# Global **sensitivity analysis** of complex numerical **landslide models** based on Gaussian-Process meta-modelling

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J. Rohmer, E. Foerster

*j.rohmer@brgm.fr*

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# Objectives of sensitivity analysis

"What model parameters have the **largest influence** on the simulation results?"

"What model parameter(s) can be fixed at any nominal values with **negligible influence** on the model results?"

"How to guide **future characterization** studies and what **effort** should I spent?"

# Variance-based global sensitivity analysis

## Basic idea:

- **Var(Y) =** Variance of the model output Y
  ~ measure of uncertainty

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  ➔ **expected reduction in the variance of Var(Y)**
  = measure of sensitivity

**Tools:** Sobol' indices (Sobol' 1993; Saltelli et al., 2008)

$$\boxed{S_1 = V_1 / V_Y = V(E(Y | X_1 = x_1^*)) / V_Y}$$

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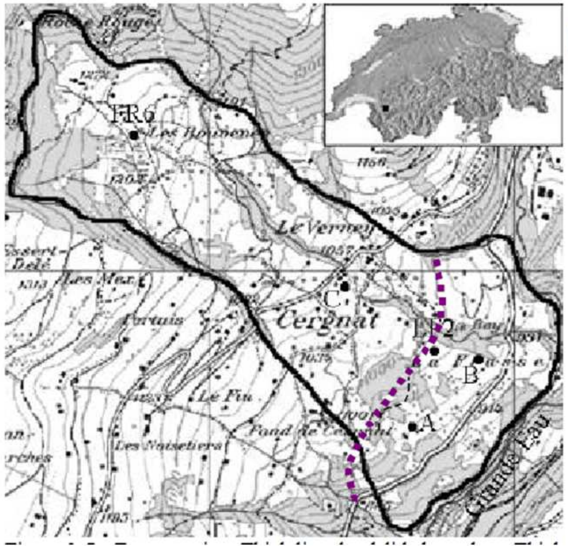
## Variance-based global sensitivity analysis

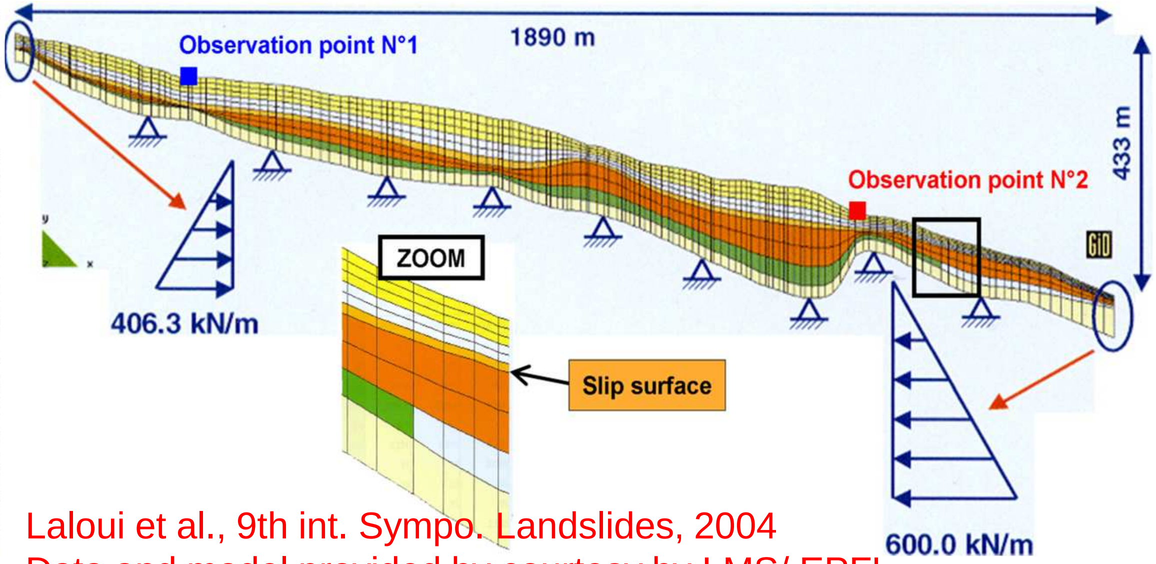
➔Explore the sensitivity to input parameters over their **whole range of variation** (i.e. in a global manner)

➔Fully account for **possible interaction** between them

➔Applicable **without** introducing a priori assumptions on the mathematical formulation of the landslide model

# Application case: La Frasse landslide






Laloui et al., 9th int. Sympo. Landslides, 2004
Data and model provided by courtesy by LMS/ EPFL

- **Evolution of the groundwater table** considered to be at the origin of the 1994 crisis (Noverraz & Bonnard, 1990);
- **Finite-element** simulations , 2D cross-section (Laloui et al., 2004) using program GEFDYN (Aubry *et al.*, 1986);
- Slip surface material modelled by the **Hujeux elastoplastic multi-mechanism constitutive model** (e.g., Lopez-Caballero and Modaressi Farahmand-Razavi, 2008)

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# Objective:

Ranking the 7 parameters of the Hujeux constitutive law

| Input factor | Vol. comp. mod. | Shear mod. | Non-linearity coeff. | Internal friction angle | Dilatancy angle | Plastic comp. | Initial critical pressure |
|---|---|---|---|---|---|---|---|
| Symbol | K | G | $n_e$ | $\phi$ | $\Psi$ | $\beta$ | $p_{c0}$ |
| Unit | MPa | MPa | - | ° | ° | - | MPa |
| Lower value | 180 | 83.25 | 0.225 | 19.125 | 14.25 | 20.625 | 0.375 |
| Upper value | 300 | 138.75 | 0.375 | 31.875 | 23.75 | 34.375 | 0.625 |

20% uniform variation around values of Laloui et al., 2004

# Computation challenge

Global sensitivity analysis ➔ **Ns.(d+2) simulations**
Saltelli (2002)

With
Number of input parameters: d =7 ;
Number of Monte-Carlo samples : Ns >1,000

