Application of a normalized Nash-Sutcliffe efficiency to improve the accuracy of the Sobol' sensitivity analysis of a hydrological model

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Outstanding Student Poster Contest

A. Introduction

- Sensitivity analysis (SA)
  - Main practice in hydrological modelling
  - Identify influential and non-influential parameters
  - Insights on the model processes
- Sobol' SA method [1]
  - Variance based method
  - For time series -> objective function is required -> scalar
  - Variance estimation assessed with Monte Carlo integrals (!!!)
  - Accuracy of variance estimation may decrease when mean value of scalar inputs f_0 for SA is large [2]
  - Nash-Sutcliffe efficiency (NSE) [3] yields more accurate results than e.g. the also commonly used Sum of Squared Residuals
  - Sufficient for flow predictions (mean NSE is -0.73)
  - Poor for water quality simulations (mean NSE up to -4E6)

=> Suitable objective function required (!!!)

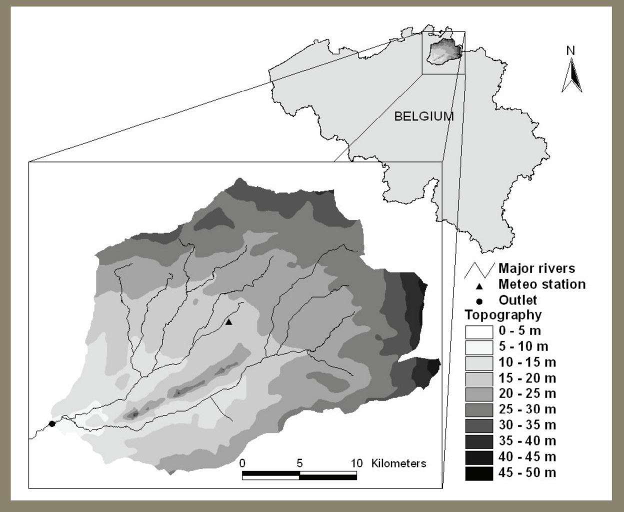
B. Sobol' sensitivity analysis

- Very robust SA technique with almost ideal properties
- No assumptions about linearity, additivity, monotonicity
- Quantifies amount of variance that each parameter contributes to unconditional variance of model output
- Expressed with sensitivity indices:
  - First order index (main effect): $S_i = V_i / V$
    - Measure for variance contribution of individual parameter X_i to total variance
  - Total index: $S_{Ti} = S_i + \sum_j S_{ij} + ... = 1 - V_{\sim i} / V$
    - Sum of main effect of X_i and all interactions with other parameters
- f_0, V, V_i, $V_{\sim i}$, ... numerically estimated with Monte Carlo integrals
  - E.g. $V = (1 / n\text{-}1) \sum_m f^2(x_m) - f_0^2$
    (with m: $1\text{->}n$ and f the scalar output of the objective function)
- A large number of random samples in the parameter hyperspace are required to evaluate model
  - A sample size (n) of 12000 is used, resulting in 336000 (=n·(p+2)) model evaluations (with p=26, the number of parameters) for S_i and S_{Ti}

C. SWAT-model of the River Kleine Nete catchment [4]

- SWAT (The Soil and Water Assessment Tool) [5]:

  Physically based, semi-distributed (HRUs), basin scale simulator for water quantity, water quality and sediment simulations
- The River Kleine Nete catchment
  - Small (580 km²), lowland (av. 23 masl) catchment
  - 56% agriculture, 26% forests
  - Mainly sandy soils (95%)



- The model
  - 1997-2007
  - Precipitation: 700 to 1100mm/y
  - PET: 700mm/y, ET: 500mm/y
  - 5°C in winter & 14°C in summer
  - Short river reaches
  - 26 parameters

D. The normalized Nash-Sutcliffe efficiency

- The regular Nash-Sutcliffe efficiency:

$$NSE = 1 - \frac{\Sigma_i (o_i - s_i)^2}{\Sigma_i (o_i - \bar{o})^2}$$

with s_i the simulated value on day i, o_i the observed value on day i and $\bar{o}$ the average of the observations

- The normalized Nash-Sutcliffe efficiency:

$$NNSE = \frac{1}{2 - NSE} = \frac{\Sigma_i (o_i - \bar{o})^2}{\Sigma_i (o_i - s_i)^2 + (o_i - \bar{o})^2}$$

- Values of the regular and normalized Nash-Sutcliffe efficiency for a perfect model (Max), a model that performs as well as the mean of the observations (Mean) and the worst possible model (Min)

	Min	Mean	Max
NSE	-∞	0	1
NNSE	0	0.5	1

E. Results

- Mean objective function values (f_0) for the NSE and NNSE for different variables

	Flow	KjN	NH_4^+	NO_2^-	NO_3^-	o-PO_4^{3-}	P_t	Sed.	CBOD
NSE	-0.7310	-289.23	-214.45	-1790.4	-4.13E6	-1691.8	-232.76	-0.7474	-2.7928
NNSE	0.3761	0.0532	0.0315	0.0022	0.0191	0.3427	0.3255	0.4223	0.2272

large

close to 0

- Improved and more realistic sensitivity indices (S_i and S_{Ti}) for NNSE for water quality variables

