Numerical and analytical solution of diffusion wave approximation for surface water routing in a distributed hydrological model

1. Introduction

The distributed rainfall-runoff model **WetSpa** was successfully used in the past 10 years for flood prediction and scenario analysis for impacts of landuse and climate change for a broad spectrum of catchment types. During the last decade more and more precise data sources became available for model parameterization. Use of them indicated some **oversimplifications in the model structure**. The major issue was the surface water routing component, which was based on an analytical solution of the diffusion wave approximation in the form of an Instantaneous Unit Hydrograph (IUH).

The IUH is calculated in the preprocessing stage and defines for all flow paths the responses between each model cell and the catchment outlet. **As a consequence, the model structure cannot account for changing conditions during a simulation** as e.g. changes in hydraulic radius of the flow paths. Moreover, the flow paths does not reflect other hydrological processes (e.g. infiltration, evapotranspiration) occurring during the routing of the water to the catchment outlet. This model characteristic easily can cause overestimation of the surface runoff component in the total runoff simulated by the model.

The here presented solution for this issue is a change from an analytical solution of the diffusion wave approximation (IUH) to a numerical solution. The numerical solution is calculated for each time step in the model computational cells while taking into account dynamics of surface water losses occurring in the catchment system. The two approaches of surface water routing are compared in a set of experimental catchments in order to evaluate the differences in results.

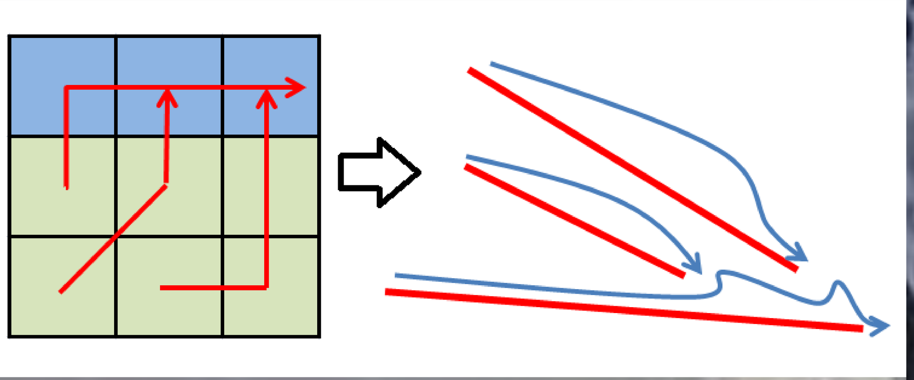

Fig 1. Conversion of a catchment to a cascade of flow paths.

2. The methods of surface water routing

1. Standard WetSpa IUH approach:

IUH is calculated for each cell to the catchment outlet, based on GIS-based flow path characteristics.

V_i is obtained by abstractions based on runoff coefficient and infiltration, so discharge is always generated.

$$U_i(t) = \frac{1}{\sqrt{2\pi\sigma_i^2 t/t_i}} \exp\left[-\frac{(t-t_i)^2}{2\sigma_i^2 t/t_i}\right]$$

$$Q_i(t) = \sum_{\tau=0}^{t-\tau} V_i(\tau) U_i(t-\tau)$$

$U_i(t)$ – flow path response at cell i

$Q_i(t)$ – cell i discharge

$V_i(\tau)$ – input runoff volume at cell i

σ – variation of the flow time

t_i – mean flow time from cell i to the flow path end

t, τ – time and time delay

2. New hydraulic approach :

Catchment is expressed as a cascade of flow paths connected by lateral inflow (Fig 1). Discharge from each flow path is calculated by the finite differences explicit solution of the diffusive wave approximation; the Control Volume Approach is used to stabilize the solution.

No runoff coefficient is used, water available for runoff is obtained by abstraction of infiltration.

$$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = K\frac{\partial^2 y}{\partial^2 x} + (i - f)$$

$$K = \frac{q}{2s}, \quad q = vy$$

$$c = \frac{5}{3}v, \quad v = \frac{1}{n} y^{2/3}\sqrt{s}$$

y – water height

t – time

x – distance

i, f – precipitation and infiltration rates

v – water velocity

s – land slope

n – Manning coefficient

3. Comparison with other models

The results of WetSpa with hydraulic routing in an "open-book" catchment (Fig 2) fits better the results obtained by other hydraulic model then the standard IUH-based routing (Fig 3). *The data presented in Fig. 3 were provided by Elga Salvadore.*

