Assilimation of Earth Rotation Parameters into the Community Atmosphere Model

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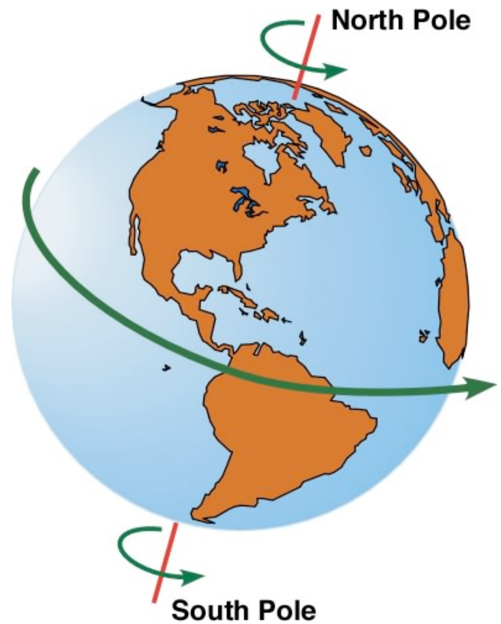

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GEOMAR Helmholtz Centre for Ocean Research Kiel

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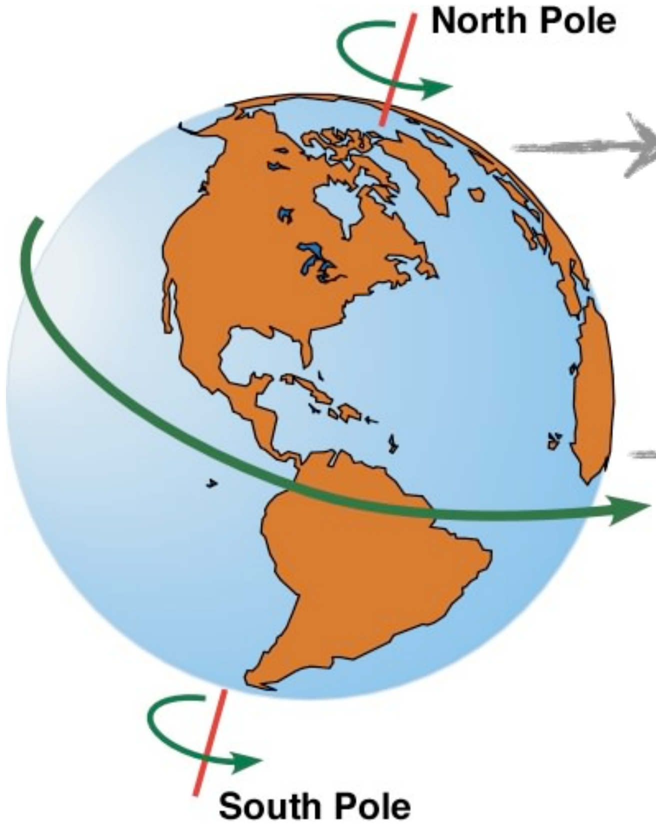
Motivation

Earth rotation varies in time.



Polar Motion Angles (PM1, PM2)

Length-of-day (LOD)

1. These changes are observed.
2. They reflect motion in the fluid shell around the Earth, primarily the atmosphere.
3. Therefore, they offer a constraint upon atmosphere models.

Atmospheric Excitation of Earth Rotation

$$
\begin{aligned}
p_1 + \frac{\dot{p_2}}{\sigma_0} &= \chi_1 = -\frac{R^3}{g(C-A)}\left[\frac{1.16}{\Omega}\int\int\int(u\sin\phi\cos\phi\cos\lambda - v\cos\phi\sin\lambda)\mathrm{d}\lambda\mathrm{d}\phi\mathrm{d}p + 1.10R\int\int p_s\sin\phi\cos^2\phi\cos\lambda\mathrm{d}\lambda\mathrm{d}\phi\right] \\
-p_2 + \frac{\dot{p_1}}{\sigma_0} &= \chi_2 = -\frac{R^3}{g(C-A)}\left[\frac{1.61}{\Omega}\int\int\int(u\sin\phi\cos\phi\sin\lambda + v\cos\phi\cos\lambda)\mathrm{d}\lambda\mathrm{d}\phi\mathrm{d}p + 1.10R\int\int p_s\sin\phi\cos^2\phi\sin\lambda\mathrm{d}\lambda\mathrm{d}\phi\right] \\
\frac{\Delta\mathrm{LOD}}{\mathrm{LOD}_0} &= \chi_3 = \frac{R^3}{C_m g}\left[\frac{0.997}{\Omega}\int\int\int u\cos^2\phi\mathrm{d}\lambda\mathrm{d}\phi\mathrm{d}p + R\int\int p_s\cos^3\phi\mathrm{d}\lambda\mathrm{d}\phi\right]
\end{aligned}
$$

Earth Rotation Variations ~ Variations in Atmospheric Angular Momentum

=

Barnes et al. (1983) Gross (2009)

Atmospheric Excitation of Earth Rotation

$$p_1 + \frac{\dot{p}_2}{\sigma_0} = \chi_1 = -\frac{R^3}{g(C-A)}\left[\frac{1\text{X}6}{\Omega}\int\int\int(u\sin\phi\cos\phi\cos\lambda - v\cos\phi\sin\lambda)\mathrm{d}\lambda\mathrm{d}\phi\mathrm{d}p + 1.10R\int\int p_s\sin\phi\cos^2\phi\cos\lambda\mathrm{d}\lambda\mathrm{d}\phi\right]$$

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Atmospheric Excitation of Earth Rotation

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Earth Rotation Variations ~ Variations in Atmospheric Angular Momentum

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Atmospheric Excitation of Earth Rotation

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Earth Rotation Variations ~ Variations in Atmospheric Angular Momentum

= Motion terms (relative AM)

Barnes et al. (1983) Gross (2009)

Atmospheric Excitation of Earth Rotation

$$p_1 + \frac{\dot{p_2}}{\sigma_0} = \chi_1 = -\frac{R^3}{g(C-A)}\left[\frac{1.\cancel{1}6}{\Omega}\int\int\int(u\sin\phi\cos\phi\cos\lambda - v\cos\phi\sin\lambda)\mathrm{d}\lambda\mathrm{d}\phi\mathrm{d}p + 1.10R\int\int p_s\sin\phi\cos^2\phi\cos\lambda\mathrm{d}\lambda\mathrm{d}\phi\right]$$

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Earth Rotation Variations ~ Variations in Atmospheric Angular Momentum

= Motion terms (relative AM)

Barnes et al. (1983) Gross (2009)

Thursday, May 3, 2012

Atmospheric Excitation of Earth Rotation

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= Motion terms (relative AM)

Barnes et al. (1983) Gross (2009)

Thursday, May 3, 2012

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Earth Rotation Variations ~ Variations in Atmospheric Angular Momentum

= Motion terms (relative AM) Mass terms (moment of inertia)

Barnes et al. (1983) Gross (2009)

Atmospheric Excitation of Earth Rotation

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Earth Rotation Variations ~ Variations in Atmospheric Angular Momentum

= Motion terms (relative AM) + Mass terms (moment of inertia)

Barnes et al. (1983)   Gross (2009)

Geographic Weighting Functions

$$p_1 + \frac{\dot{p}_2}{\sigma_0} = \chi_1 = -\frac{R^3}{g(C-A)}\left[\frac{1.16}{\Omega}\int\int\int(u\sin\phi\cos\phi\cos\lambda - v\cos\phi\sin\lambda)\mathrm{d}\lambda\mathrm{d}\phi\mathrm{d}p + 1.10R\int\int p_s\sin\phi\cos^2\phi\cos\lambda\mathrm{d}\lambda\mathrm{d}\phi\right]$$