Sed.	S_i		S_{Ti}	
	NSE	NNSE	NSE	NNSE
CH_N	0.765	0.874	0.957	1.024
CN2	0.046	0.027	0.274	0.048
ALPHA_BF	0.013	0.009	0.060	0.005
...	...	...	...	...
ESCO	0.000	0.001	0.041	0.000
SMTMP	0.000	0.001	0.040	0.000
...	...	...	...	...
SUM	0.830	0.939	2.248	1.090

overall realistic

But values of insensitive parameters too high

NO_3^-	S_i		S_{Ti}	
	NSE	NNSE	NSE	NNSE
CN2	0.000	0.272	0.937	0.447
ALPHA_BF	0.000	0.063	-0.073	0.145
SURLAG	0.000	0.044	0.642	0.092
...	...	...	...	...
SMTMP	0.000	0.002	-0.845	0.003
ESCO	0.000	0.001	-18.390	0.003
...	...	...	...	...
SUM	0.002	0.624	-40.656	1.591

Very low values for most sensitive parameters

unrealistic!

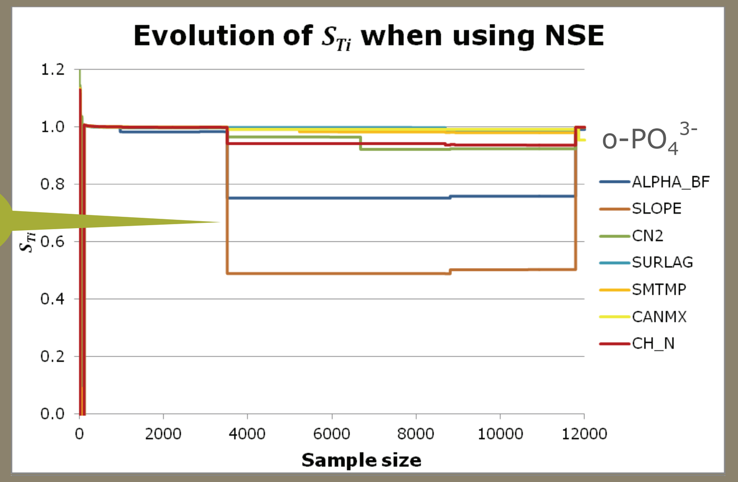
o-PO_4^{3-}	S_i		S_{Ti}	
	NSE	NNSE	NSE	NNSE
CN2	0.000	0.483	1.000	0.593
SURLAG	0.000	0.051	1.000	0.241
CANMX	0.000	0.040	0.955	0.189
...	...	...	...	...
CH_N	0.000	0.000	1.000	0.026
SMTMP	0.000	0.000	1.000	0.014
...	...	...	...	...
SUM	0.000	0.736	25.267	1.706

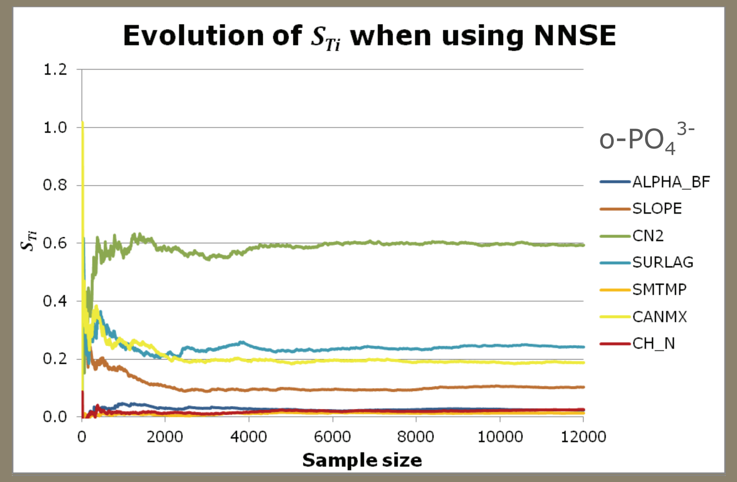
Very low values for most sensitive parameters

unrealistic!

- Graphical representation of the evolution with increasing sample size of the total sensitivity index (S_{Ti}) for o-PO_4^{3-} when applying the regular NSE and the normalized NSE as an objective function

unrealistic jumps





F. Conclusions

- A normalized Nash-Sutcliffe efficiency (NNSE) has been introduced with similar properties as the regular Nash-Sutcliffe efficiency (NSE) to evaluate model output, but with values between 0 and 1
- The NNSE values between 0 and 1 provide a more intuitive interpretation of the model evaluations
- The NNSE can be successfully applied for the Sobol' sensitivity analysis of flow and water quality variables

G. References

[1] Sobol', I.M., 1990. On sensitivity estimation for nonlinear mathematical models. Matematicheskoe Modelirovanie, 2(1): 112-118.

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[3] Nash, J.E. and Sutcliffe, J.V., 1970. River flow forecasting through conceptual models part I -- A discussion of principles. Journal of Hydrology, 10(3): 282-290.

[4] Nossent, J., Elsen, P. and Bauwens, W., 2011. Sobol' sensitivity analysis of a complex environmental model. Environmental Modelling & Software. 26 (2011), 1515-1525.

[5] Arnold, J.G., Allen, P.M. and Bernhardt, G., 1993. A Comprehensive Surface-Groundwater Flow Model. Journal of Hydrology, 142(1-4): 47-69.