Long-range persistence in the global mean surface temperature and the global warming "time bomb"

Martin Rypdal and Kristoffer Rypdal

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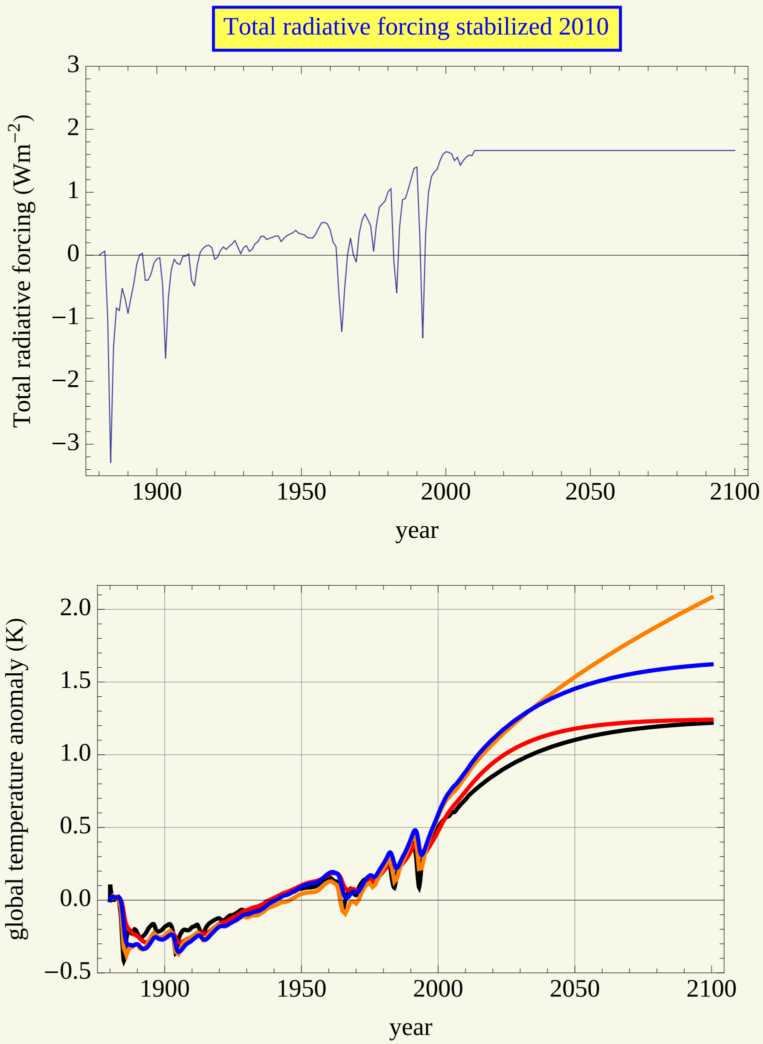
Motivation



- K. Rypdal, J. Geophys. Res. (2012)**117**, D06115
  Poster 3330 on Wednesday:

  Global temperature evolution under prescribed radiative forcing for different dynamic response models, including scale-free (power-law) response.

- Here we introduce stochastic forcing and demonstrate that power-law response yields much better agreement with observed temperature record than an exponential response.

Outline of the talk

- Analysis of scaling properties in global temperature data
    - on decadal to centennial time scales we have strong long-range memory: *pink* rather than *white* noise
- Linear-response modeling
    - *power-law* rather than *exponential* impulse response
- Implications
    - more global warming "in the pipeline"

Notation

- Let $X(t)$ be a continuous-time process with stationary increments.
- Assume $\mathbb{E}[X(t)^2] \sim t^{2H}$.
- Denote $x(n) = X(n+1) - X(n)$.