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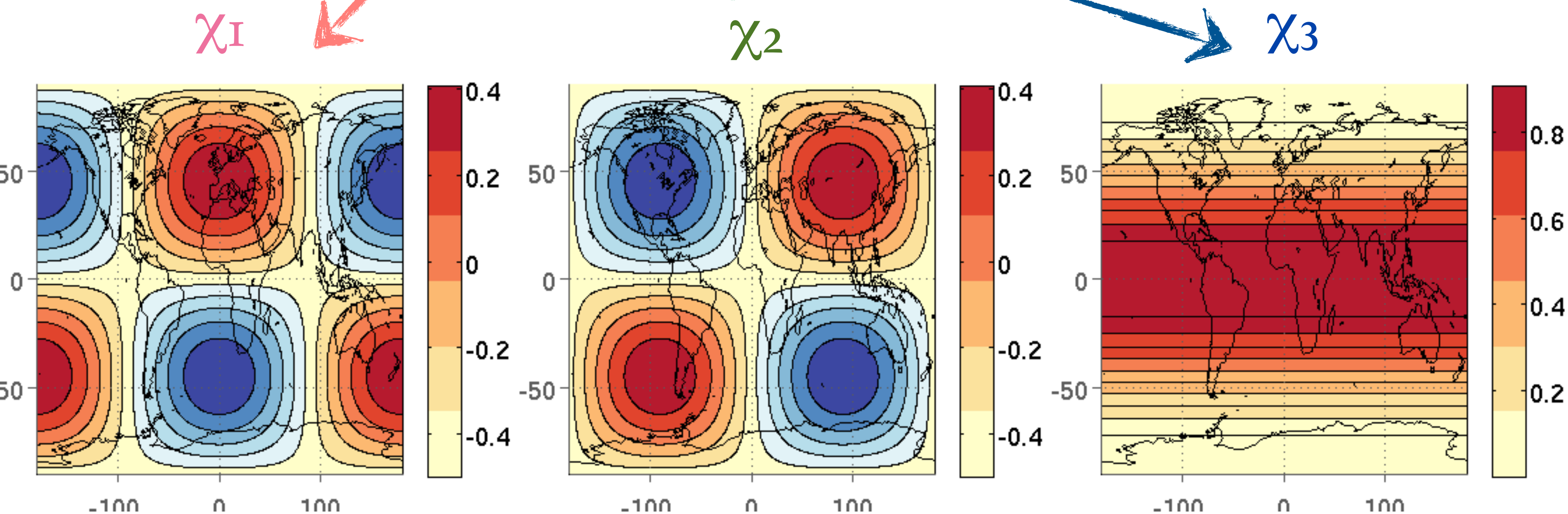
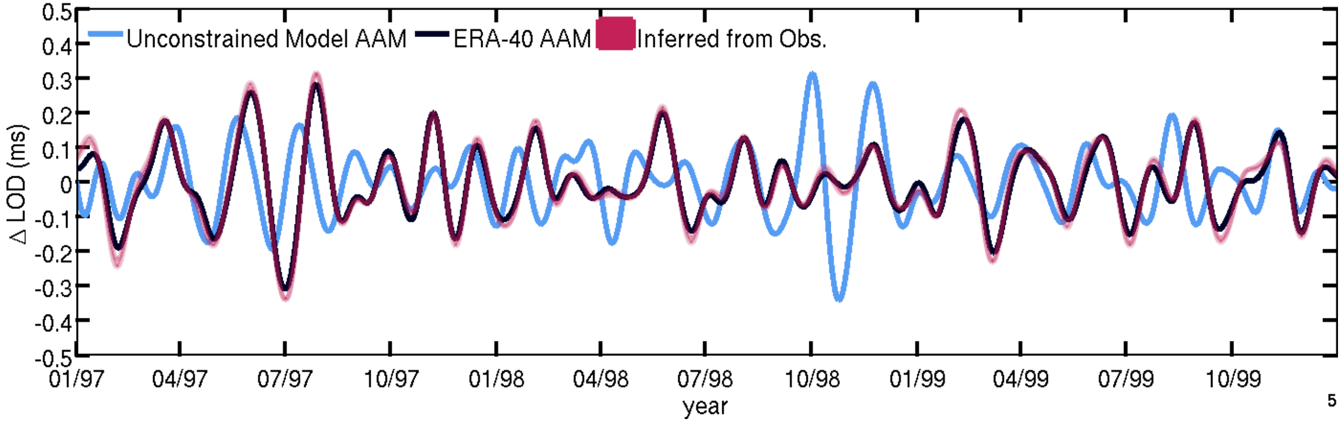


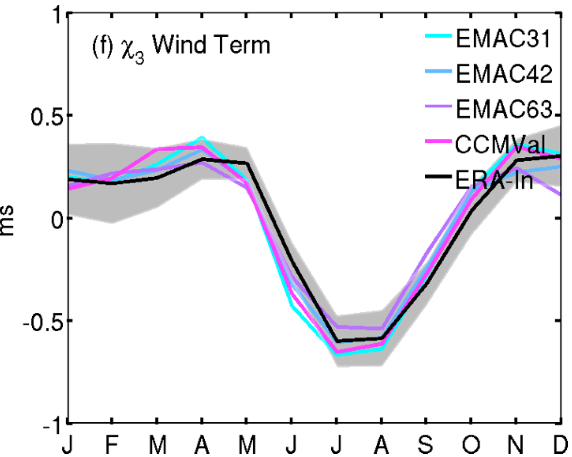
Barnes et al. (1983) Gross (2009)

Atmosphere Models Simulate Earth Rotation Excitation, but how depends on timescale.

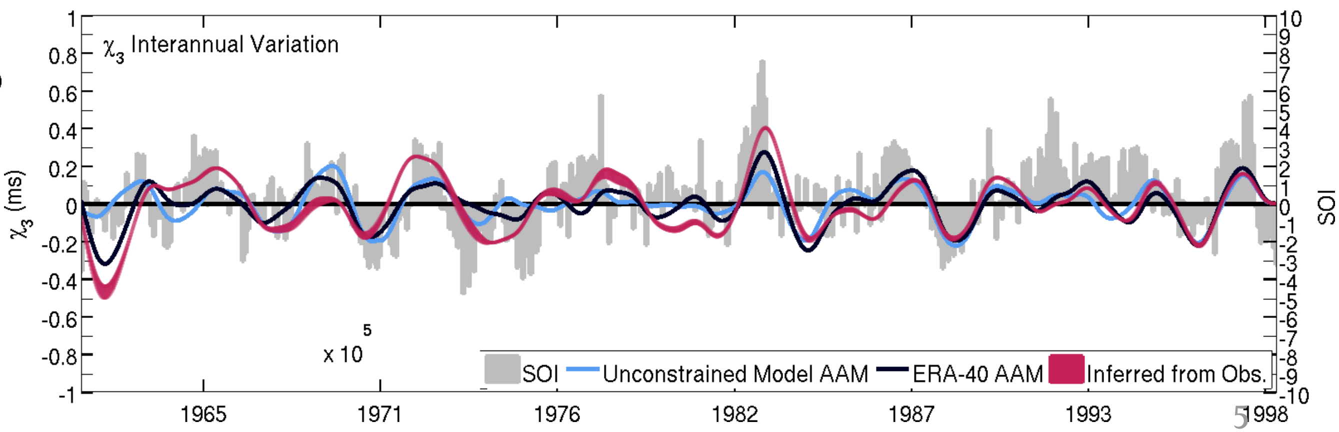
Subseasonal timescales



Annual Cycle

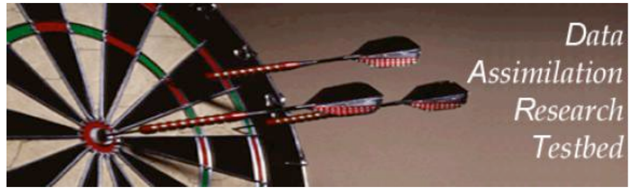


Interannual timescales



Neef & Matthes (2012)

Assimilating ERPs Into an Atmosphere Model Using DART-CAM



Anderson et al. (2009), Raeder et al. (2012)

