at the orography surface
– the **geopotential** Φ_s at the orography surface
– the **temperature** T_k for each level
– the **specific humidity** q_k for each level

Ecmwf model-levels geometry

Equations of motion which are solved by the weather forecast model, have a simpler form in terms of geopotential than geodetic height : using geopotential eliminates centrifugal force and air density in the equations and makes analytical calculations more convenient. **The influence of gravitional attraction inside the ECMWF model derives from the geopotential**.

Hybrid coordinates are defined from the time variable surface pressure on the orography. The discrete formulation of hydrostatic equation for the moist air relies geopotential and pressure at each level. So **no assumption is made on the geometry of the atmosphere** in the ECMWF model except to define the orographic geopotential.

The orography is derived by averaging and filtering the GTOPO30 terrain elevation data set. GTOPO30 elevations are referenced vertically to the EGM96 geoid of WGS84. As ECMWF model approximates gravity by a mean gravity to define orography, $\Phi_s(\phi,\lambda,t)$ is defined as orographic orthometric height multiplied by the constant standard gravity $g_0 = 9.80665$ m.s^{-2}. Therefore, we can conclude that **orography is realistic and properly defines with respect to the WGS84.**

How to determine the geometry of refractivity field

Geopotential and pressure at each level are rebuilt from the model-levels data and the hydrostatic equilibrium using a method detailed in the IFS documentation, part 3. The **total refractivity** of moist air for all levels is expressed in terms of the total pressure p_k and the pressure of water vapor p_k^v :

$$N_k(\phi,\lambda,t) = k_1\frac{p_k(\phi,\lambda,t)}{T_k^v(\phi,\lambda,t)} + k_2'\frac{p_k^v(\phi,\lambda,t)}{T_k(\phi,\lambda,t)} + k_3\frac{p_k^v(\phi,\lambda,t)}{T_k(\phi,\lambda,t)^2}$$

where k_1, k_2' and k_3 are empirical coefficients (Rüger, 2002) and T_k^v denotes the virtual temperature. Using this formula, we have the refractivity for all levels.

GNSS ray tracing applications use geodetic coordinates : we have to transform geopotential at ECMWF levels Φ_k into geodetic height h_k (height above the WGS84 ellipsoid). **To make the conversion between geopotential and geodetic height, it is necessary to choose a model of gravity acceleration.** For the following explanation, we consider the normal gravity as defined in the WGS84. The geopotential is defined by :

$$\Phi(\phi,\lambda,h) = \int_0^h \gamma(\phi,\lambda,z)dz$$

where $\gamma(\phi,\lambda,z)$ is the normal gravity which is given on the ellipsoid surface by the Somigliana formula :

$$\gamma_s(\phi) = \gamma_e\frac{1+k\sin^2(\phi)}{\sqrt{1-e^2\sin^2(\phi)}} \text{ where } k = \frac{b\gamma_p}{a\gamma_e} - 1$$

where γ_e and γ_p are respectively theoretical normal gravity at equator and poles, a and b are the semi-major and minor axes of the WGS84 ellipsoid.

We can realize γ above the ellipsoid by γ_h at any level using a truncated Taylor series expansion on height :

$$\gamma_h(\phi,\lambda,z) = \gamma_s(\phi)\left[1 - \frac{2z}{a}\left(1+f+m-2f\sin^2(\phi)\right) + \frac{3z^2}{a^2}\right]$$

$$\text{where } m = \frac{\omega^2 a^2 b^2}{GM}$$

and f is the WGS84 ellipsoid flattening.

As the geopotential is the integral of γ, the normal height $H_k(\phi,\lambda)$ of the geopotential $\Phi_k(\phi,\lambda)$ at each level has to satisfy the following equation :

$$\Phi_k(\phi) = \gamma_s(\phi)H_k\left[1 - \frac{H_k}{a}\left(1+f+m-2f\sin^2(\phi)\right) + \frac{H_k^2}{a^2}\right]$$

The normal height H_k of each level can be solved iteratively from Φ_k using this formula and geodetic heights h_k can be obtained adding H_k and $\zeta(\phi,\lambda)$, the EGM96 height anomaly. Thus we have geodetic coordinates of all levels and hence **the geometry of the refractivity field is rebuilt.**

In this section we have explained how to retrieve the geometry of refractivity field from model levels data. We have explained the method with a model of gravity acceleration but other models can be used. In the next column, we will transform geopotential in geodetic heights with other models of gravity acceleration and will investigate the influence of this change on the tropospheric delay.