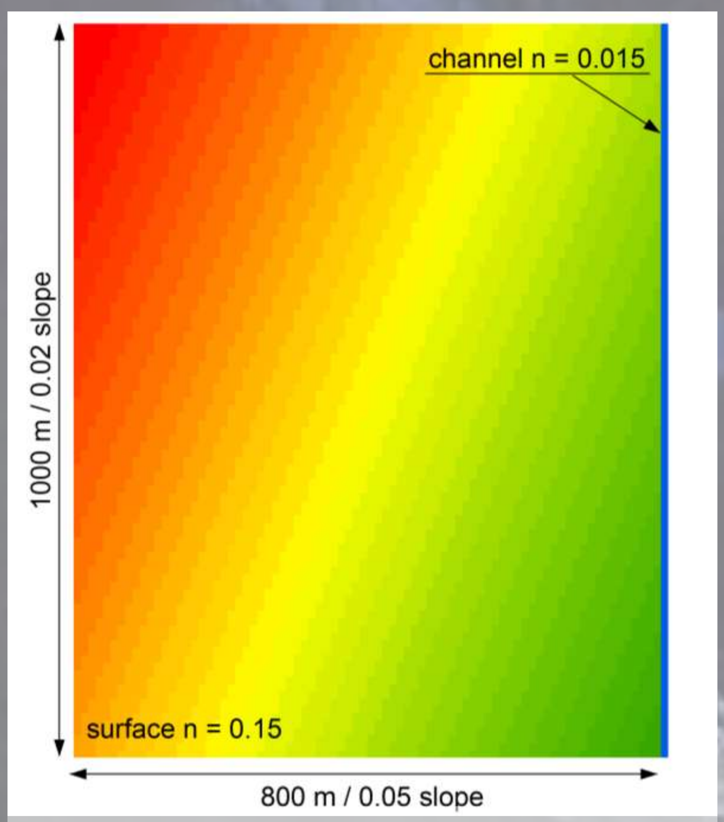

Fig 2. Elevation of the "open-book" catchment

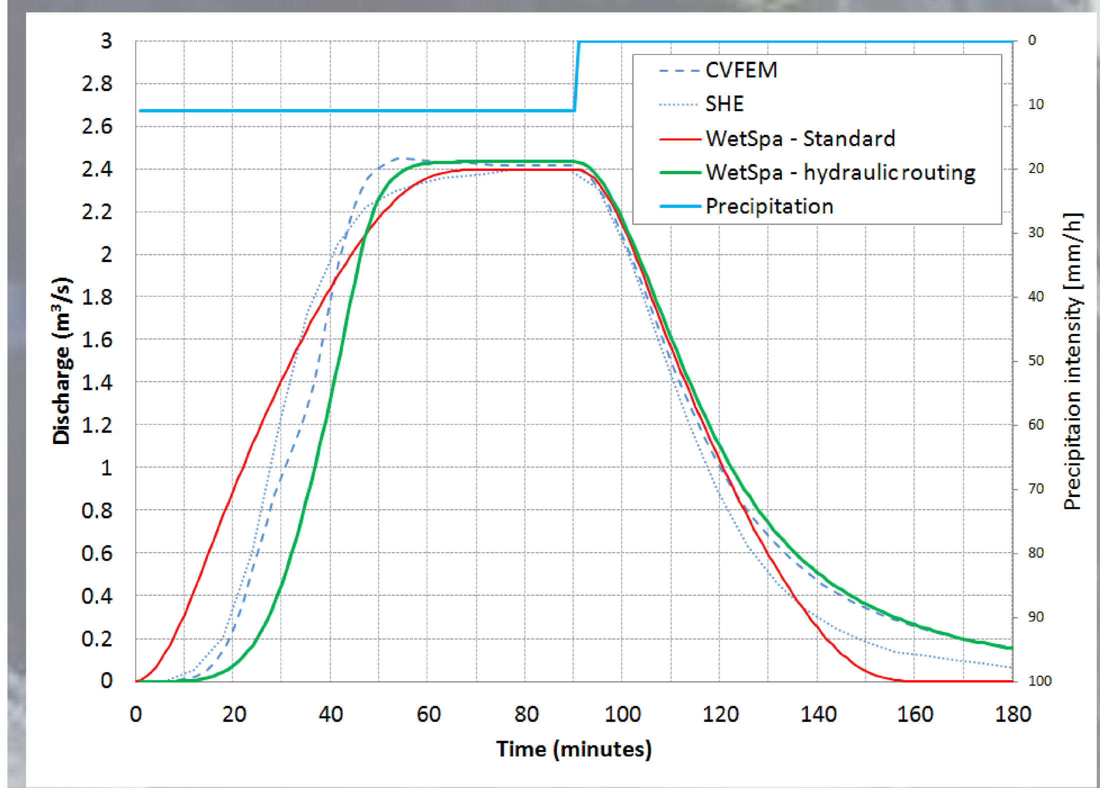

Fig 3. Discharge obtained from different models from the "open-book" catchment

4. The standard vs. hydraulic approach

Both surface water routing methods were tested in the 5 catchments presented in Fig 4. The catchments were different only in a soil type. The discharge simulation results, presented in the right column of Fig 4, shows that the linear IUH combined with a runoff coefficient provide just a rough approximation of the runoff simulated by the hydraulic model with an infiltration component.