➔ >9,000 simulations

# Computation challenge

Global sensitivity analysis ➔ **Ns.(d+2) simulations**
Saltelli (2002)

With d =7 ; Ns >1000

- If CPU time of 1 simulation = **1s,** sensitivity analysis ➔ **2.5 hours** of calculation ;
- If CPU time = **1min** ➔**>6 days** ;
- If CPU time = **1 hour** ➔**375 days**;

…

# Computation challenge

Global sensitivity analysis ➔ **Ns.(d+2) sim**
Saltelli (2002)

with d =7 ; Ns >1000

➢ If CPU tim… …sitivity analysis ➔
2.5 **hours** o…

…h ➔>**6 days** ;

**1 hour ➔375 days**;

**1 simulation of « La Frasse » model=4 days**
**➔ 36,000 days …**

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# Meta-model-based strategy

**Basic idea:**

replace the computationnally intensive landslide model G by an approximation *g* (= meta-model)

➔ **Costless-to-evaluate** analytical function

➔ Constructed using a **very limited** set of input parameters' configurations

$$y = G(x) \approx g(x)$$

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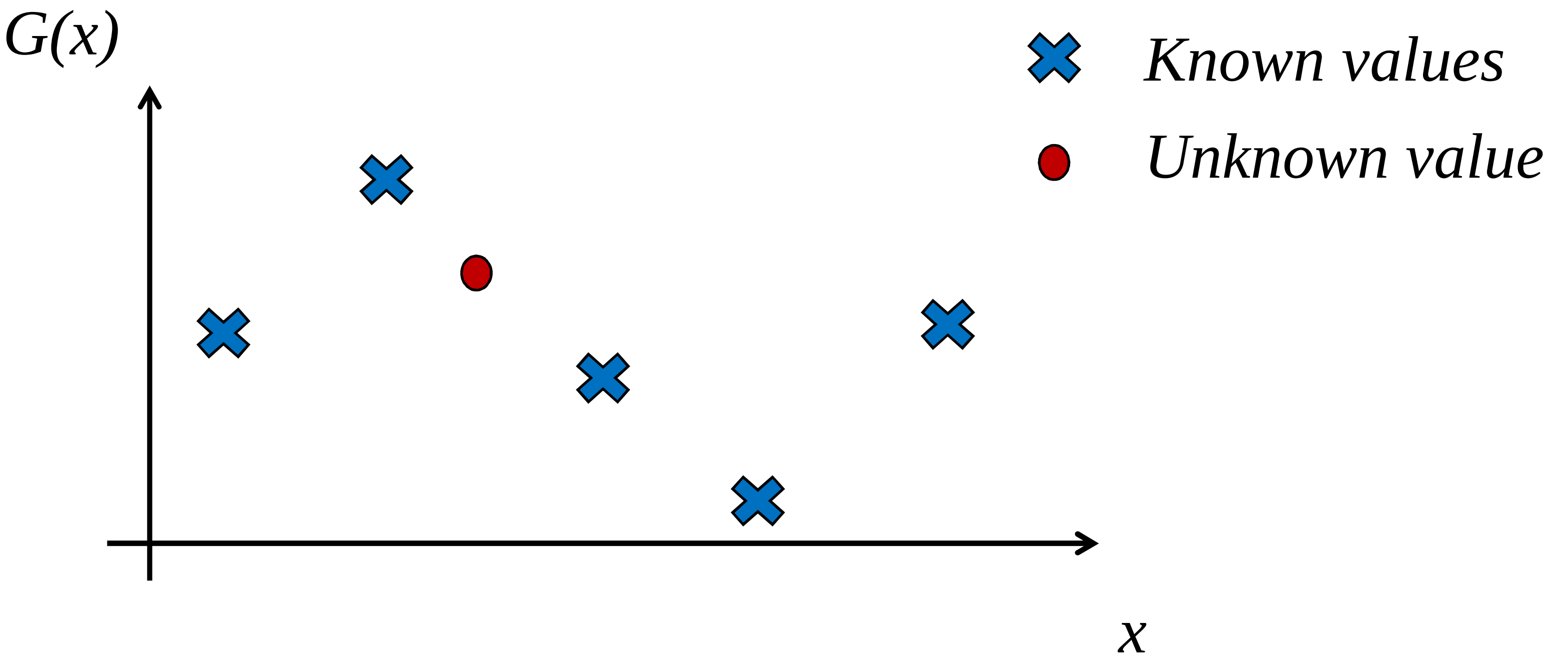
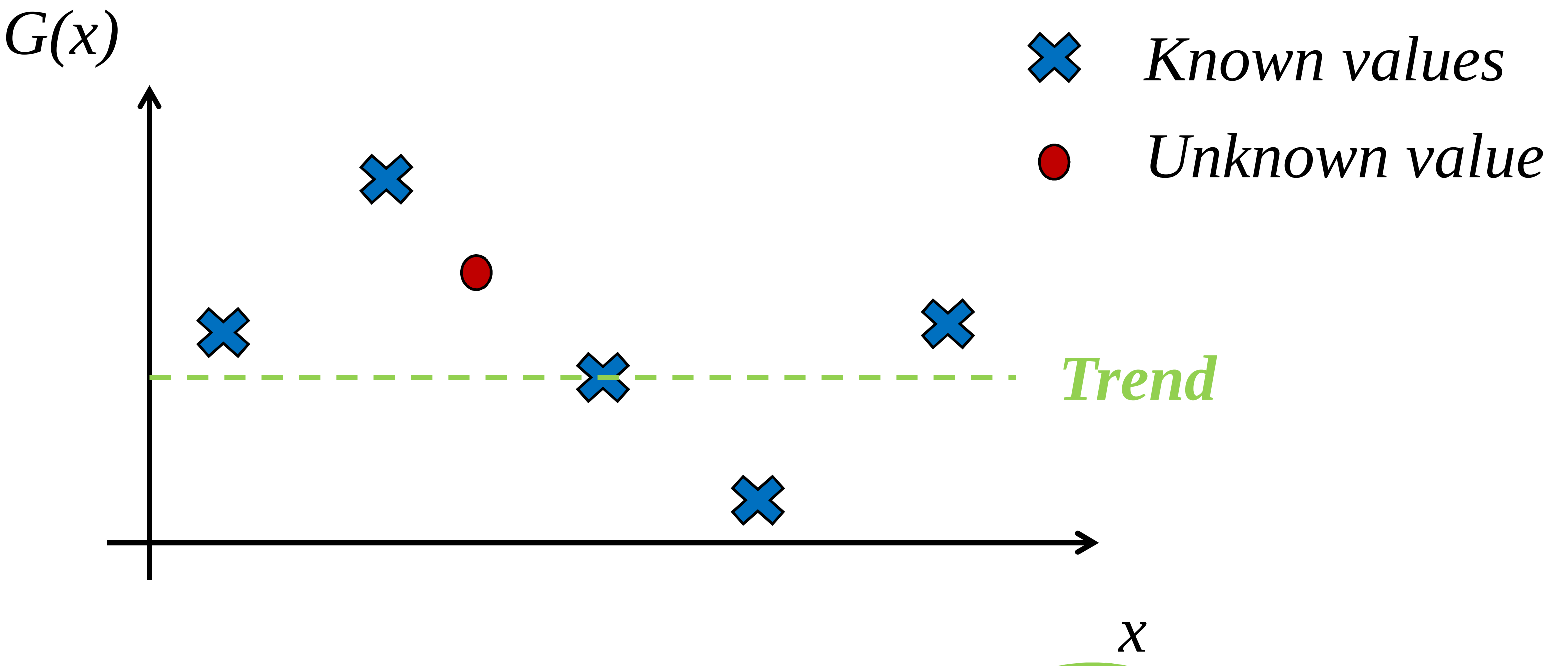
# Gaussian Process – intuitive definition