$y = (\chi_1, \chi_2, \chi_3)$

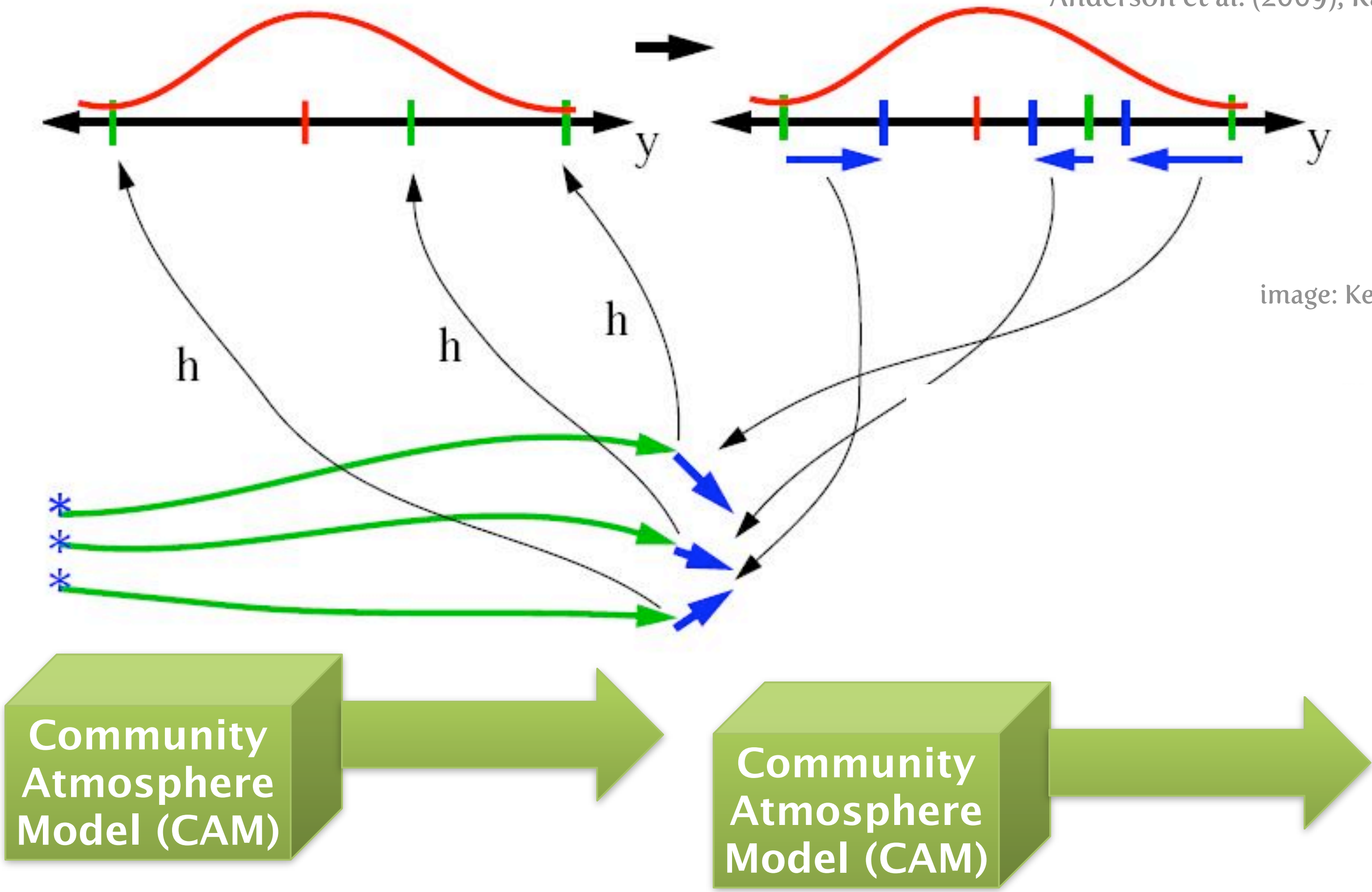
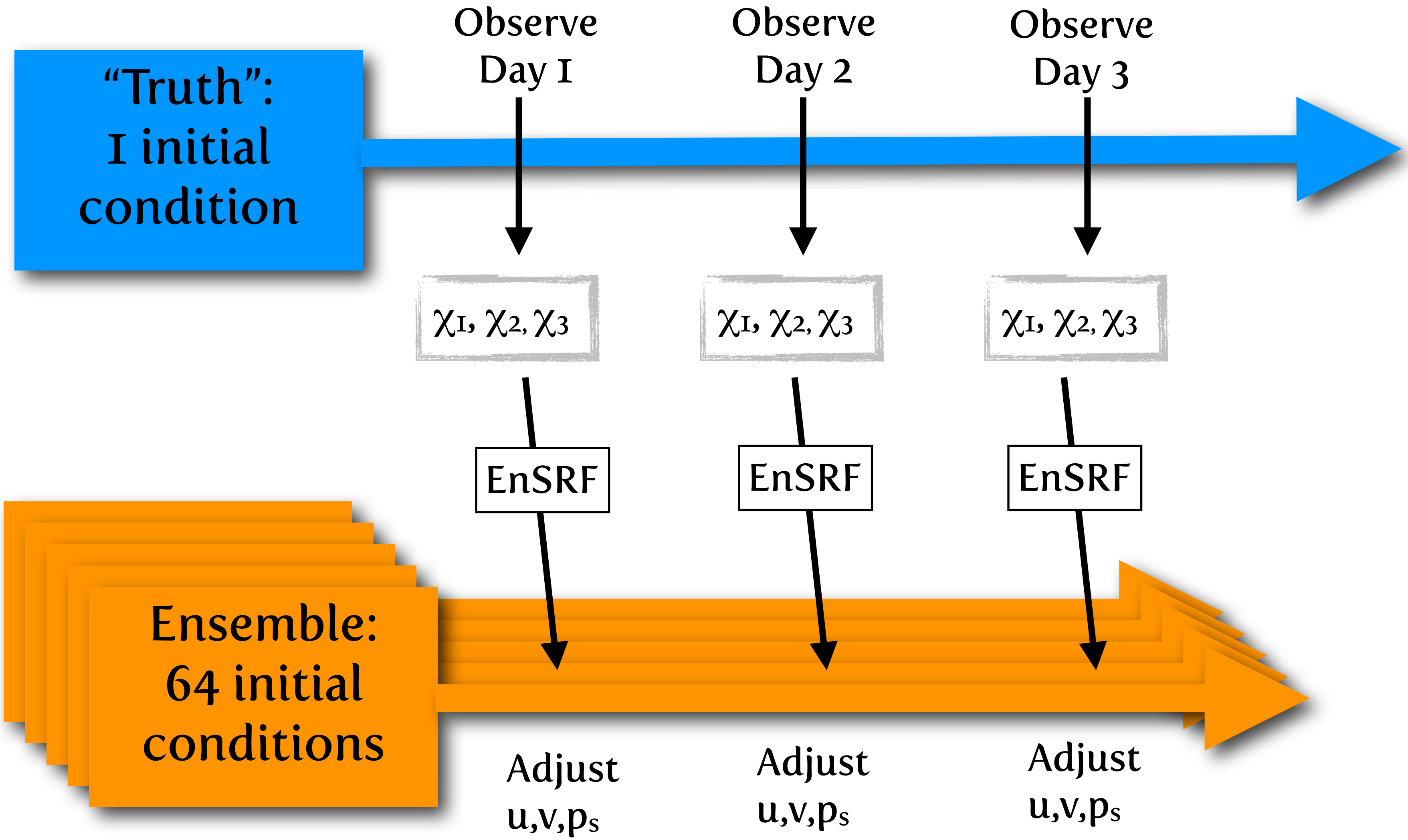


