On ocean-atmosphere time scales

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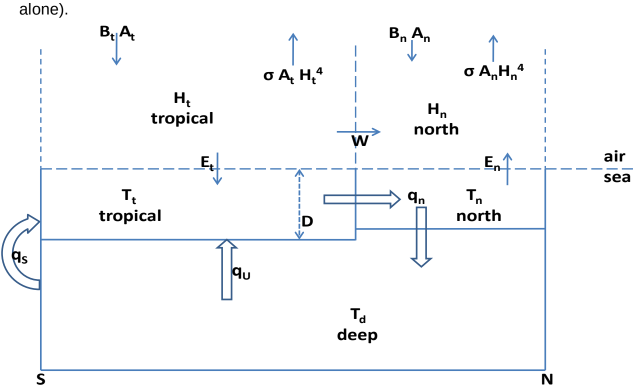
Abstract

We consider an elementary model coupling a three-“box” ocean and two-“box” atmosphere. The present form of largest-scale overturning ocean circulation is built in. Coupling adds two degrees of freedom, hence two new time-scales compared with ocean or atmosphere alone. With temperatures and transports as the only variables, the model has a unique, stable, steady state: all perturbations on the steady state decay. The new time-scales are some decades, corresponding to adjustment of ocean temperature above the main thermocline. [Atmosphere-only relaxation times are days to months; ocean-alone overturning relaxation times are centuries]. No oscillatory modes are found, probably because the model has little spatial description or anomaly propagation. Adding salinities as oceanic variables, and exchanges of water with the atmosphere, allows instability in a limited range of relative “box” sizes. Growing oscillations with time-scales of centuries are possible if the density effect of fresh-water flux via the atmosphere is significant and tropical “upwelling” is small relative to the total transport from the tropical to the northern ocean. Overall this evidence suggests that (ocean) models should allow spatial propagation if internal (climate) variability is to be represented.

1. Introduction

Climate is a function of the coupled ocean and atmosphere. Ability to predict future climate depends on good understanding and modelling of this coupled system. Climate projections for future decades are complicated by inter-decadal “internal” variability as large as several decades of average climate trend. What are the possible origins of this variability? The question is considered here by coupling the ocean model of Johnson et al. (2007), reduced for the present form of meridional overturning and uniform salinity, with a two-“box” atmosphere. Because salinity anomalies have been suggested as contributors to oscillatory oceanic behaviour, salinity is also considered as another variable in each box of the ocean (treated alone).



2. Formulation

Ocean

In the figure, the q are transports with the present form of meridional overturning, T are temperatures, D is tropical thermocline depth, box volumes are V_n, V_d, A_tD, surface areas are A_t, A_n. As in Johnson et al. (2007)

$q_U = KA_t/D$, $q_n = q_n'(T_t - T_d)$ where $q_n' = g\alpha D^2/(2f_n)$.

Here K is a vertical diffusivity, empirically $O(10^{-4} m^2/s)$ (Munk, 1966), g is acceleration due to gravity, α is thermal expansion coefficient, f_n is northern Coriolis parameter.

Atmosphere

Shorter-wave solar radiation is B ($B_t > B_n$), atmospheric temperatures H_t, H_n, radiation to space is σH^4 ($\sigma = 5.67 \times 10^{-8} Wm^{-2}\,°K^{-4}$ is Stefan’s constant). Heat fluxes warming the tropical ocean and northern ocean are

$$E_t = M_t C_p (H_t - T_t) / t_e, \qquad E_n = M_n C_p (H_n - T_n) / t_e.$$

Here M_t, M_n are masses of tropical and northern atmosphere, C_p is specific heat of air, $t_e = O(3.10^5 s)$ is a relaxation time to equilibrate atmospheric to oceanic temperature.

Northward atmospheric heat flux $W = (M_t + M_n) C_p (H_t - H_n) / t_w$

with time-scale $t_w = O(10^7 s)$ (months) to equilibrate tropical & northern atmospheres.

Equations

Ocean volumes: $A_t \partial_t D = q_S + q_U - q_n = q_S + KA_t/D - g\alpha D^2(T_t - T_d) / (2f_n)$ (O1)

Deep-ocean heat: $V_d \partial_t T_d - T_d A_t \partial_t D = \partial_t(T_d V_d) = q_n T_n - (q_S + q_U) T_d$. (O2)

Atmosphere heat (two boxes)

$$M_t C_p \partial_t H_t = B_t A_t - MC_p(H_t - H_n)/t_w - M_t C_p(H_t - T_t)/t_e - \sigma A_t H_t^4 \quad \text{(A1)}$$

$$M_n C_p \partial_t H_n = B_n A_n + MC_p(H_t - H_n)/t_w + M_n C_p(T_n - H_n)/t_e - \sigma A_n H_n^4 \quad \text{(A2).}$$

T_t and T_n are fixed if the ocean and atmosphere are treated separately, but variables in the coupled system. Upper-ocean heat (two boxes):

$$\rho_w C_w (A_t D \partial_t T_t + A_t T_t \partial_t D) = \rho_w C_w [T_d(q_S + q_U) - T_t q_n] + M_t C_p (H_t - T_t)/t_e \quad \text{(E1)}$$

$$\rho_w C_w V_n \partial_t T_n = \rho_w C_w (T_t - T_n) q_n - M_n C_p (T_n - H_n)/t_e. \quad \text{(E2)}$$

Here ρ_w, C_w are the density and specific heat of sea water.