**References:**
**Sacks et al., 1989; Santner et al., 2003; Rasmussen and Williams, 2006; Forrester et al., 2008**

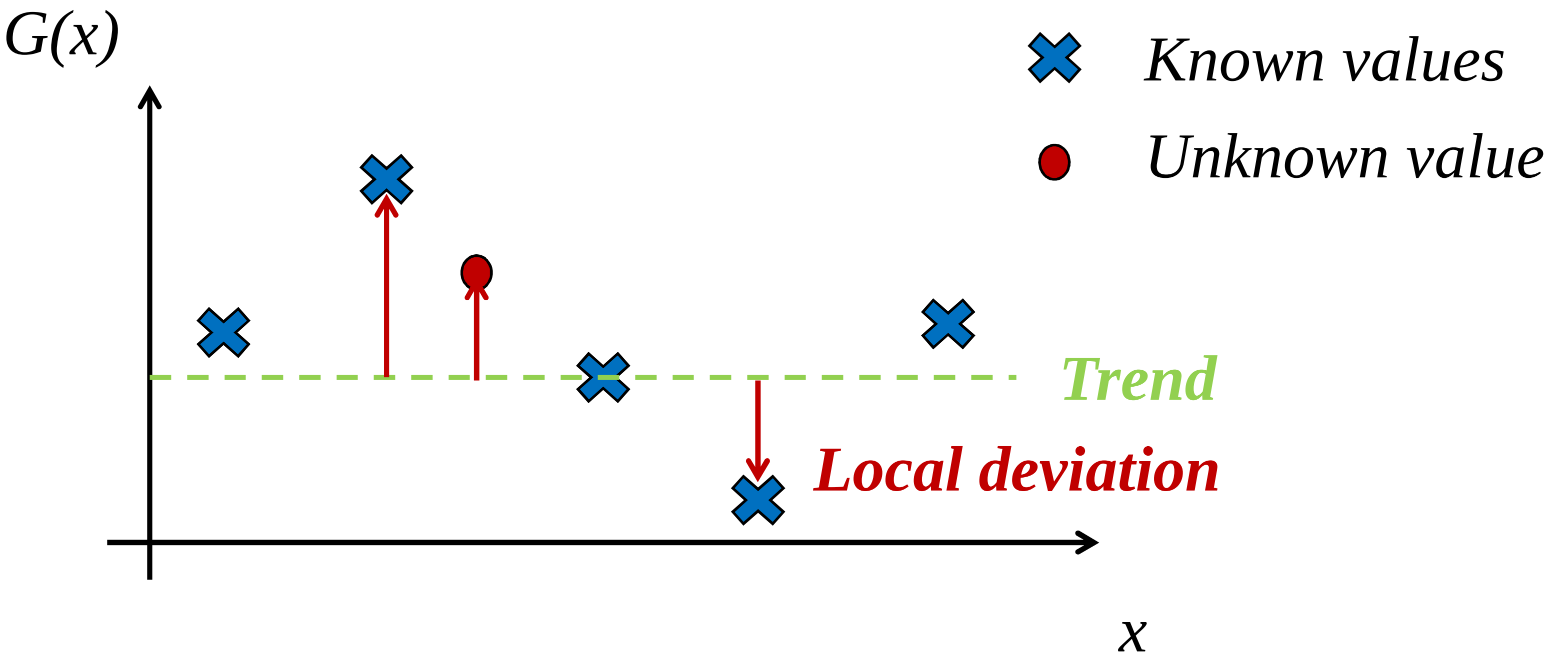
# Gaussian Process – intuitive definition