Experiment's description

To study the importance of gravity acceleration model used and see only the influence of this model regardless of geoid or geoid undulations, we compute tropospheric delays in **an idealized Earth** : the WGS84 reference ellipsoid **without topography and geoid**. We make calculations in the following atmosphere :
– Earth's atmosphere is static without humidity and has the same pressure and temperature all over the ellipsoid.
– Atmosphere is divided in 1680 regular geopotential layers.
– The upper layer is defined as the isogeopotential surface with the value of 82,3758.6 m^2.s^{-2}
– We consider the vacuum state above the upper layer.
– Pressure and temperature at each level are interpolated from pressure and temperature profiles defined in the US Standard Atmosphere 76.
These assumptions of no topography and no geoid lead to that : pressure is constant on the reference surface, levels are isogeopotential surfaces and geopotential levels are parallel surfaces of the Earth's surface.

Alternative ways of modelling gravity acceleration

Here the idea is to examine in the aforementioned conditions, how tropospheric delays change with respect to the model of gravity acceleration which is unavoidable to convert geopotential from model-levels data to geodetic height. As height is defined with the respect of the considered gravity, it is necessary to define dynamic $H^{(d)}$, orthometric $H^{(o)}$ and normal H height accordingly with the chosen model. We have selected the following models for our experiment :
– the **constant** model : Gravity is constant anywhere on and above the Earth :

$$g^{(1)} = g_0 = 9.80665 \text{ m.s}^{-2} \Rightarrow \Phi^{(1)} = g_0 H^{(d)}$$

– the **ellipsoidal** model : Gravity is determined on the WGS84 ellipsoid surface by the Somigliana formula but constant with altitude :

$$\gamma^{(2)}(\phi) = \gamma_s(\phi) \Rightarrow \Phi^{(2)}(\phi) = \gamma_s(\phi)H^{(d)}$$

– the **spherical and altitude dependent** model : Gravity is calculated for an idealized Earth : a sphere with Earth's mean radius. Gravity is equal to g_0 at the Earth's surface and decreases with altitude. This formula is obtained analytically using the inverse square law of gravitation :

$$g^{(3)}(H^{(o)}) = g_0\frac{r_0^2}{(r_0+H^{(o)})^2} \Rightarrow \Phi^{(3)}(H^{(o)}) = g_0\frac{r_0 H^{(o)}}{r_0+H^{(o)}}$$

– the **ellipsoidal and altitude dependent** model : Gravity as defined in the WGS84 :

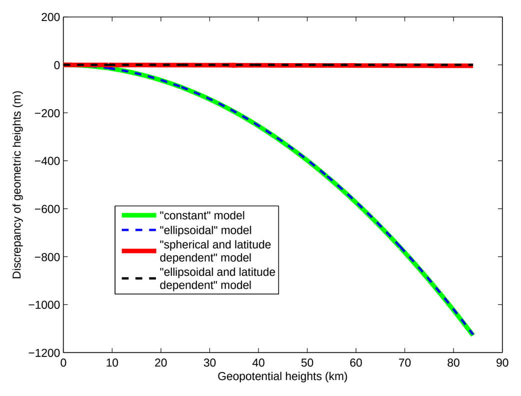
$$\Phi^{(4)}(\phi,H) = \gamma_s(\phi)H\left[1 - \frac{H}{a}\left(1+f+m-2f\sin^2(\phi)\right) + \frac{H^2}{a^2}\right]$$

It is usual to approximate the geodetic height by the sum of the normal height and the height anomaly or by the sum of the orthometric height and the geoidal undulation. As we work with an idealized Earth without geoid, geoidal undulation and height anomaly are equal to zero. This implies that in this context, orthometric, dynamic and normal heights are equal to the geodetic height.

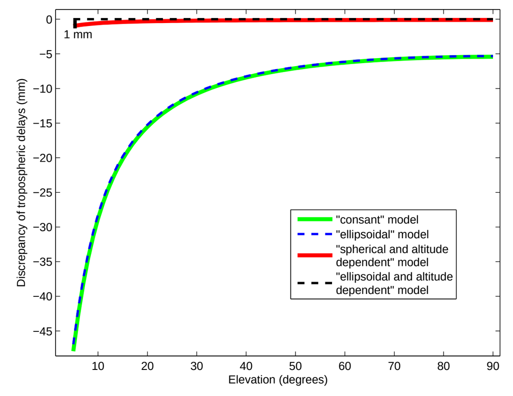
As the "ellipsoidal and altitude dependent" model is more realistic among the three models, we have chosen it as reference. So, to the following figures, we represent geodetic heights and tropospheric delay with the respect to the "ellipsoidal and altitude dependent" model which have the value 0.

We have a geodetic height difference of 1.1 km for the upper layer (i.e geopotential height of 84 km) between models which are altitude dependent and not.