These are six evolution equations for the six variables D, T_d, H_t, H_n, T_t, T_n.

There is a unique steady solution ($\partial_t \equiv 0$).

2'. Departures from steady state

Small departures D_1, T_{d1}, T_{t1}, T_{n1}, H_{t1}, H_{n1}: subtract the steady state from the six equations. This gives six linear equations for the six perturbation quantities. The other quantities are constant with their steady-state values. Hence we expect solutions as $\exp(\lambda t)$. Substituting λ for ∂_t in gives six homogeneous equations with non-zero solution only if the determinant of coefficients (for D_1, T_{d1}, T_{t1}, T_{n1}, H_{t1}, H_{n1}) is zero. This is a 6th order polynomial equation $P(\lambda)$ which we cannot expect to solve explicitly. However, we use the wide range of time-scales (days to centuries) to seek $P(\lambda) = 0$ in ranges of λ.

The results are **six negative roots for λ**.

For λ of order $10^{-10} s^{-1}$: $\lambda = -q_n/V_d$ and $\lambda = -(q_U + 2q_n)/V_t$

These are the two solutions in the ocean-alone model, representing: (i) T_d relaxes to its equilibrium value (T_n) with rate q_n/V_d or time scale V_d/q_n (several centuries); (ii) D to its equilibrium value with rate $(q_U + 2q_n)/V_t$ or time scale $A_tD/(q_U + 2q_n)$ of order one century. Atmospheric temperatures are well-adjusted to ocean temperatures and radiation and hardly affect the heat balance.

For λ of order $10^{-9} s^{-1}$: $\lambda \approx -4\sigma H_t^3 A_t/(\rho_w C_w V_t)$ and $\lambda \approx -4\sigma H_n^3 A_n/(\rho_w C_w V_n)$

to lowest order (in particular for small exchange between the tropical and northern atmosphere, i.e. small t_w^{-1}). These solutions are the two “new” ones; the values λ are real and negative for the parameter ranges considered. They represent adjustment of the upper tropical and northern ocean temperature to solar heating; they depend mainly on the temperature-sensitivity of heat radiation rate, relative to the thermal capacity (depth V/A) of the ocean. The heat is primarily in the ocean; atmospheric temperatures H_n, H_t are well-adjusted.

For λ of order $10^{-6} s^{-1}$: $\lambda = -t_e^{-1} - (\omega_n + \omega_t)/2 \pm \{(\omega_n - \omega_t)^2/4 + r\,\Omega_w^2/t_e^2\}^{1/2}$

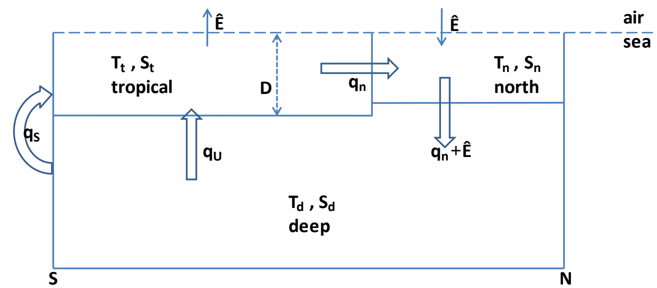
for the atmosphere alone. The solutions represent adjustment of the atmospheric temperature to the underlying upper-ocean temperature. [The ω are adjustment rates for the respective atmospheric box to solar heating; modifications by W appear in the form of ω and the Ω_w^2 term]. For coupled atmosphere and ocean, the λ are formally the same, but the ω are modified by oceanic adjustment (only slight as oceanic thermal capacity is large).

3. Ocean alone with salinity.

All the modes in section 2 show decay without oscillation. Can salinity affect this? Consider the three-box ocean (only) as illustrated. As in Johnson et al. (2007)

$q_U = KA_t/D$, $q_n = q_n'[\alpha(T_t - T_d) - \beta(S_t - S_d)]$ where $q_n' = gD^2/(2f_n)$.

Here the S are salinities and $\hat{E}$ is a fresh-water transfer from the upper tropical to northern ocean via the atmosphere. Equations are:



Ocean volumes:

$$A_t \partial_t D = q_S + q_U - q_n - \hat{E} = q_S + KA_t/D - \hat{E} - gD^2[\alpha(T_t - T_d) - \beta(S_t - S_d)] / (2f_n) \quad \text{(OS1).}$$

Deep-ocean heat [T_d(OS1) added]: $V_d \partial_t T_d = (q_n + \hat{E})(T_n - T_d)$ (OS2).