**Assumption:** $y = G(x) \approx g(x) = g_0(x) + Z(x)$

*Central tendency*

# Gaussian Process – intuitive definition




**Assumption:** $$y = G(x) \approx g(x) = g_0(x) + Z(x)$$

*Gaussian centered stationary stochastic process*

➔ **Measures departure** from g0

➔ Linked with the **dependencies** betw. the input vectors ***x***

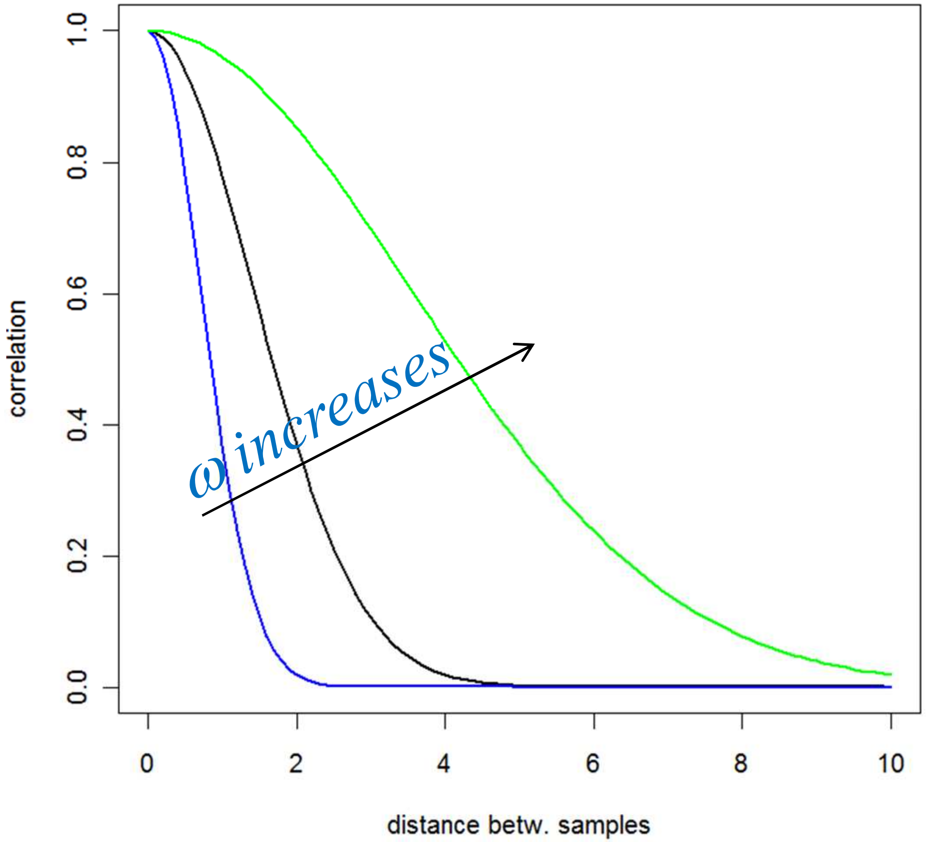
# Gaussian Process – intuitive definition



**Basic idea:**

*« The closer the unknown value from a known value, the more similar »*

➔ *Correlation betw. random variables*




$$R(\boldsymbol{u};\boldsymbol{v}) = \exp(-\sum_{i=1}^{d} \left\| u_i - v_i \right\|^2 / \omega_i)$$

Gaussian correlation model

***ω = correlation length***

*If u=v ➔ correlation = 1*

*If |u-v| → ∞ ➔ correlation → 0*

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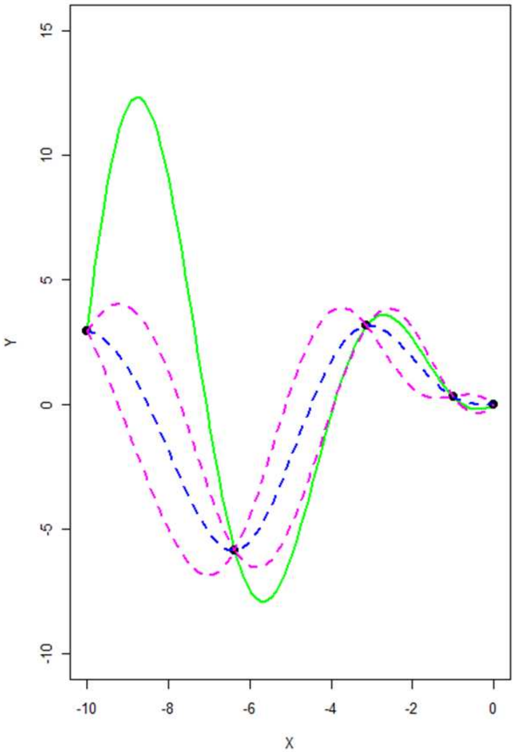
# Gaussian Process – 1d example

$$y = G(x) = x \cdot (\cos(x) + \sin(x))$$

**5 samples**




True function ; Best estimate ; Confidence interval (95%)

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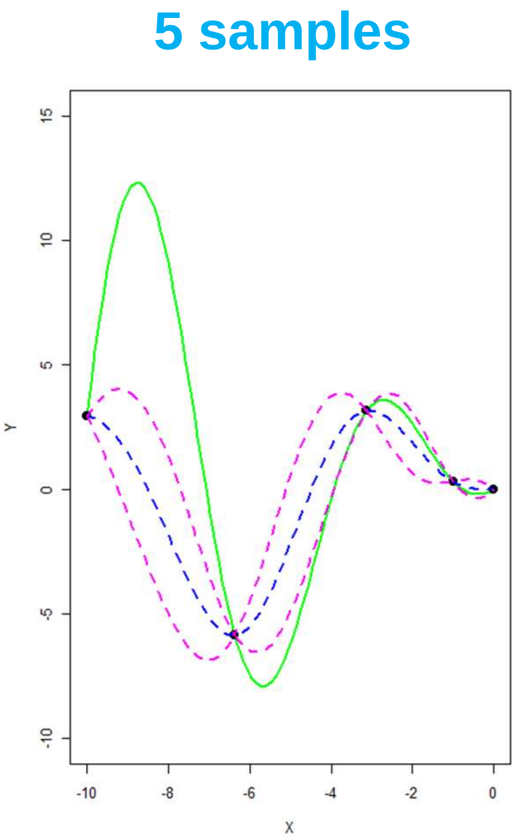