image: Kevin Raeder, NCAR

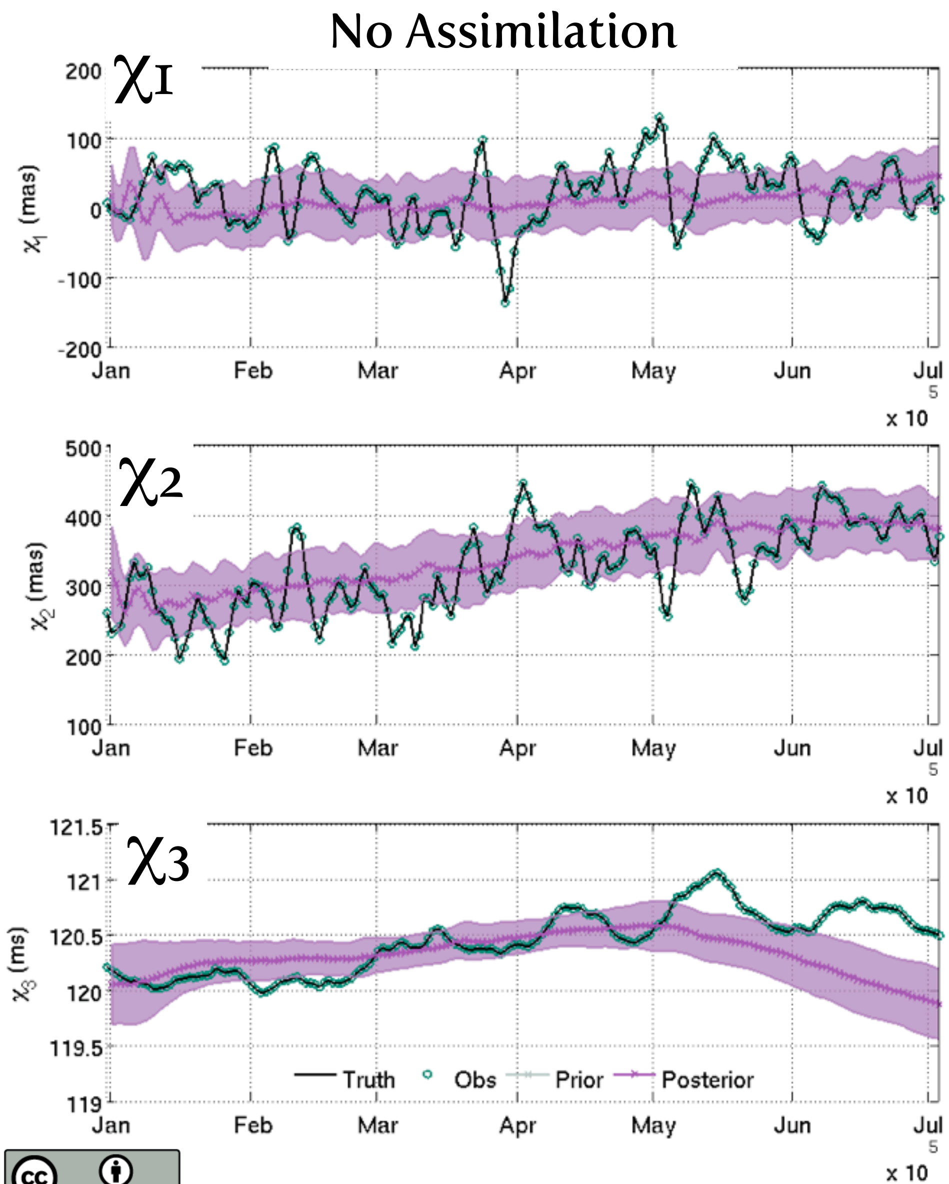
Perfect-Model Experiments



Given a perfect model and perfect observations, can we recover the truth?