Discrepancy of atmospheric levels variation with respect to the "ellipsoidal and latitude dependent" model



Discrepancy tropospheric delay for a base point at 45° geodetic latitude



We have a difference of 4.5 cm at low elevation between models which depend on altitude ("ellipsoidal and altitude dependent" and "spherical and altitude dependent" models) and these which are not dependent ("constant" and "ellipsoidal" models). This difference depends on latitude : at the equator, it is equal to 12 cm.

The difference between "ellipsoidal and altitude dependent" and "spherical and altitude dependent" models is only of 1 mm at low elevation.

Conclusion :

– At low elevation, the impact on the tropospheric delays between models are at least millimetric so **it is important to have for computations the more realistic description of the gravity field as possible.**
– But to this study, we can also conclude that **the principal influence on slant tropospheric delays is the altitude dependency and not the geometry of the gravity field**.

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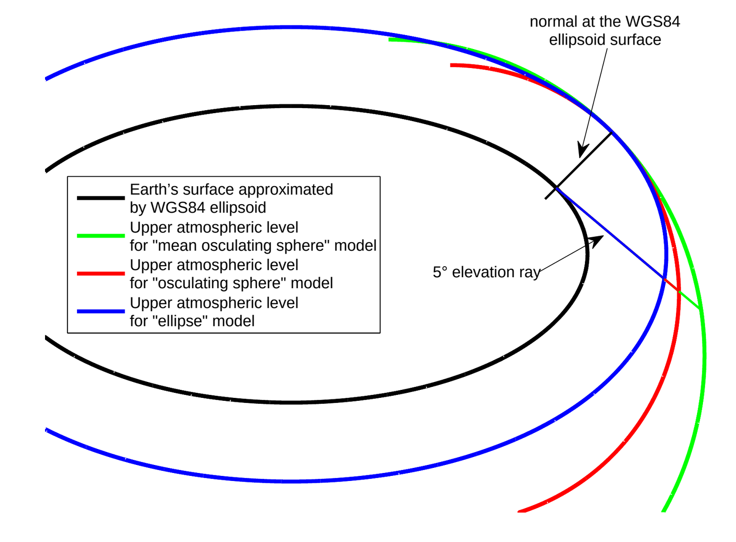
Earth's shape influence

To make calculations more convenient, we can approximate Earth's surface. This modifies the geometry of the atmosphere since we have considered that atmospheric levels are parallel surfaces to the Earth's surface. But what is the influence of changing atmosphere shape on slant tropospheric delays ? To answer, we compute with a 2D ray tracing algorithm using Snell's law at each level, slant tropospheric delays with several Earth's surface model in the ray direction :

Representation of the upper layer of atmosphere with the different models of Earth's surface for a ray having south direction and a geodetic latitude of 45°.



We have not considered the Earth as a concentric sphere with a mean radius (not dependent on latitude) because of we know that this model is not sufficient to have millimetric accuracy (it introduces a centimetric error) on slant tropospheric delays.

– the **mean osculating sphere** which is only latitude dependent : atmospheric levels are parallel to the circle which has the mean radius of curvature of the WGS84 ellipsoid at the latitude ϕ where ray tracing begins. This mean radius of curvature is :

$$R_g(\phi) = \sqrt{R_n(\phi)\times R_m(\phi)}$$

where $R_m(\phi) = \frac{a(1-e^2)}{\sqrt{1-e^2\sin^2(\phi)}(1-e^2\sin^2(\phi))}$ and $R_n(\phi) = \frac{a}{\sqrt{1-e^2\sin^2(\phi)}}$ are respectively the WGS84 ellipsoid's radii of curvature in the meridian plane and in the prime vertical normal section.

– the **osculating sphere** in the direction of the ray propagation which is latitude and azimuth dependent : atmospheric levels are parallel to the osculating circle which has the same radius of curvature than the WGS84 ellipsoid in the ray direction. The radius of the osculating circle can be computed using the Euler's formula :

$$R_e(\phi,\alpha) = R_n(\phi)\times\sin^2(\alpha) + R_m(\phi)\times\cos^2(\alpha)$$

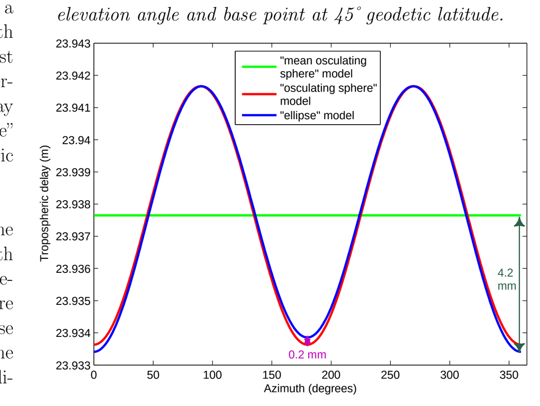
where α is the initial azimuth of the ray.