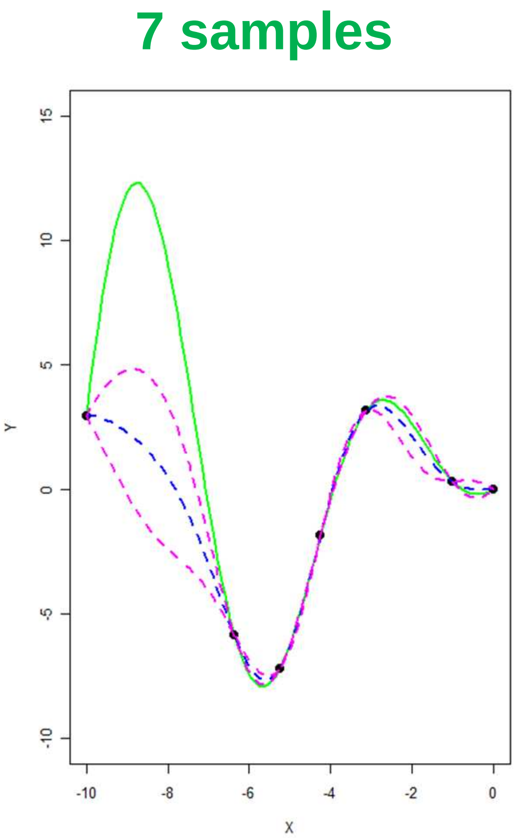


True function ; Best estimate ; Confidence interval (95%)



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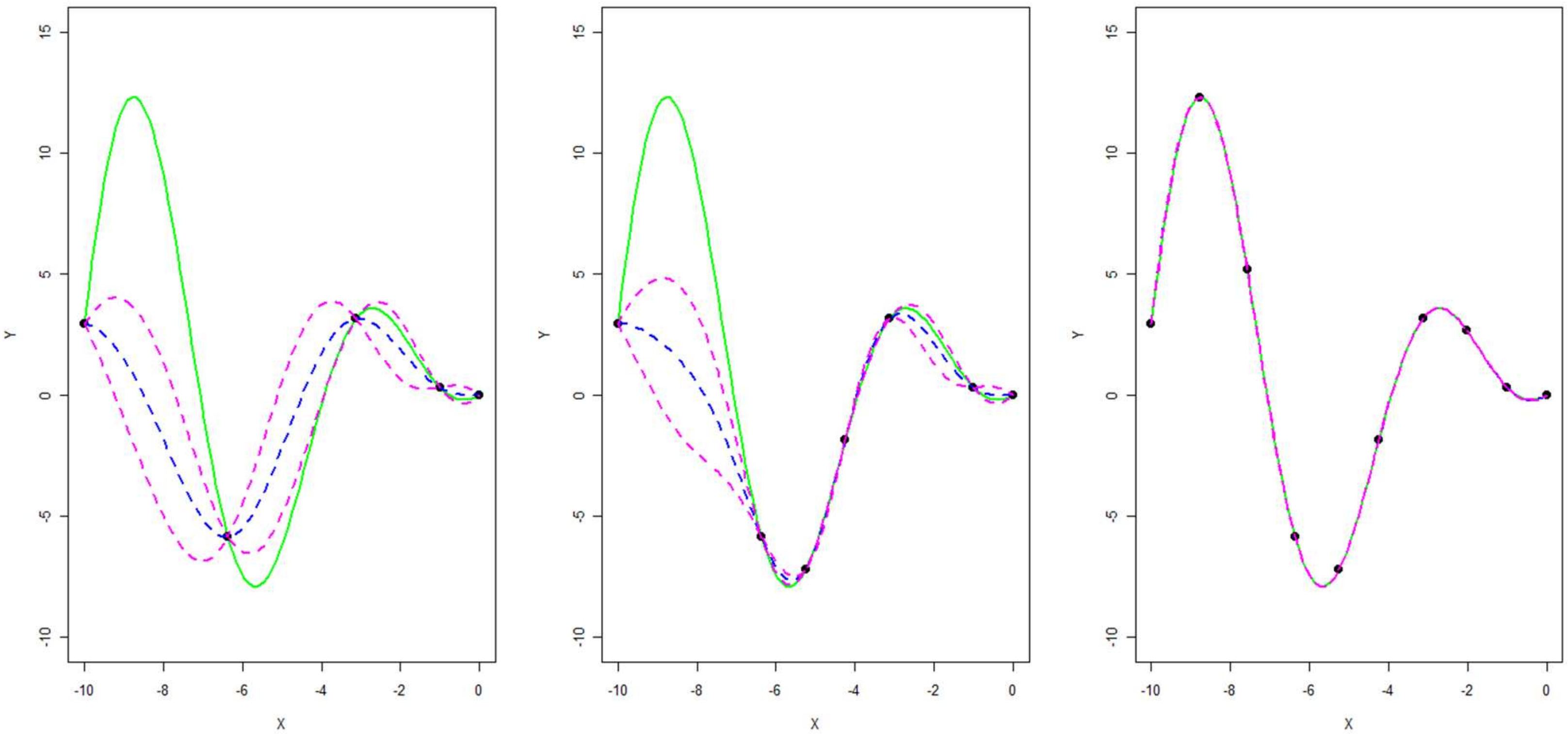
# Gaussian Process – 1d example

$$y = G(x) = x \cdot \left(\cos(x) + \sin(x)\right)$$

**5 samples** **7 samples** **10 samples**




True function ; Best estimate ; Confidence interval (95%)

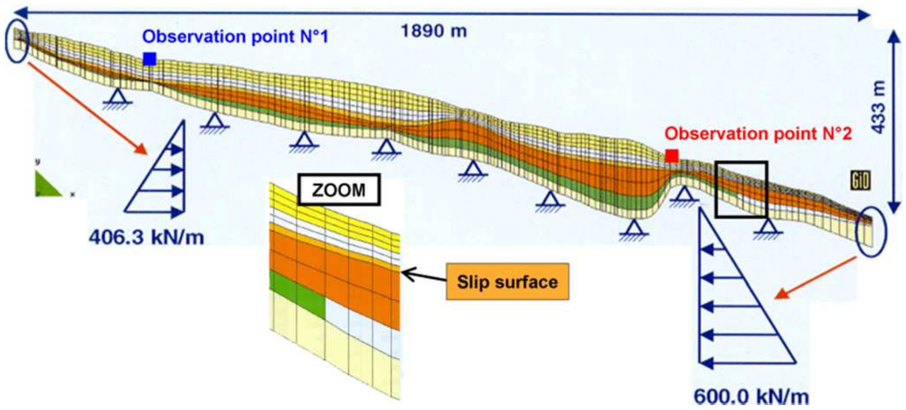
# Application to the La Frasse landslide