Tropical salt [S_t(OS1) subtracted]: $A_t D \partial_t S_t = (q_S + q_U)(S_d - S_t) + \hat{E} S_t$ (OS3).

Northern salt: $V_n \partial_t S_n = q_n (S_t - S_n) - \hat{E} S_n$ (OS4).

Deep-ocean salt [S_d(OS1) added]: $V_d \partial_t S_d = (q_n + \hat{E})(S_n - S_d)$ (OS5).

Steady state ($\partial_t \equiv 0$)

$T_d = T_n$ from (OS2), $S_d = S_n$ from (OS5), $S_t = S_n(1 + \hat{E}/q_n)$ from (OS4), $q_n + \hat{E} = q_S + q_U$ from (OS3). S_n is specified by the total salt in the ocean. Then

$$q_n = gD^2[\alpha(T_t - T_n) - \beta S_n \hat{E}/q_n]/(2f_n) \quad \text{(quadratic for } q_n\text{)} \quad \text{with roots}$$

$$q_n \approx gD^2\alpha(T_t - T_n)/(2f_n) - \beta S_n \hat{E}/(\alpha(T_t - T_n)) \quad \text{and} \quad q_n \approx \beta S_n \hat{E}/[\alpha(T_t - T_n)]$$

for realistic parameter values. The larger root (first formula) applies if density driving of present overturning is mainly driven by temperature gradient.

3'. Departures from steady state

Small departures D_1, T_{d1}, S_{t1}, S_{n1}, S_{d1}: subtract the steady state from (OS1-5). This gives five linear equations for the five perturbation quantities. The other quantities are constant with their steady-state values. Hence we expect solutions as $\exp(\lambda t)$. Substituting λ for ∂_t gives five homogeneous equations with non-zero solution only if the determinant of coefficients (for D_1, T_{d1}, S_{t1}, S_{n1}, S_{d1}) is zero.

(OS2) stands alone and gives $\boldsymbol{\lambda = -(q_n + \hat{E})/V_d}$.

Adding all, $\boldsymbol{\lambda = 0}$ is a solution budgeting total salt.

Write $a \equiv A_t D/V_n$, $\text{đ} \equiv V_d/V_n$, $\Lambda \equiv \lambda V_n/q_n$, $U \equiv q_U/q_n$, $\hat{e} \equiv \hat{E}/q_n$, $B \equiv q_n'\beta S_n \hat{E}/q_n^2$

If $\hat{E} = 0$ (no evaporative transfer from the tropics to the north), then

$\boldsymbol{\Lambda = -(U+2)/a}$ **or** $\boldsymbol{\Lambda = -[a^{-1} + \text{đ}^{-1} + 1]/2 \pm \{[a^{-1} - (\text{đ}^{-1} + 1)]^2 - 4\text{đ}^{-1}\}^{1/2}/2}$

i.e. unconditional decay although the pair of roots may be oscillatory.

If $\hat{E} \neq 0$ but (realistically) $B < 1$, all modes decay.

For larger *B*, there can be growing oscillatory modes, if U is small enough:

$(0 <)\ U < \tfrac{1}{2}\{[9 + 8a/(\text{đ}+1)]^{1/2} - 3\}$ if $\hat{e} \ll 1$ is a necessary condition.

E.g. $U = 0$, small $\hat{e}$, $a = 1$, $\text{đ} = 2$, $B = 110/9$: $0 = (\Lambda + 2^{3/2}i/3)(\Lambda - 2^{3/2}i/3)(\Lambda + 9/2)$.

The oscillatory modes grow if B is a little larger than 110/9.

Oscillations may grow if fresh-water flux $\hat{E}$ is large and upwelling q_U is small.

4. Conclusions

Combining ocean & atmosphere inevitably introduces new time-scales (2 more here).

All modes decay in this coupled model of atmosphere and ocean (temperature only).

The two new time-scales & modes represent adjustment of the tropical and northern upper ocean temperatures to balance solar heating and radiation.

The new time-scales (decades) are less than the centuries for overturning circulation.

Undamped oscillatory modes are enabled by including (i) salinity as a variable, (ii) a significant density effect of fresh-water flux via the atmosphere, provided (iii) tropical “upwelling” q_U is small enough relative to q_n (the tropical-to-northern ocean total).

The time-scale for these salinity-enabled undamped oscillatory modes is V_n/q_n.

To represent internal variability, ocean models should allow spatial propagation.

References

Johnson, H.L., D.P. Marshall and D.A.J. Sproson, 2007. Reconciling theories of a mechanically-driven meridional overturning circulation with thermohaline forcing and multiple equilibria. *Climate Dynamics*, **29**, DOI:10.1007/s00382-007-0262-9.

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