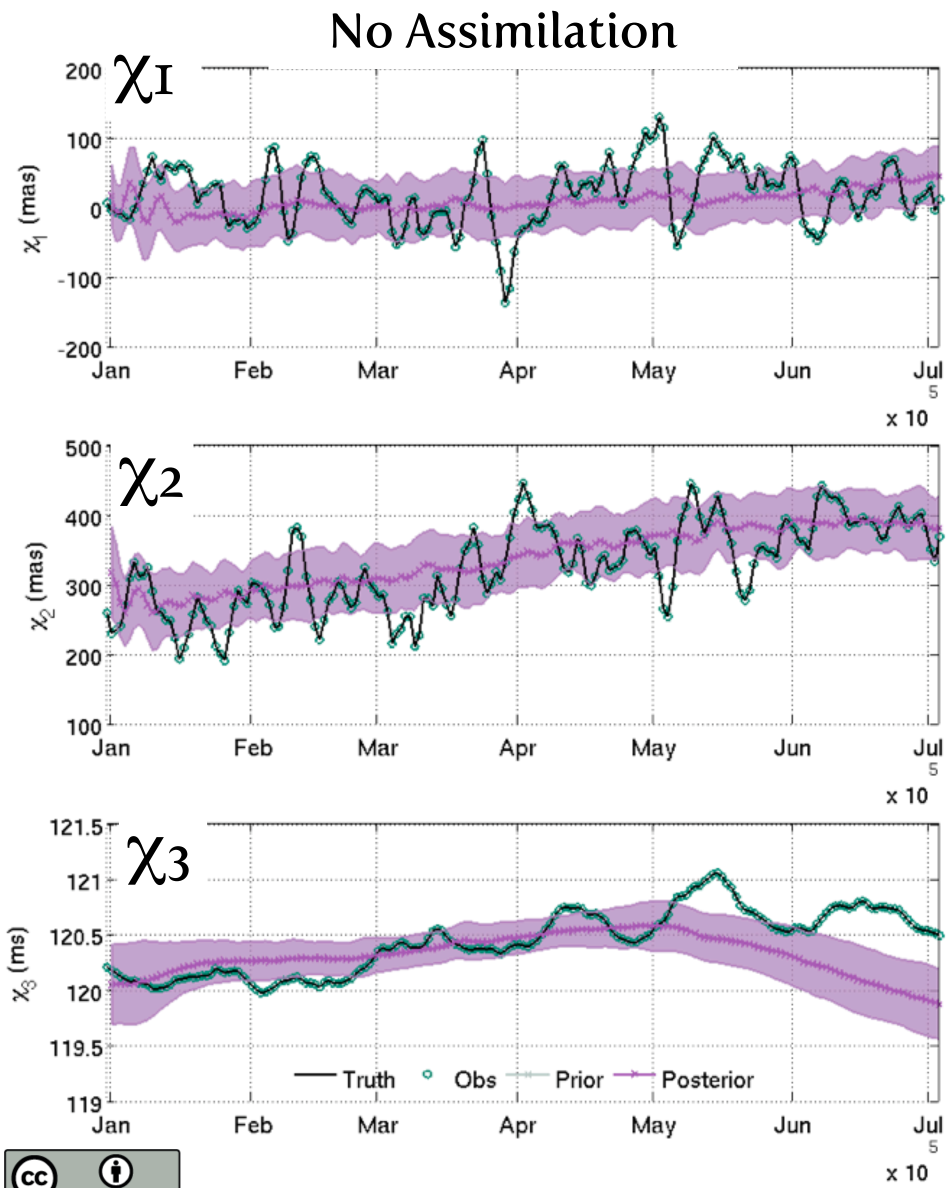
Observation Space

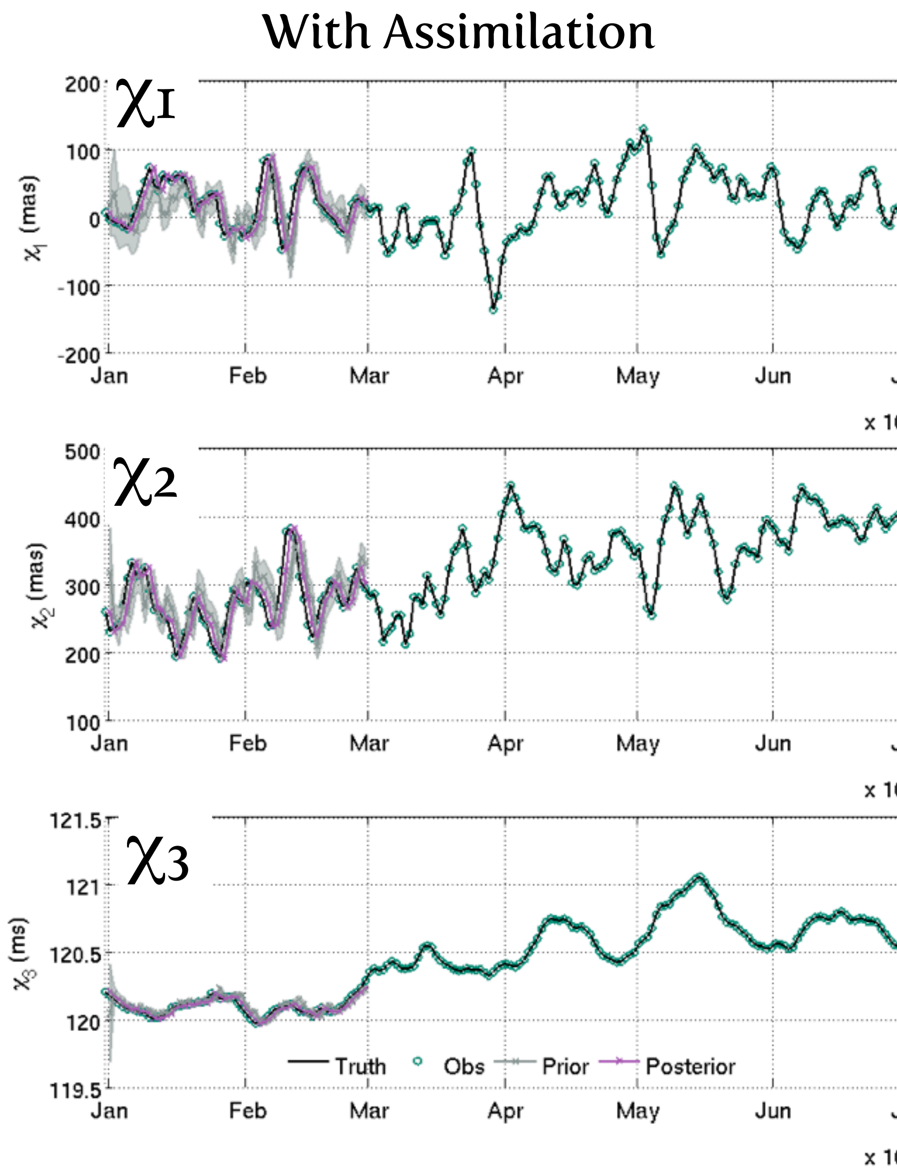
Truth Obs Prior Posterior



Observation Space

Truth Obs Prior Posterior

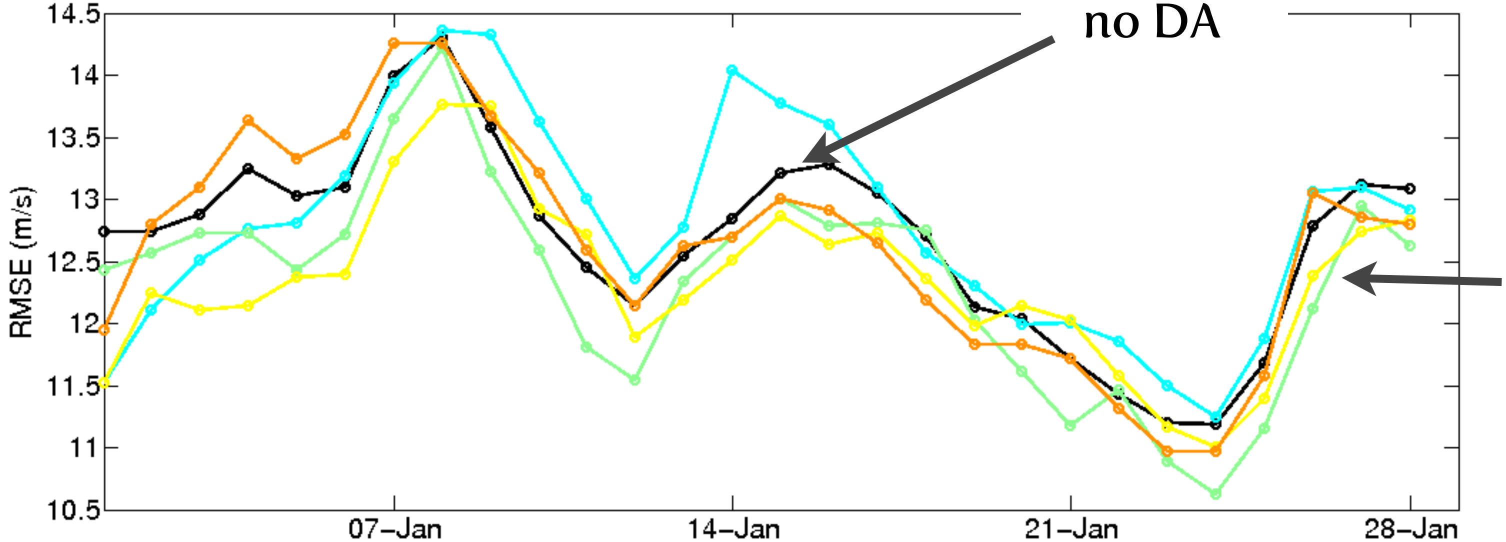




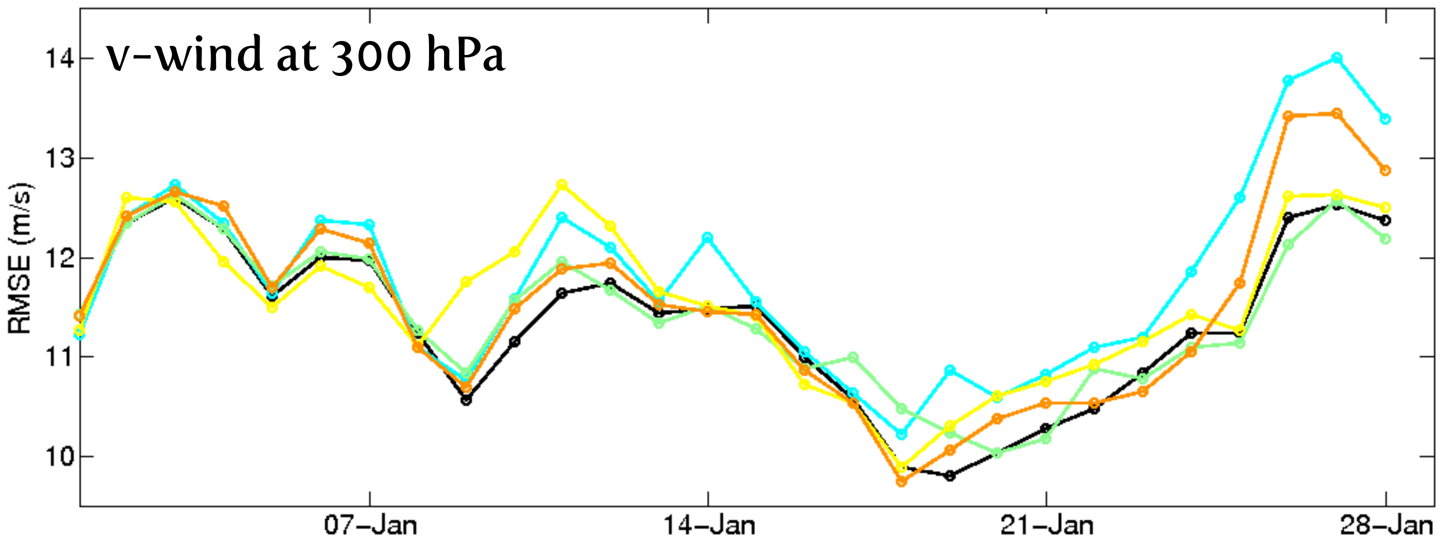
State Space Error

<u>Assimilation variables</u>
All ERPs
χ_1 only χ_2 only χ_3 only

u-wind at 300 hPa



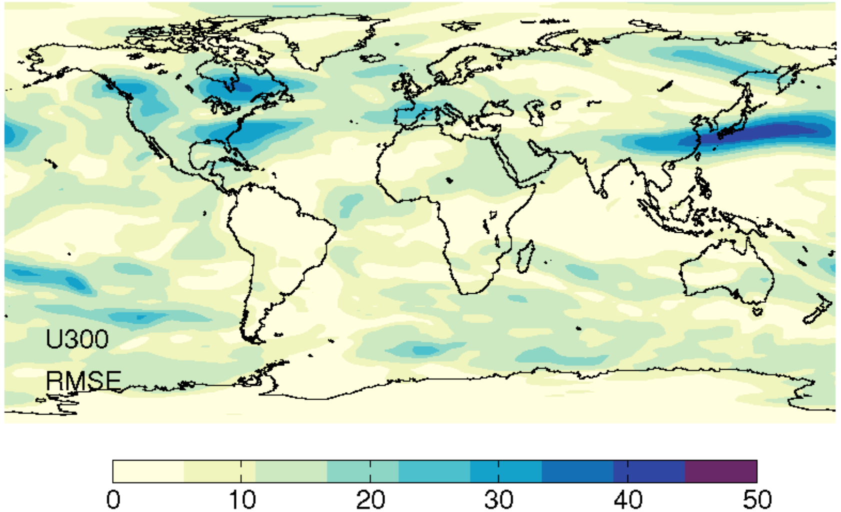
Zonal Wind:
Improvement if only polar motion components are observed.



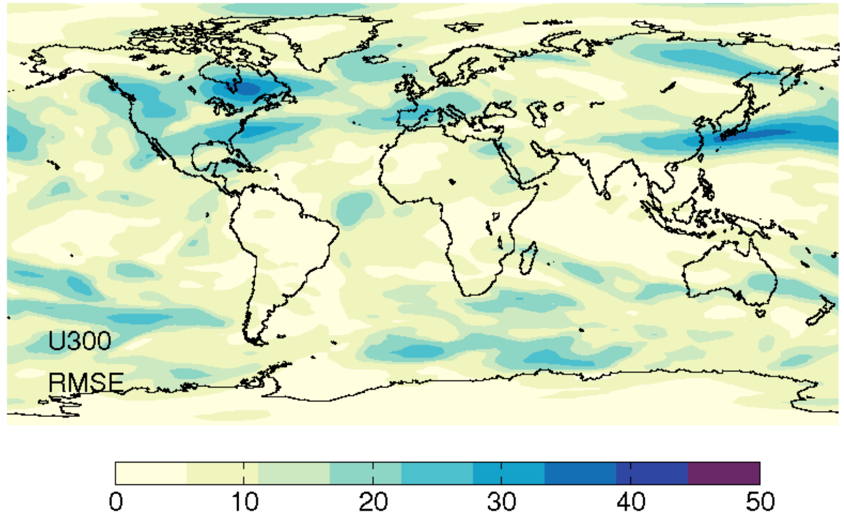
Meridional wind:
No clear improvement in any case.