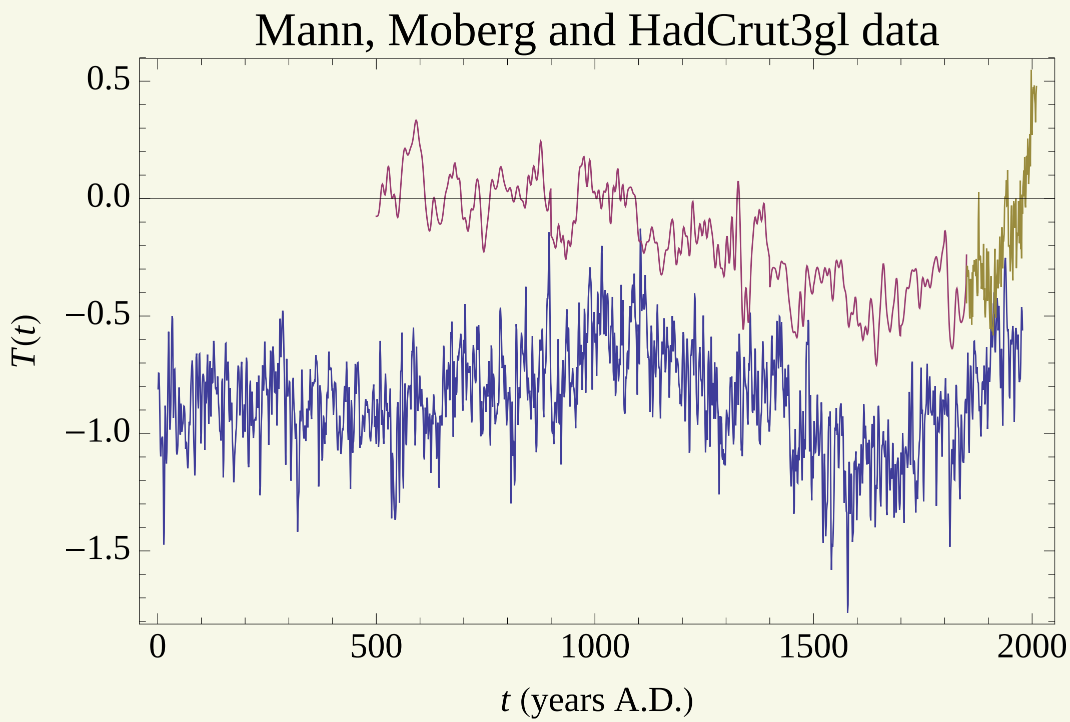
Definition.
The Hurst exponent of $x(n)$ is H and the Hurst exponent of $X(t)$ is $H+1$.

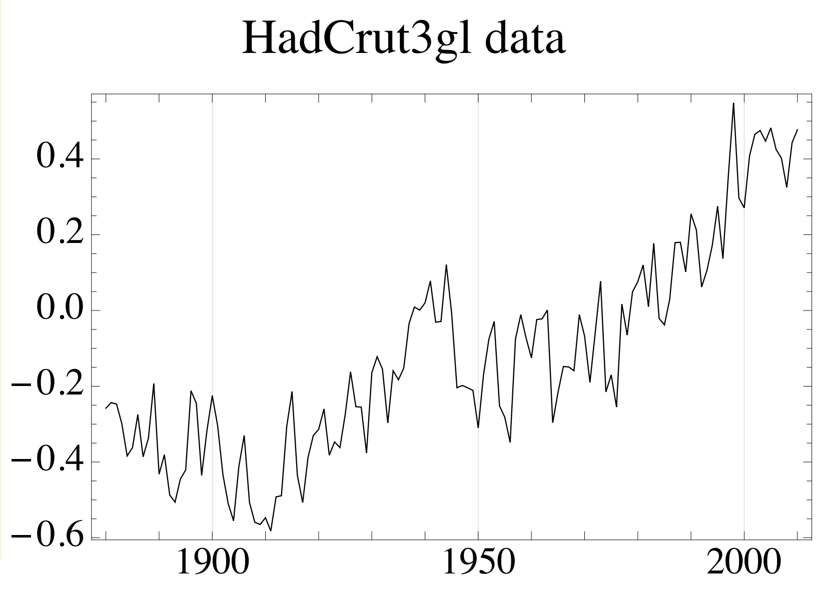
Remark.
If $X(t)$ is selfsimilar, then its selfsimilarity exponent is $h = H - 1$.

