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Maximum Likelihood Estimates of trend- and memory-coefficients in climatic time series

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Maximum Likelihood Estimation (MLE)

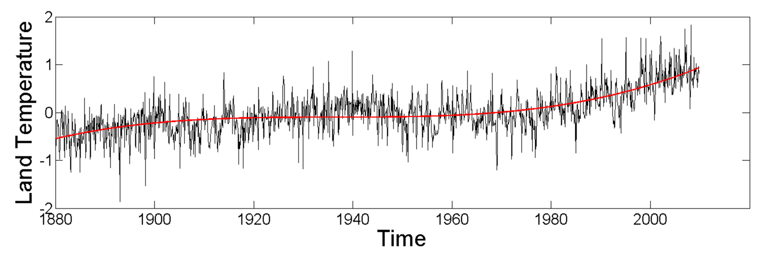
To use the Maximum Likelihood Estimation method to find the Hurst exponent, H, we need a model. In our study we have used

$$x_t = \sum_{k=0}^{m} a_k \left(\frac{t}{n}\right)^k + \sigma\epsilon_t^{(H)},$$

where

- n: number of observations
- m: polynomial order
- $\epsilon_t^{(H)}$: fractional Gaussian noise with Hurst exponent H and st.d. σ

Land Temperature



$a.0$	$a.1$	$a.2$	$a.3$	H	σ
-0.55	3.12	-7.19	5.56	0.75	0.33

Fig. 1: MLE on land temperature with $m = 3$. The red line is the polynomial fit with coefficients from the table.

Ocean Temperature

Ocean temperature: $H > 1 \rightarrow$ motion.
The *increments*: $H_{\text{inc}} = H_{\text{ocean}} - 1 < 1 \rightarrow$ noise.

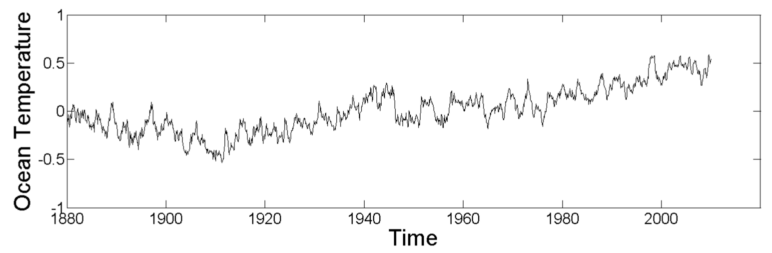
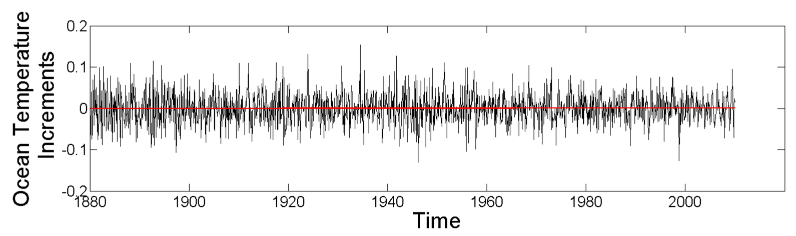


Fig. 2: Ocean temperature.



$a.0$	$a.1$	H	σ
-5.48×10^{-4}	1.84×10^{-3}	0.50	0.037

Fig. 3: MLE on ocean temperature increments with $m = 1$. The red line is the polynomial fit with coefficients from the table.