– the **ellipse** resulting of the intersection between the WGS84 reference ellipsoid and the plane of ray propagation : atmospheric levels are parallel to this ellipse.

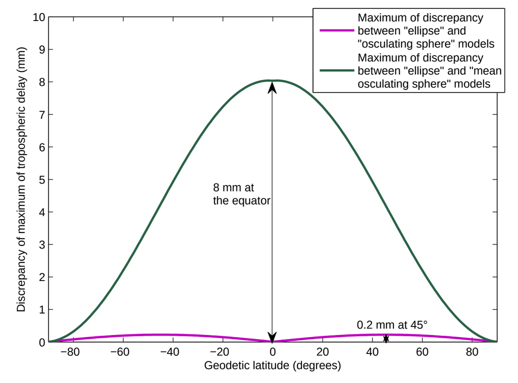
The discrepancy between the "mean osculating sphere" model and others reaches a minimum (-4.2 mm) along the north-south direction and a peak along the east-west direction. The difference is zero in the intermediate directions : slant tropospheric delay computed with the "mean osculating sphere" model corresponds to a mean tropospheric delay at each latitude.

Slant tropospheric delays computed with the "osculating sphere" model are equal in north and south directions while tropospheric delays computed with the "ellipse" model are not the same in these two directions because the ellipsoidal surface is not symmetric. The difference is maximal in the north-south direction with a value of 0.2 mm.

Tropospheric delay in function of azimuth for 5° elevation angle and base point at 45° geodetic latitude.



Maximum of discrepancy of tropospheric delay for 5° elevation between models in function of geodetic latitude



To construct this figure, we have computed at each geodetic latitude slant tropospheric delays in function of azimuth. We have picked up the maximum of discrepancy between the "ellipse" and "mean osculating sphere" models and between the "ellipse" and "osculating sphere" models and have verified that these maxima are always in the same directions.

This figure shows us that the maximum of discrepancy between the "ellipse" and "osculating sphere" models is always sub-millimetric and maximum at 45°. This can be explained in term of geometry : 45° of geodetic latitude is the place where ellipsoid have the highest asymmetry.

Concerning the maximum of discrepancy between the "ellipse" and "osculating sphere" models, it is equal to zero at the poles and increases towards the equator. This can be explain in term of geometry : at poles, the mean osculating sphere and ellipsoid are coincident in all direction ($R_n(\pm 90) = R_m(\pm 90)$) at the contrary of the equator where the discrepancy between R_n and R_m is maximal and the ellipsoid is the least similar to a sphere.

As the "ellipse" model is the better realistic Earth's surface model among the three models, we have chosen it as reference. We can see that slant tropospheric delay computed with the "mean osculating sphere" model corresponds to a mean tropospheric delay at each latitude. The difference in terms of slant tropospheric delays between this model and our reference has a maximum of 8 mm in north-south direction on the equator and a mean value of 4 mm.

Conclusion :

– As we have the objective to achieve millimetric accuracy on slant tropospheric delays, we will conclude that **the "mean osculating sphere" model is not sufficient** in our case.
– **The ellipsoidal atmosphere has an azimuthal anisotropy which is not modelled by atmospheric surfaces parallel to osculating spheres. But the error on slant tropospheric delays of this anisotropy is always sub-millimetric if osculating spheres are used.**

Conclusions and perspectives

In the first part, we have seen how rebuilt the geometry of the refractivity field from ECMWF model-levels data and why it is necessary to choose an accurate model of gravity acceleration. We have studied several gravity models and seen that dependence on altitude of models is important to compute tropospheric delays. However, the considered gravity acceleration models which are altitude dependent, are mathematically valid only in the vicinity of the Earth's surface and we use those up to an height of 84 km. Now we might legitimately wonder if our gravity model has to be changed or not. We plan to provide a more realistic mathematical description to have a physical model valid at all heights and compare with models which have been studied in this poster.

In a second part, we have considered several geometries of the Earth's shape at a given site and determined their impact on tropospheric delays. We have seen that the geometry where atmospheric levels are placed relatively to a sphere with the mean radius of curvature at a given latitude is not sufficient to achieve our objective which is a millimetric accuracy on slant tropospheric delays.

We have also seen that an osculating sphere to the ellipsoid in a specific direction does model the asymmetry induced by the ellipsoid. The modelling difference is sub-millimetric on the slant tropospheric delays for all latitudes. Here we worked under a 2D approximation without considering horizontal tropospheric gradients. If we want to consider these gradients, we have to use a 3D interpretation. So the "ellipse" model is better suited to have the shape of the refractivity field in all directions without rebuilding the atmospheric structure over different osculating spheres for each direction.