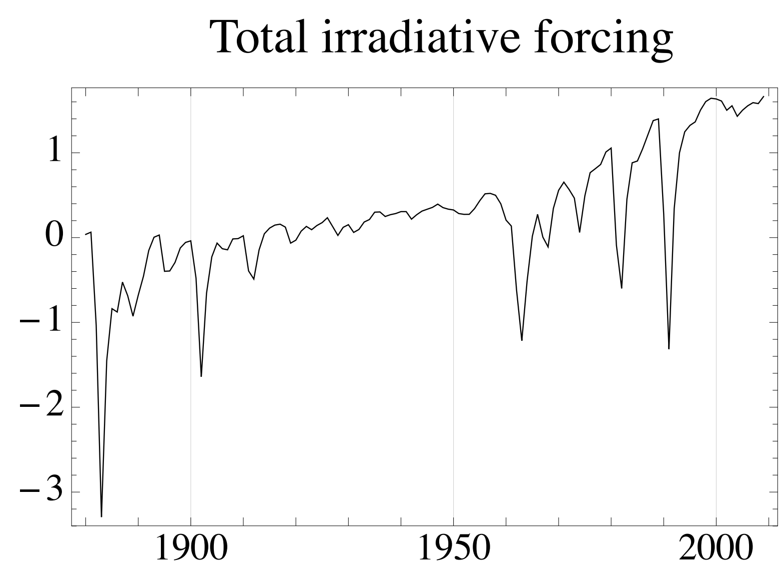
- We denote $\beta = 2H - 1$.
- $S_x(f) \sim 1/f^{\beta}$
- $\mathrm{ACF}_x(\tau) \sim \beta(\beta+1)\tau^{\beta-1}$ $(\beta < 1, \beta \neq 0)$

Exponent	Example
$\beta = 0$ $(H = 1/2)$	White noise
$\beta = 1$ $(H = 1)$	Pink noise
$\beta = 2$ $(H = 3/2)$	Brownian motion

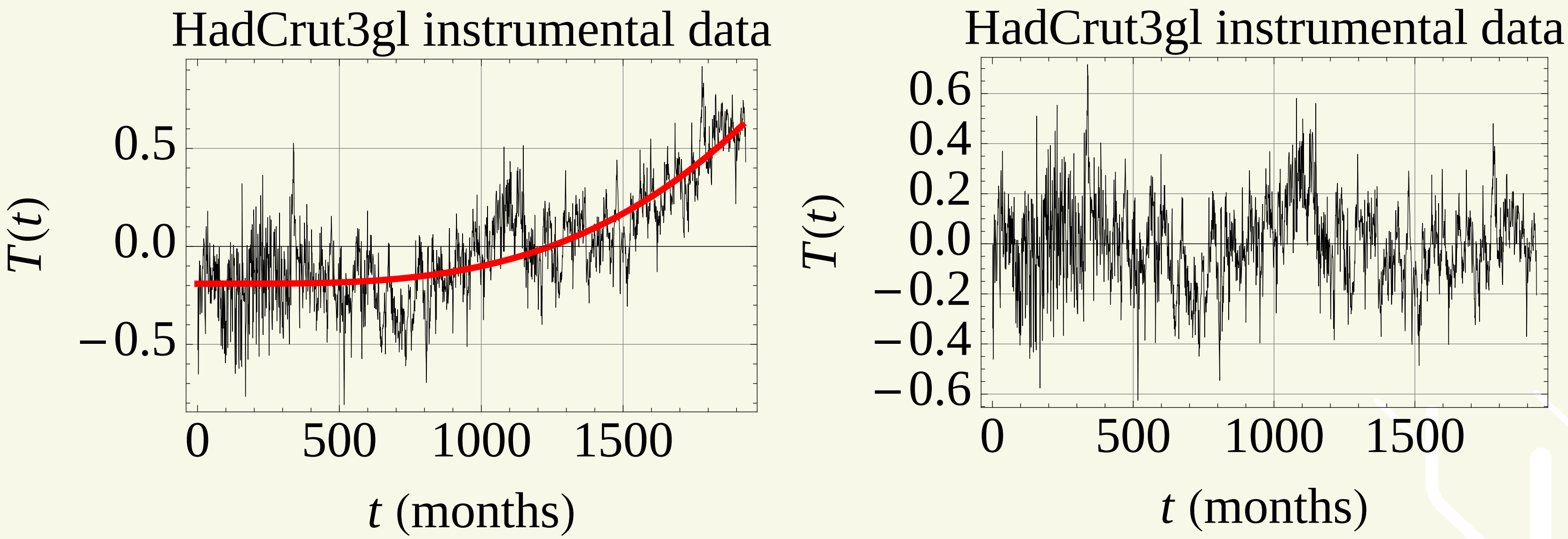
Time series


Mann, Moberg and HadCrut3gl data
0.5
0.0
−0.5
−1.0
−1.5
T(t)
0
500
1000
1500
2000
t (years A.D.)