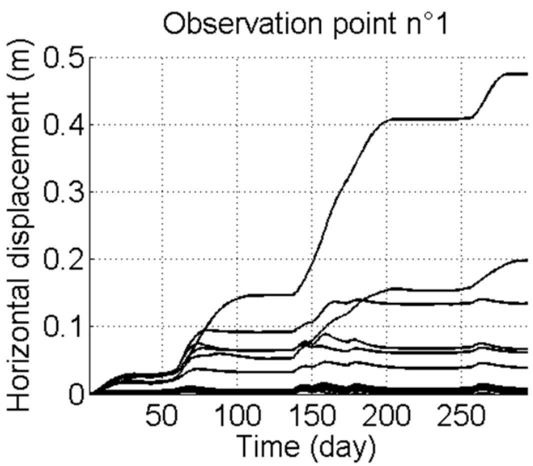
**Using the GP model**

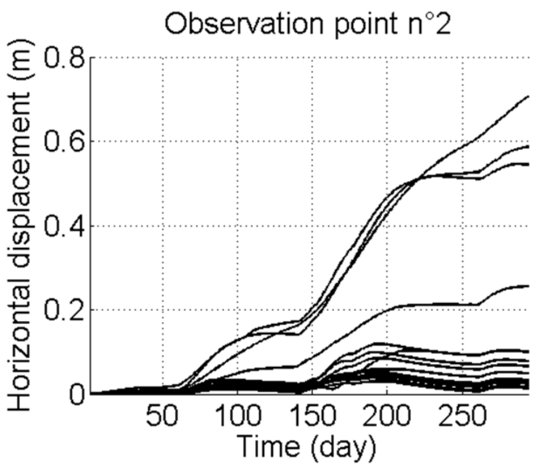
➔ Selection of 30 input configurations : ≈ 99 hours (with a cluster of 30 CPU);

➔Construction of the GP model and validation : ≈6 hours.

➔**~4.5 days**




**30 samples**



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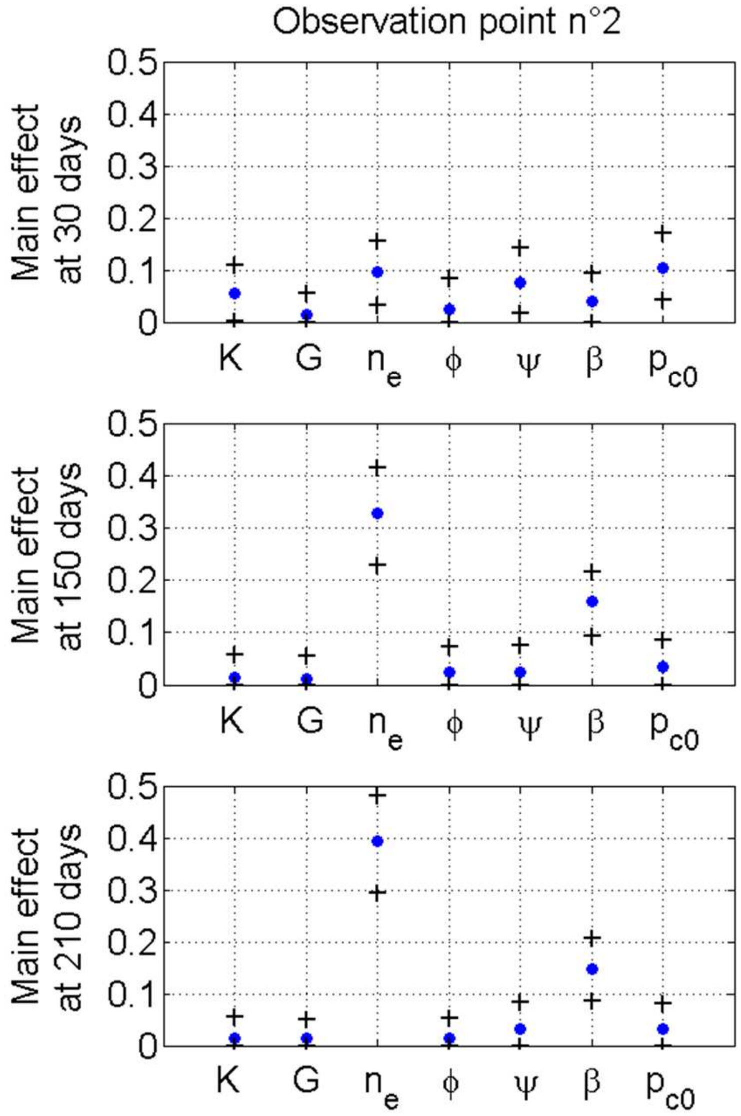
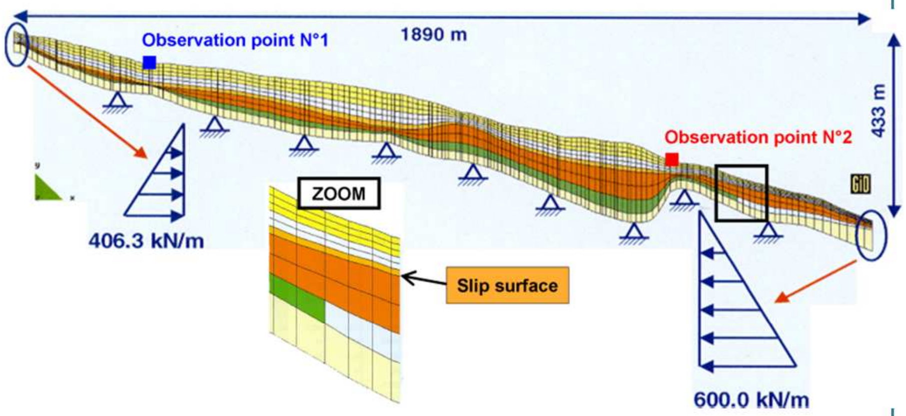
# Application to the La Frasse landslide