Regional Error vs. Ensemble Spread

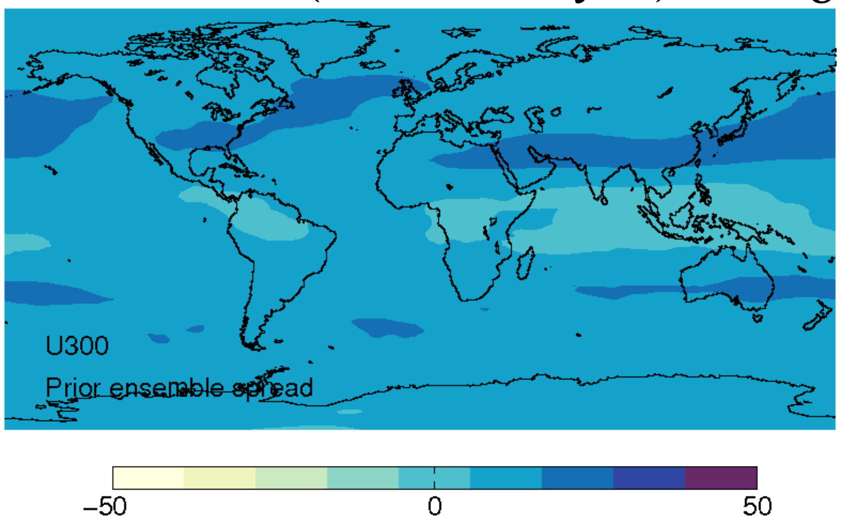
u(300 hPa) – no assimilation

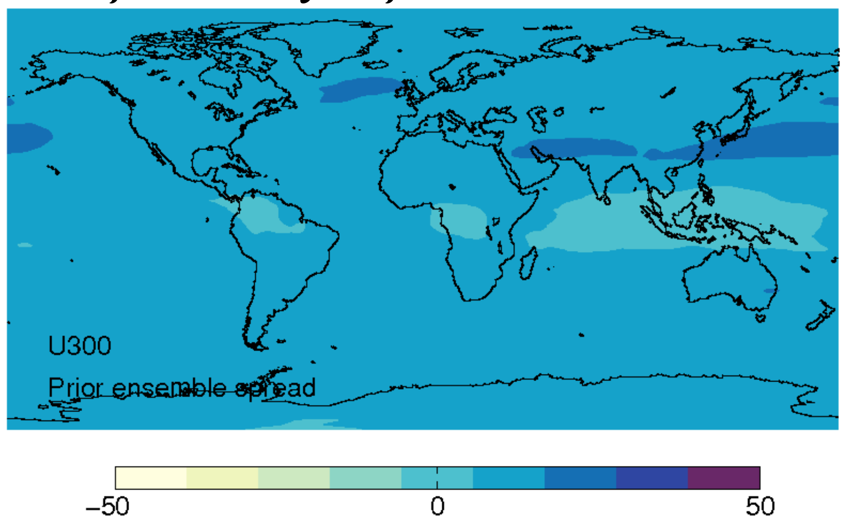


u(300 hPa) – with assimilation



RMSE (truth - analysis), averaged over first 7 days of assimilation





Ensemble spread, averaged over first 7 days of assimilation

Summary and Conclusions

- Observed Earth Rotation Parameters (ERPs) translate into atmospheric excitation functions, which are an integral constraint on the state.
- Assimilation of ERPs constrains the zonal wind field.
- Excitation functions χ_1 and χ_2 (polar motion) offer a stronger constraint than χ_3 (length-of-day) because they are weighted more heavily over midlatitudes.
- Ongoing work
  - Overall value of this constraint relative to localized observations
  - Effects of localizing the analysis increment to maxima in transfer functions

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- Anderson, J., et al. (2009), The data assimilation research testbed: A community facility, Bull. Am. Met. Soc., pp. 1283–1296, doi: 10.1175/2009BAMS2816.1.
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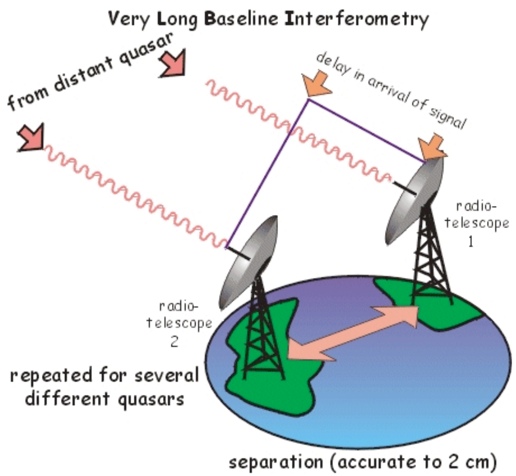
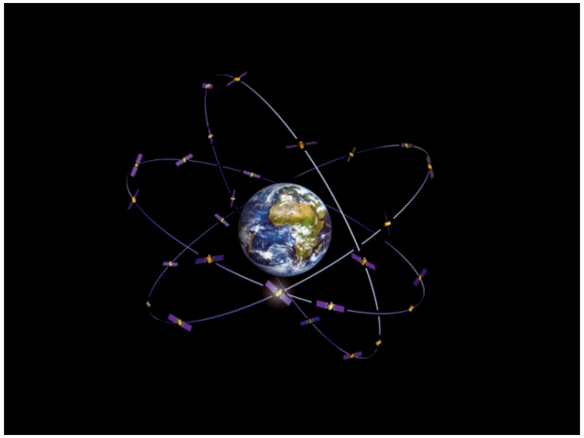
- Neef and Matthes (2012), Comparison of Earth rotation excitation in data-constrained and unconstrained atmosphere models, J. Geophys. Res., 117, D02(107), doi: 10.1029/2011JD016555.
- DART Website: http://www.image.ucar.edu/DAReS/DART/

Extras

Earth Rotation Measurements





Very Long Baseline Interferometry (VLBI)

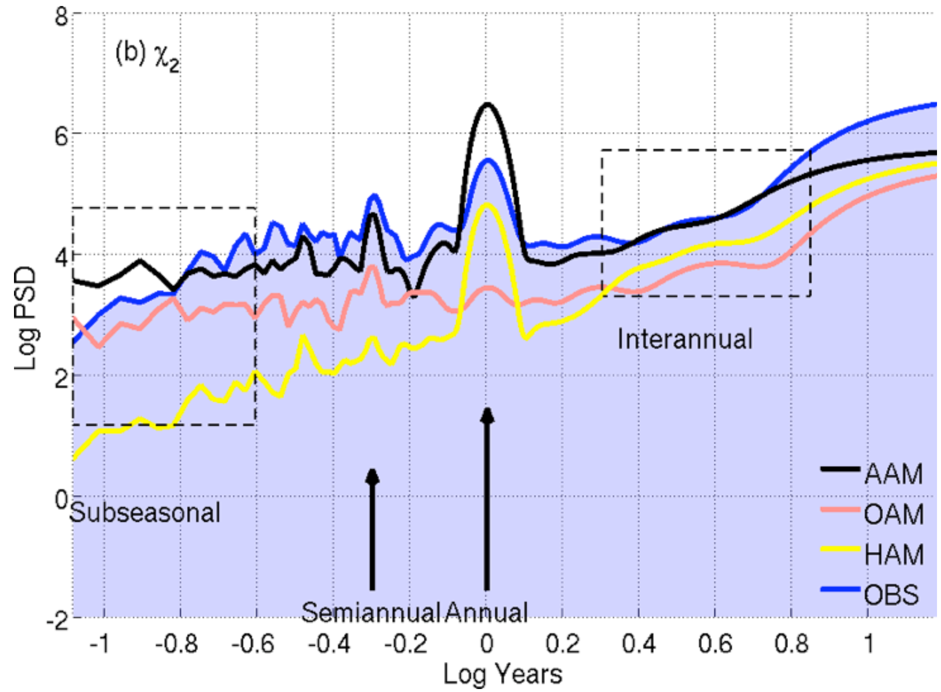
Satellite & Lunar Laser Ranging (SLR, LLR)