HadCrut3gl data
0.4
0.2
0.0
−0.2
−0.4
−0.6
1900
1950
2000

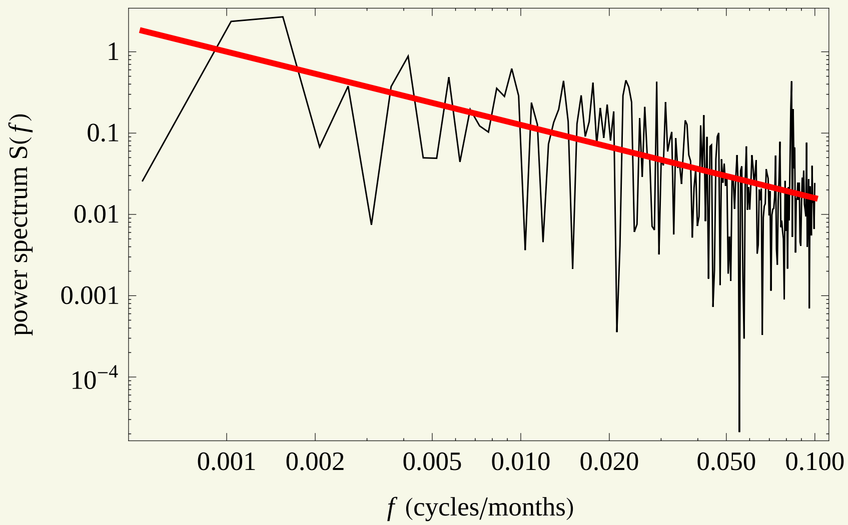

Total irradiative forcing
1
0
−1
−2
−3
1900
1950
2000

Detrended instrumental temperature record

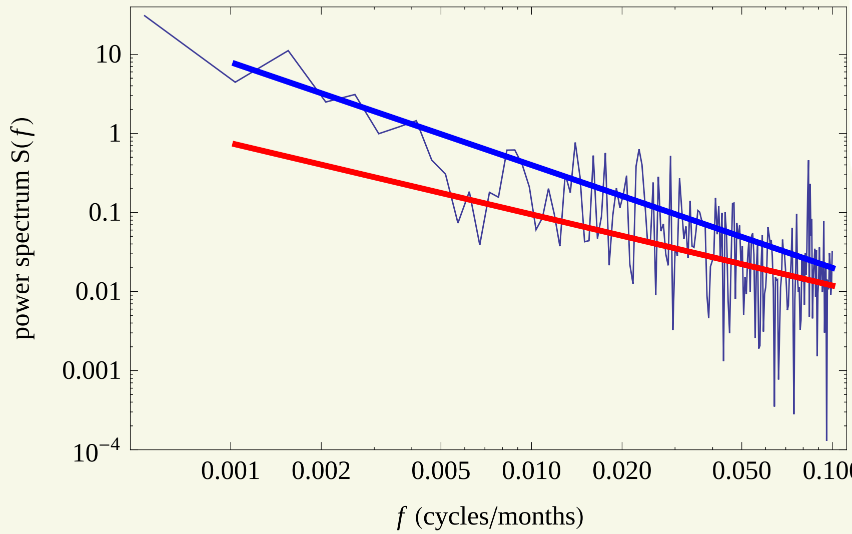


Power Spectral Densities (PSD)