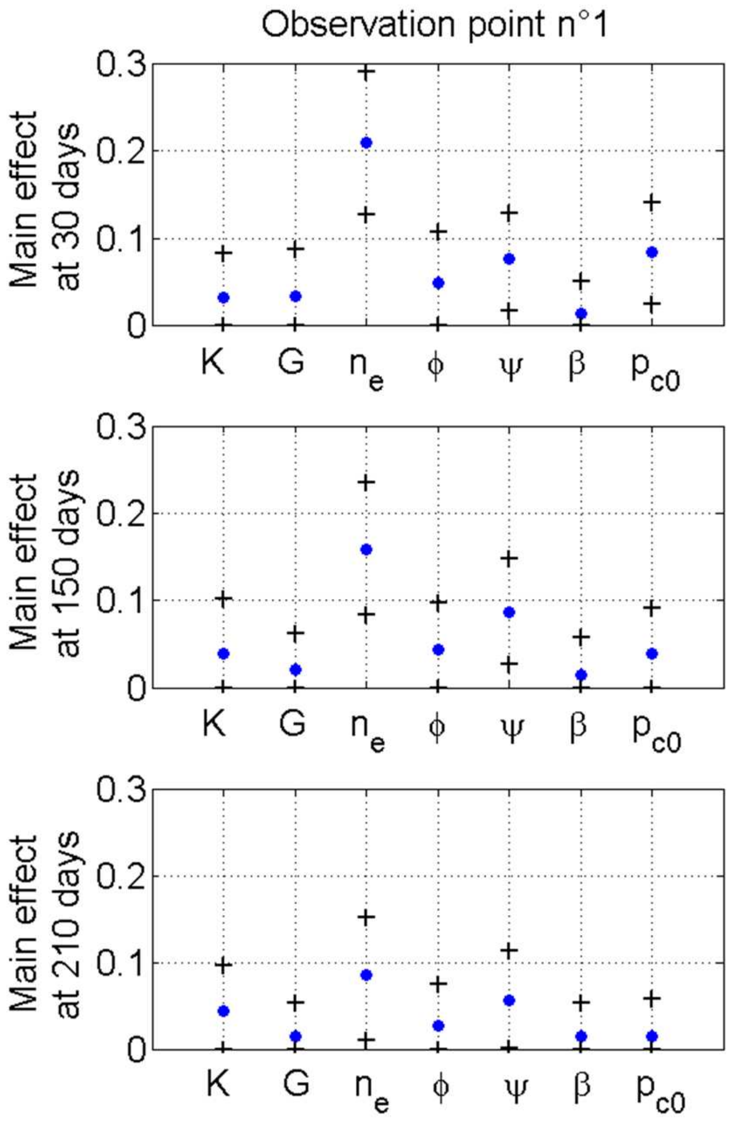


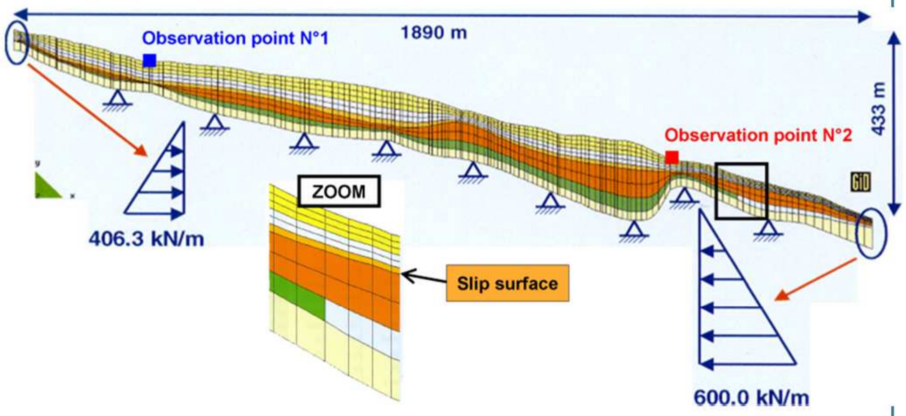




✚ Bounds of the confidence intervals (5% and 95% quantile)