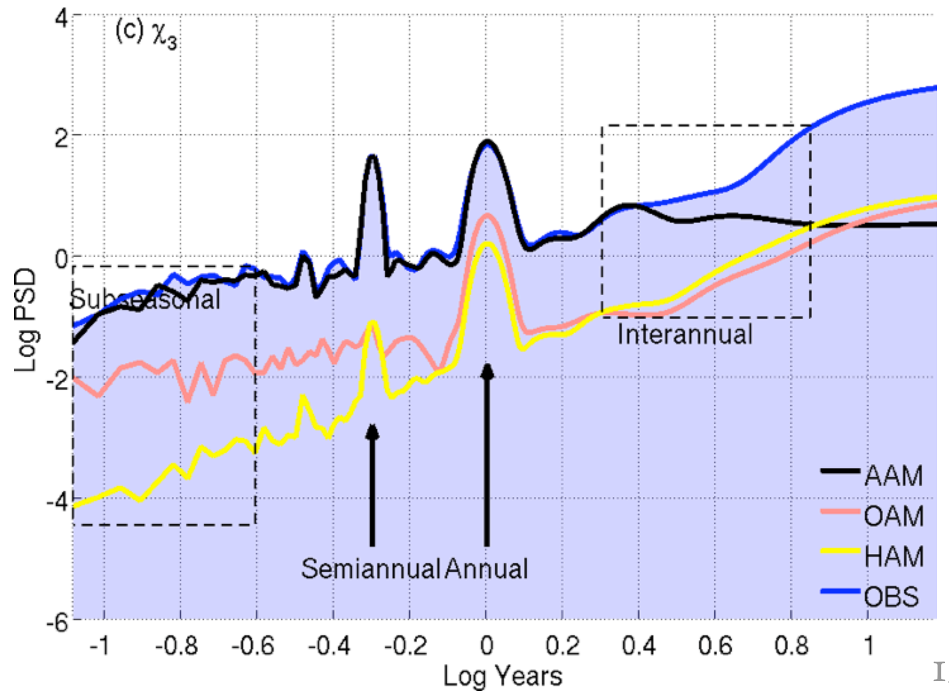
Global Positioning System (GPS)

Two angles of Polar Motion
p_1, p_2

Length-of-day anomalies
LOD

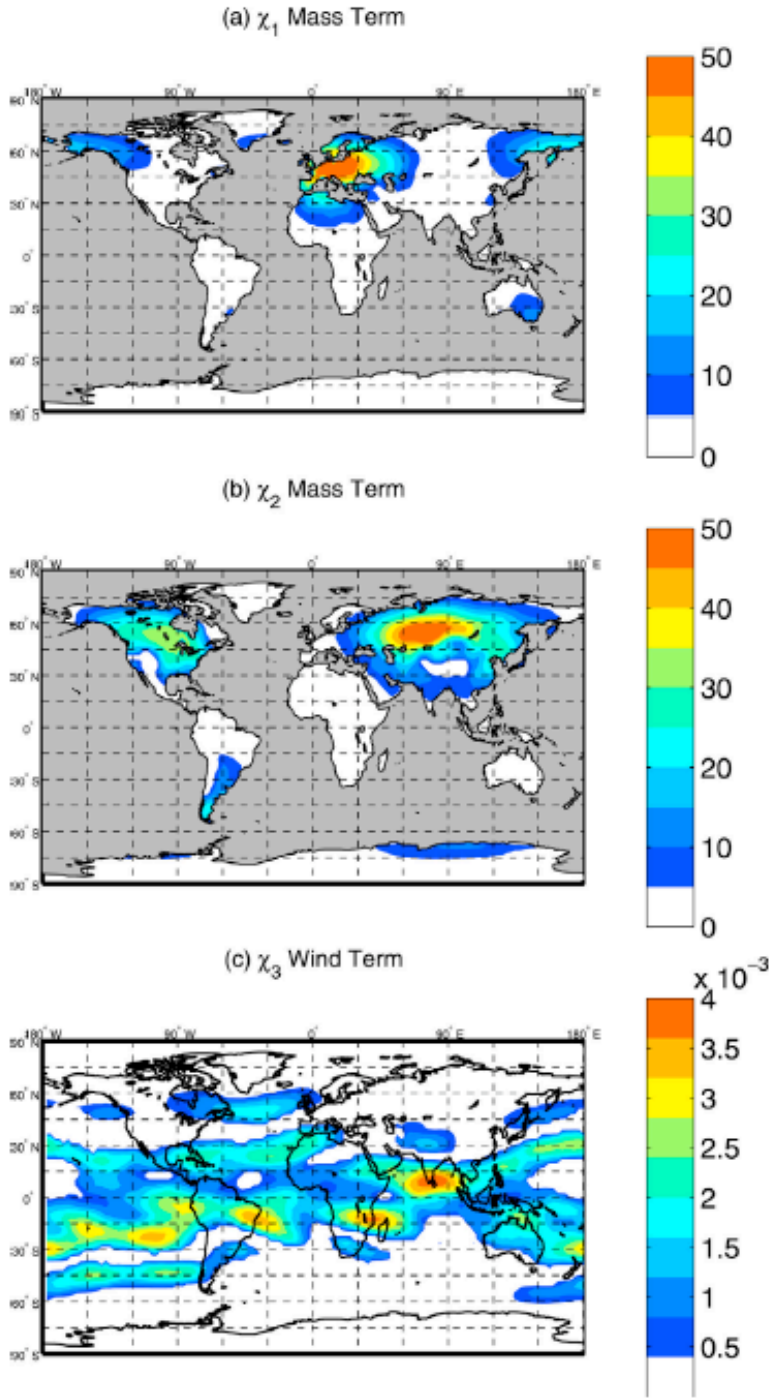
Role of the Atmosphere in Excitation of AAM