Ensemble Study

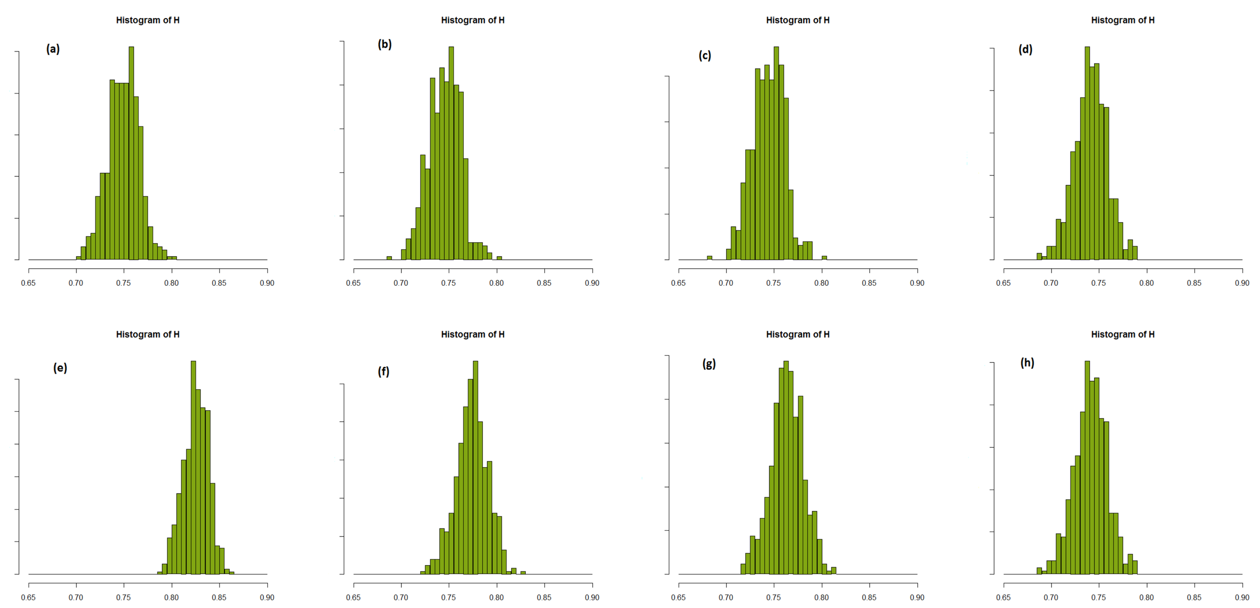


Fig. 4: Hurst exponents from ensemble study with properties from land temperature. (a)-(d) are fGn, (e)-(h) are fGn+polynomial, studied with, from left to right, $m = \{0, 1, 2, 3\}$

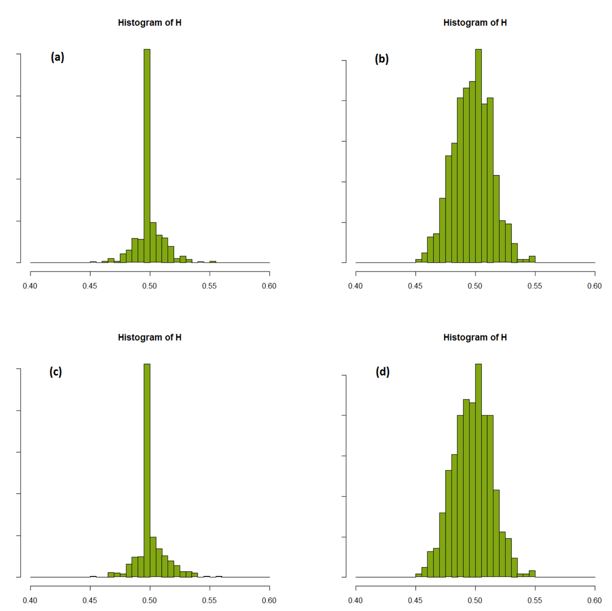


Fig. 5: Hurst exponents from ensemble study with properties from ocean temperature increments. (a) and (b) are fGn, (c) and (d) are fGn+polynomial, studied with, from left to right, $m = \{0, 1\}$

Ensemble properties

- H and σ from MLE on the temperature record
- 500 realizations
- Each realization has the same length as the temperature record
- fGn (pure noise)
- fGn + polynomial with coefficients from MLE on the temperature record

Temperature Data

The temperature data used in this study are downloaded from the the NCDC/NOAA public data webpage,

ftp://ftp.ncdc.noaa.gov/pub/data/.

- Global land temperature/global ocean temperature
- Anomalies from 1901-2000 mean
- Monthly data

Detrended Fluctuation Analysis (DFA)

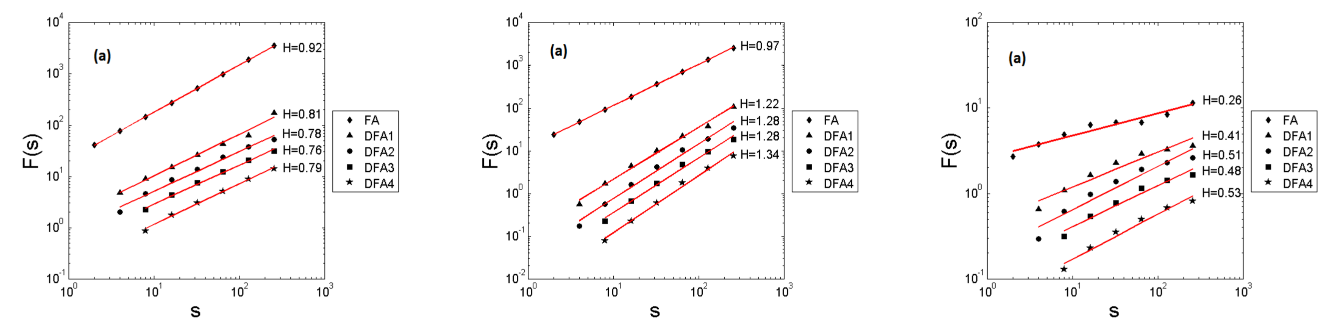


Fig.6: DFA on (a) land temperature, (b) ocean temperature and (c) ocean temperature increments

Summary

- MLE yields smaller Hurst exponents and is more accurate than DFA with a sufficient polynomial order.
- Good model fit $\rightarrow$ MLE performs great!
- This model does not take possible cross-overs into consideration
- The data may not fit well with the model (physical relationship)

References

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