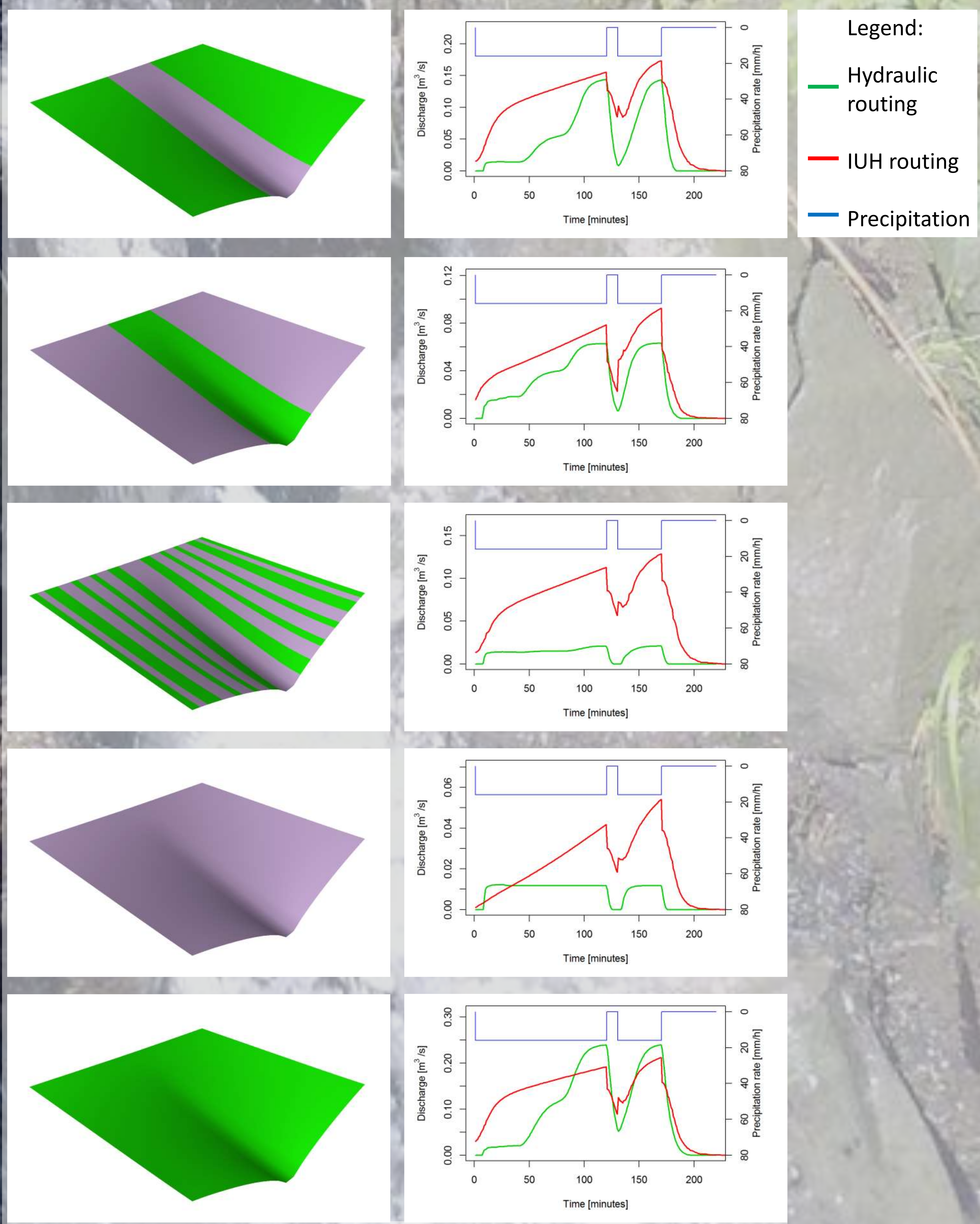

Fig 4. Experimental chatchment with different soils: **green – loamy sand**, **grey – clay loam** (left), output hydrograms using the IUH and hydraulic approaches (right).

5. Summary

The proposed here model, using diffusive wave approximation coupled with an infiltration component, shows very good performance when compared to other hydraulic models. The missed timing of rising and falling limbs of the hydrograph, observed in the IUH-based routing, is correct in the new, hydraulic approach (Fig 3).

The set of experiments presented in Fig 4 shows the issues of the IUH approach with runoff coefficient. Surface runoff is always generated at the catchment outlet when rainfall starts, while it should be delayed until the soil is fully saturated, as in the new approach. Moreover, by the overestimation of the falling limb in all five cases, it is clear, that the surface water can not infiltrate during it's routing, what is also solved in the new approach.

The new, hydraulic approach should be further tested in the real catchments and validated against true discharge, to prove that the new approach is useful and improves the WetSpa model performance, although the preliminary results are already promising.

Warsaw University of Life Sciences

Fundacja im. Anny Pasek

Vrije Universiteit Brussel

EGU General Assembly 2012

T. Berezowski (1)*, O. Batelaan (2,3), J. Chormanski (1)
(1)Dept. of Hydraulic Structures, Warsaw University of Life Sciences, Warsaw, Poland, E-mail: t.berezowski@levis.sggw.pl
(2) Dept. of Hydrology and Hydraulic Engineering, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium,
(3) Dept. of Earth and Environmental Sciences, Katholieke Universiteit Leuven, Celestijnenlaan 200E, 3001 Heverlee, Belgium,
*Tomasz Berezowski is a scholarship holder of the Anna Pasek Foundation.