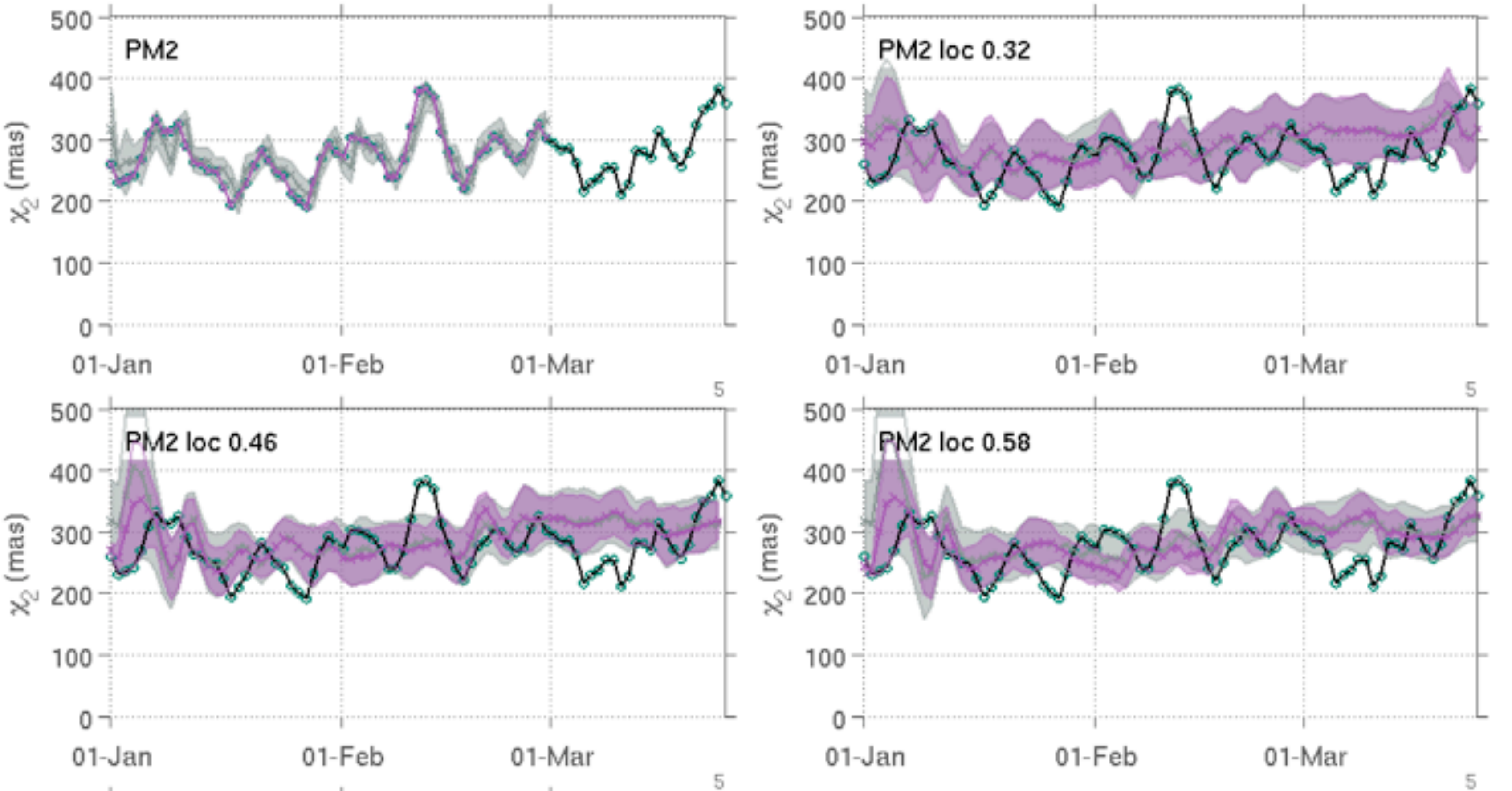


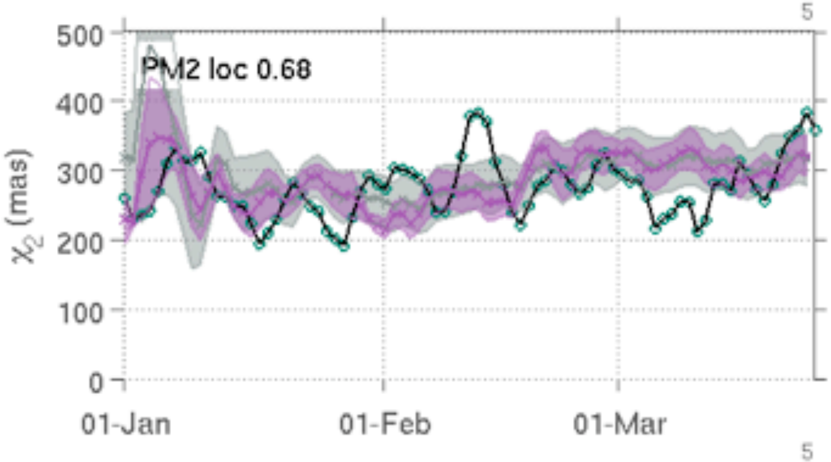
Localizing the Analysis Increment

Impact of assimilation is greatest where the initial ensemble variance is greatest …compare to spatial correlation between excitation functions and local regions (1)



Correlations between subseasonal variations in the three excitation functions and local contributions (Neef and Matthes, 2012)





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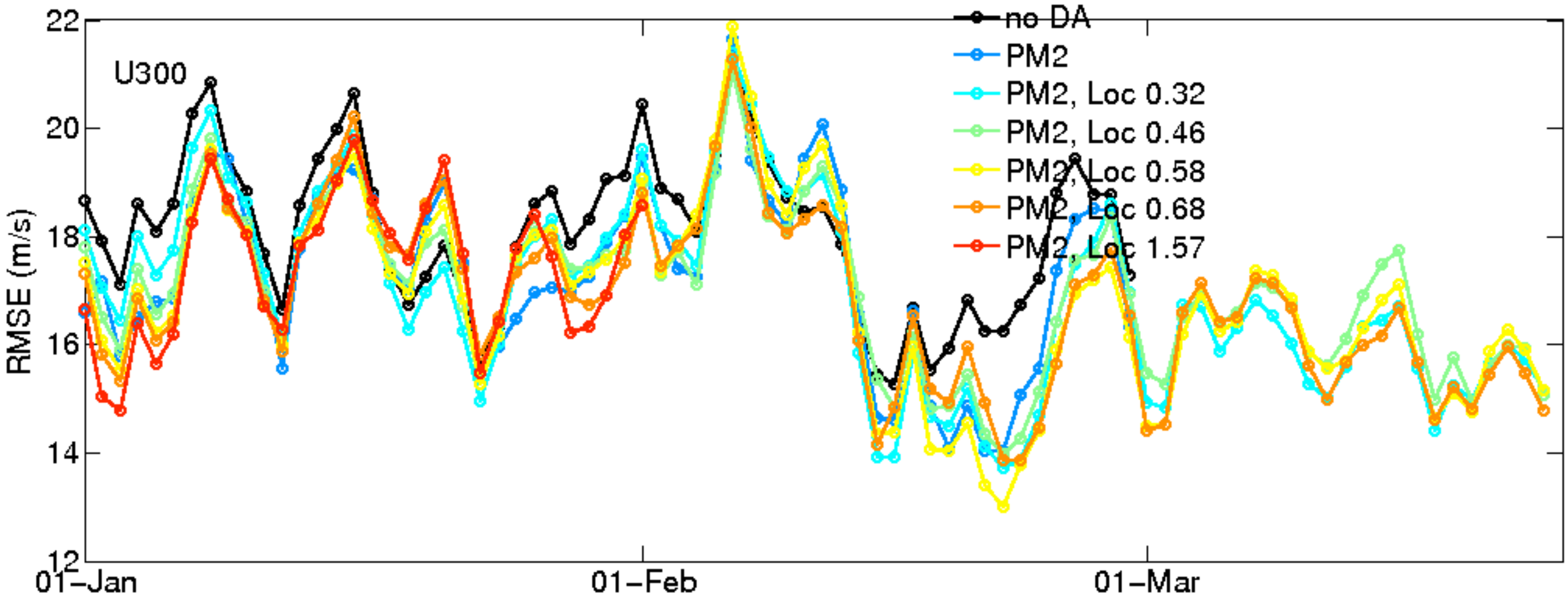

Earth Angular Momentum

$$H_i(t) = I_{ij}(t)\omega_j(t) + h_j(t)$$

moment of inertia:
Depends on distribution of mass around the Earth

relative angular momentum:
Movements relative to the the rotation vector ω

$$\frac{\mathrm{d}\vec{H}(t)}{\mathrm{d}t} = \vec{\tau} \Rightarrow \mathrm{d}(I_{ij}\omega_j + h_i)/\mathrm{d}t + \epsilon_{ijk}\omega_j(I_{kl}\omega_l + h_k) = \tau_i$$

Liouville equation

⇒ If net external torques are zero, changes in relative AM and mass distribution are evened out by changes in the rotation vector.

Changes in Earth Angular Momentum

$$\mathrm{d}(I_{ij}\omega_j + h_i)/\mathrm{dt} + \epsilon_{ijk}\omega_j(I_{kl}\omega_l + h_k) = \tau_i$$

* Now assume very small perturbations in the MOI and relative AM (in each vector component!)

$$I_{ij}(t) = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix} + \Delta I_{ij}(t)$$

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \\ 1 + m_3 \end{pmatrix} \Omega$$

⇒

$$\begin{aligned} \dot{m}_1/\sigma_c + m_2 &= \chi_2 - \dot{\chi}_1/\Omega \\ \dot{m}_2/\sigma_c - m_1 &= -\chi_1 - \dot{\chi}_2/\Omega \\ \dot{m}_3 &= -\dot{\chi}_3 \end{aligned}$$

Excitation functions χ_i:
nondimensionalized angular momentum
exchange with Earth`s fluid shell

Barnes et al. (1983)

Excitation of Earth Rotation by Angular Momentum Exchange

Two PMO components

Length-of-day changes

$$p_1 + \frac{\dot{p}_2}{\sigma_0} = \chi_1$$

$$-p_2 + \frac{\dot{p}_2}{\sigma_0} = \chi_2$$

$$\frac{\Delta \mathrm{LOD}}{\mathrm{LOD}_0} = \Delta\chi_3$$

Equatorial excitation functions

Axial excitation

Barnes et al. (1983)
Gross (2009)

Atmospheric Angular Momentum

$$
\begin{aligned}
p_1 + \frac{\dot{p}_2}{\sigma_0} &= \chi_1 \\
-p_2 + \frac{\dot{p}_2}{\sigma_0} &= \chi_2 \\
\frac{\Delta \mathrm{LOD}}{\mathrm{LOD}_0} &= \Delta\chi_3
\end{aligned}
$$

$$
\begin{aligned}
\chi_1(t) &= \frac{1.608}{\Omega\,(C - A')}\left[0.684\Omega\Delta I_{13}(t) + \Delta\mathbf{h}(t)\right] \\
\chi_2(t) &= \frac{1.608}{\Omega\,(C - A')}\left[0.684\Omega\Delta\mathbf{I}_{23}(t) + \Delta\mathbf{h}(t)\right] \\
\chi_3(t) &= \frac{0.997}{\Omega C_m}\left[0.750\Omega\Delta I_{33}(t) + \Delta h_{33}(t)\right]
\end{aligned}
$$

Mass terms (moment of inertia)

$$
\begin{aligned}
I_{13} &= -\int R^2 \cos\phi \sin\phi \cos\lambda dM \\
I_{23} &= -\int R^2 \cos\phi \sin\phi \sin\lambda dM \\
I_{33} &= \int R^2 \cos^2\phi dM
\end{aligned}
$$

Motion terms (AM relative to Earth)

$$
\begin{aligned}
h_1 &= -\int R\left[u \sin\phi \cos\lambda - v \sin\lambda\right] dM \\
h_2 &= -\int R\left[u \sin\phi \sin\lambda + v \cos\lambda\right] dM \\
h_3 &= \int R u \cos\phi dM
\end{aligned}
$$