● Best estimate

# Application to the La Frasse landslide



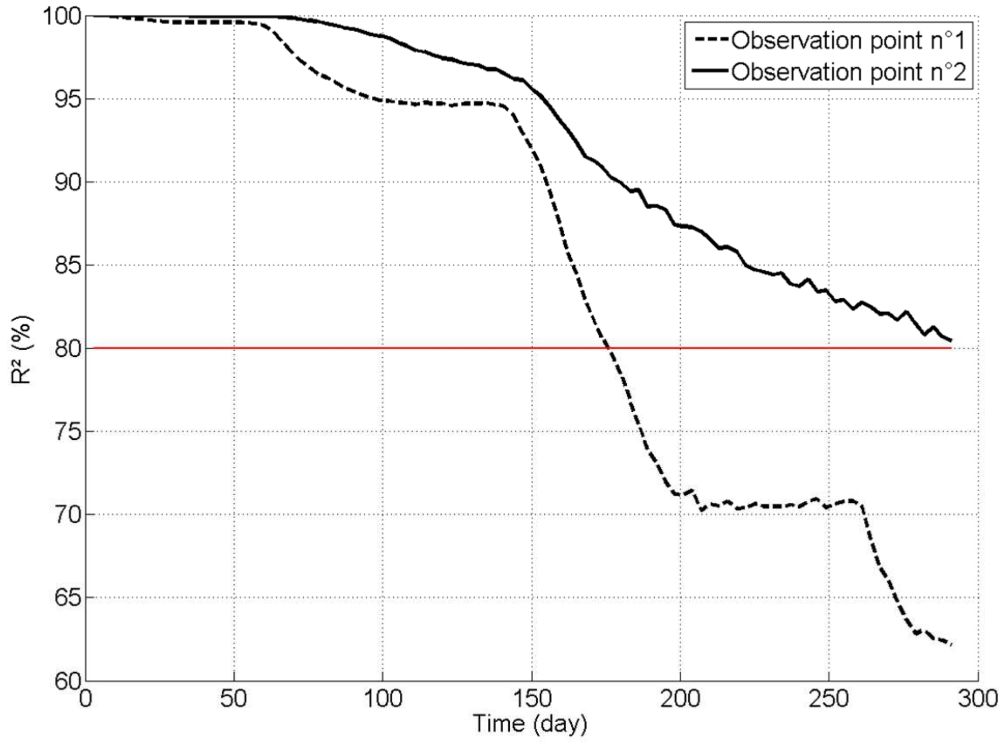



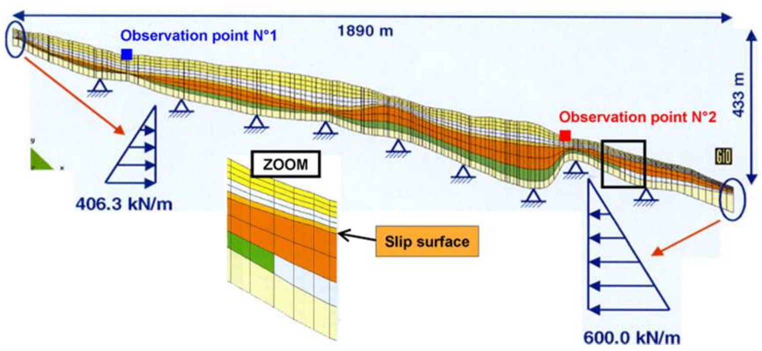

Bounds of the confidence intervals (5% and 95% quantile)

Best estimate

# Application to the La Frasse landslide

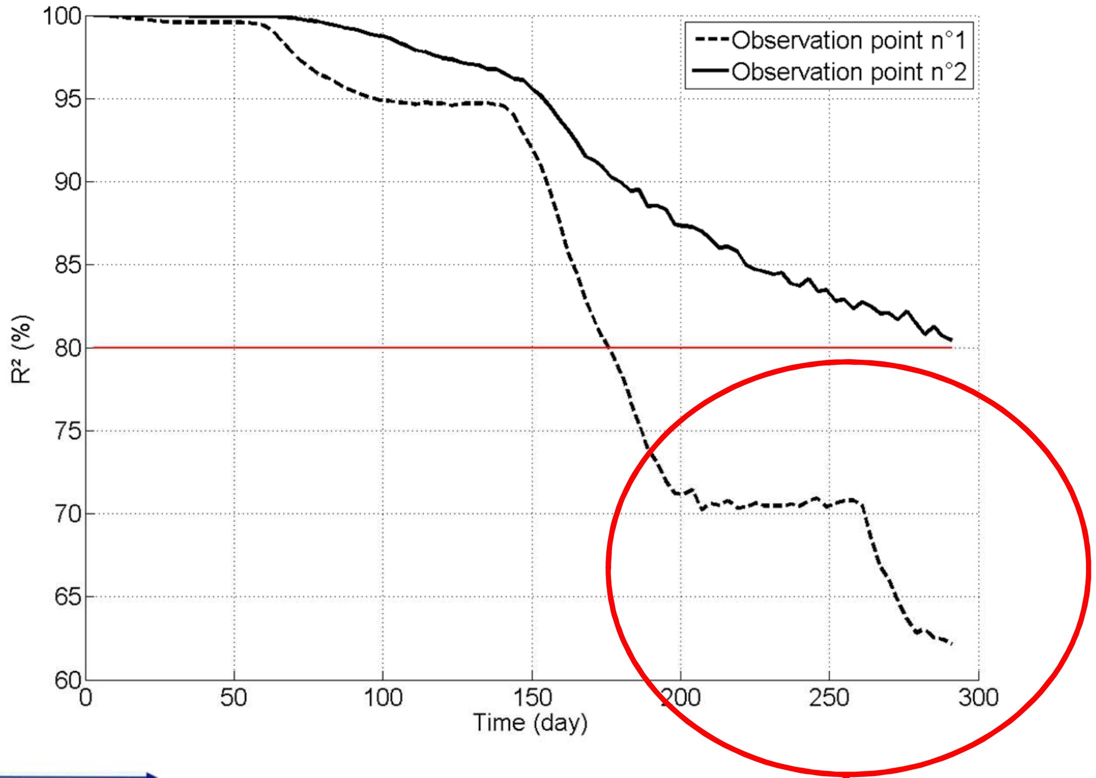
**Coefficient of determination of the Cross Validation procedure (Hastie et al., 2002)**

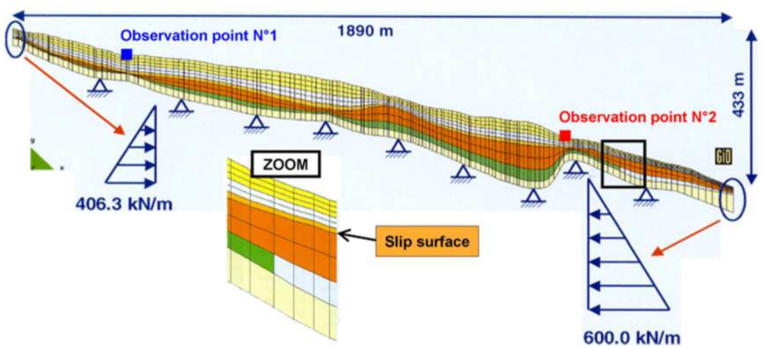


# Application to the La Frasse landslide

## Coefficient of determination of the LOO-CV




**Poor approximation**

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# In summary…

**Global sensitivity analysis ➔** very useful to guide further investigations

**BUT: requires a large number of simulations !**

➔Use of meta-model (here Gaussian Process)
= approximate using a few model evaluations

**BUT:**
➔ Careful evaluation of the approximation quality
➔ Account for the approximation error in the sensitivity analysis

# Thank you for your attention !

**Some advertising**…
Rohmer, J., Foerster, E., 2011. Computers & Geosciences, Vol. 37, Issue 7, 917–927

# Acknowledgments

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**The application is based on the finite-element landslide model built by the LMS of EPFL.**



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