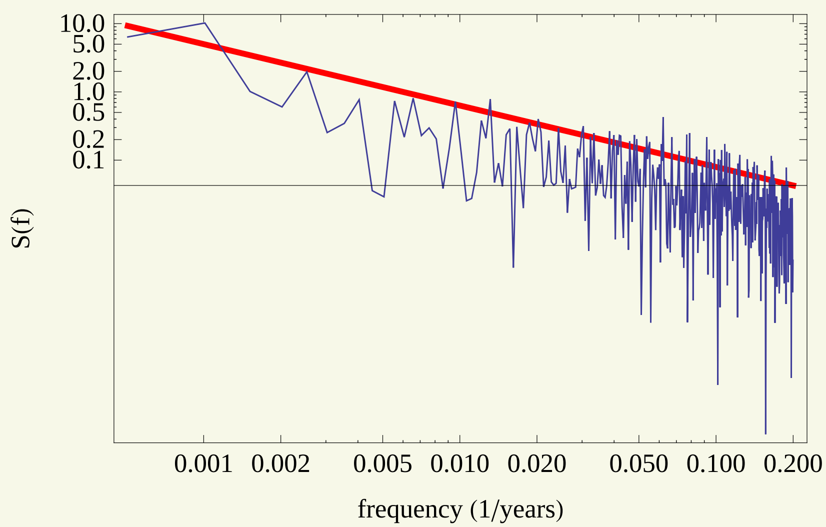
PSD HadCrut3gl, β=0.9



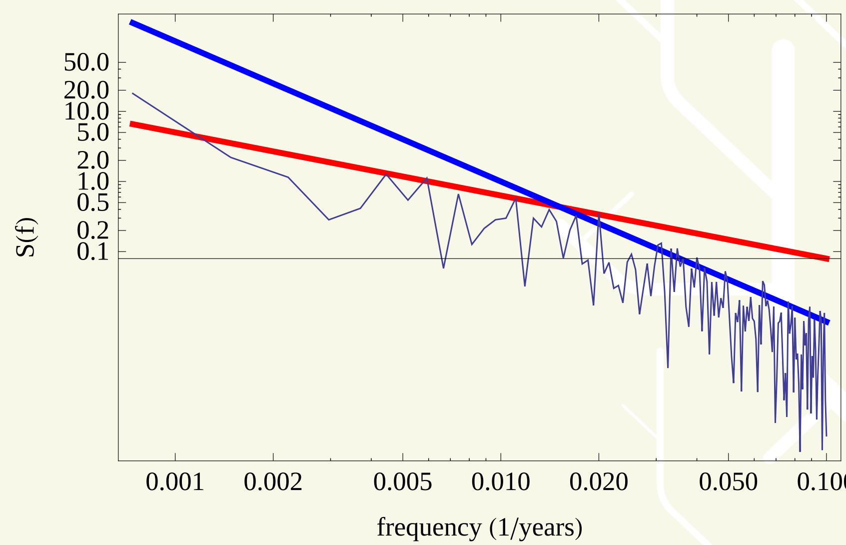
PSD HadCrut3gl, β_1=0.9, β_2=1.3,



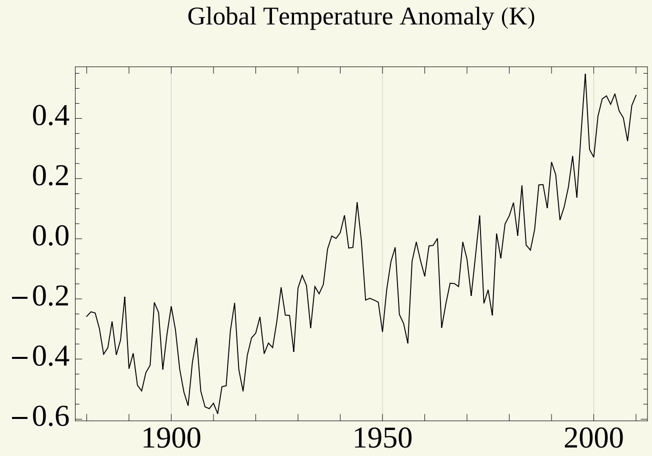
PSD undetrended Moberg, β=0.9

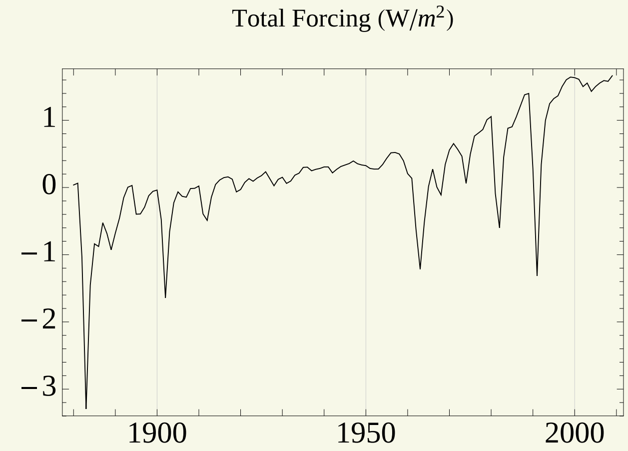


PSD undetrended Mann, β_1=0.9, β_2=2.0



Linear Energy Balance Model





- Simplest possible energy balance model:

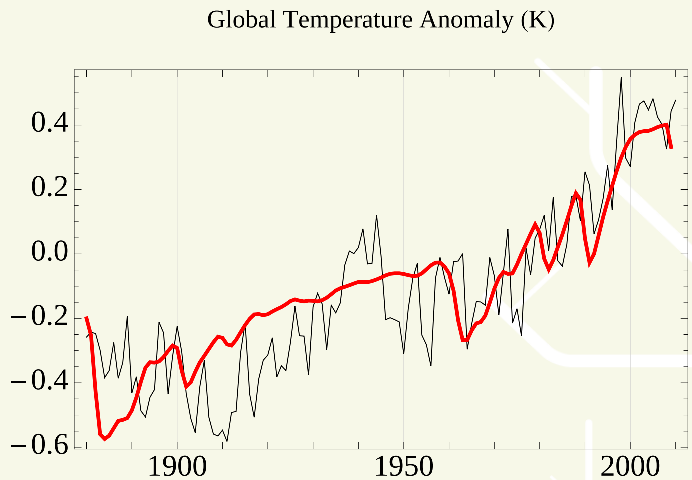
$$\frac{dT}{dt} + \frac{1}{\tau}T = \frac{1}{C}F(t)$$

Climate sensitivity: $S \equiv \tau/C$

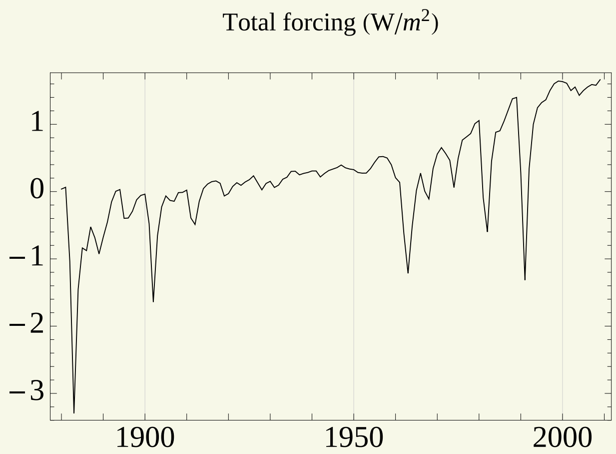
- Linear response:

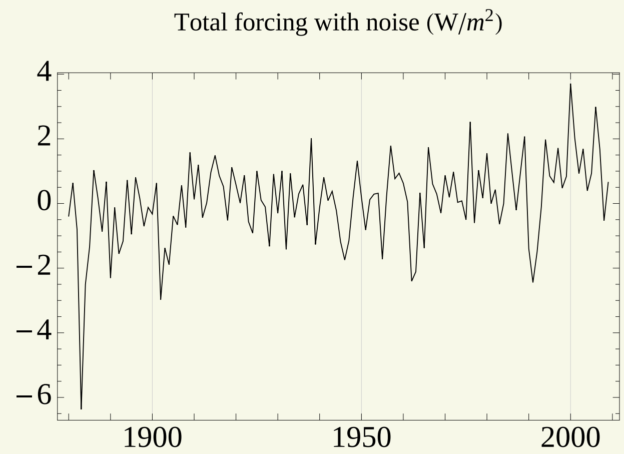
$$T(t) = \frac{1}{C}\int G(t-s)F(s)\,ds\,,$$

Impulse response: $G(t) = e^{-t/\tau}\,\Theta(t)$



Introduce stochastic forcing





- Linear stochastic differential equation:

$$\frac{dT}{dt} + \frac{1}{\tau}T = \frac{1}{C}[F(t) + \sigma w(t)]$$

- Solution:

$$T(t) = \underbrace{\frac{1}{C}\int G(t-s)F(s)\,ds}_{\text{solution of deterministic equation}} + \underbrace{\frac{\sigma}{C}\int G(t-s)\,dw(s)ds}_{\text{Ornstein-Uhlenbeck process}}$$

Fractional model with stochastic forcing

- Model:

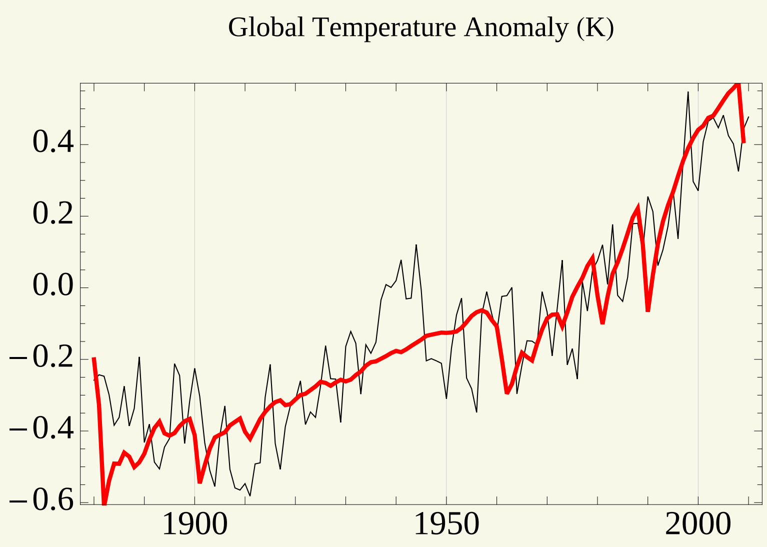
$$\frac{1}{\Gamma(\beta/2)}(\mathcal{D}^{\beta/2}T)(t) = \frac{1}{C}[F(t) + \sigma w(t)]$$

where $\mathcal{D}^{\beta/2}$ is the Liouville fractional derivative.

- Solutions:

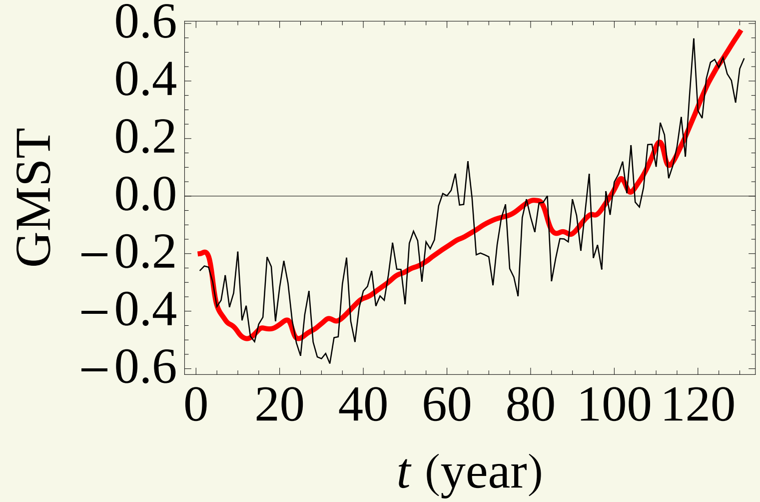
$$T(t) = \frac{1}{C}[\underbrace{\int (t-s)_+^{\beta/2-1} F(s)\, ds}_{\text{deterministic solution}} + \underbrace{\sigma \int (t-s)_+^{\beta/2-1} dw(s)\, ds}_{1/f^{\beta}\text{ noise}}]$$

- Solution with no stochastic forcing:

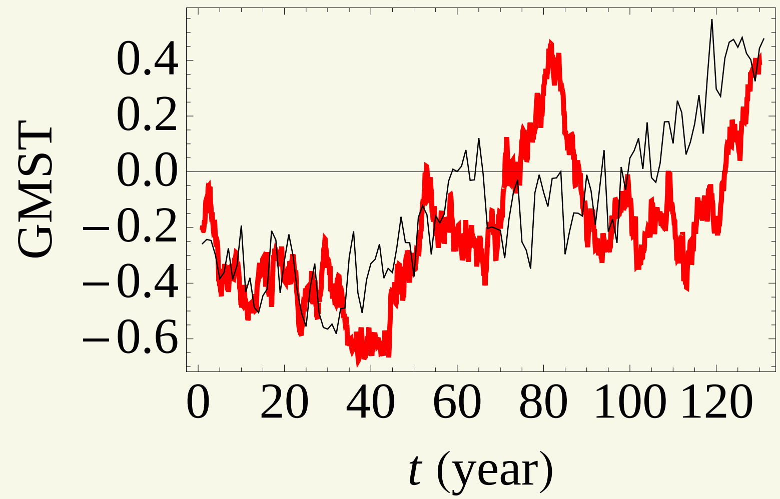


Model realizations instrumental period

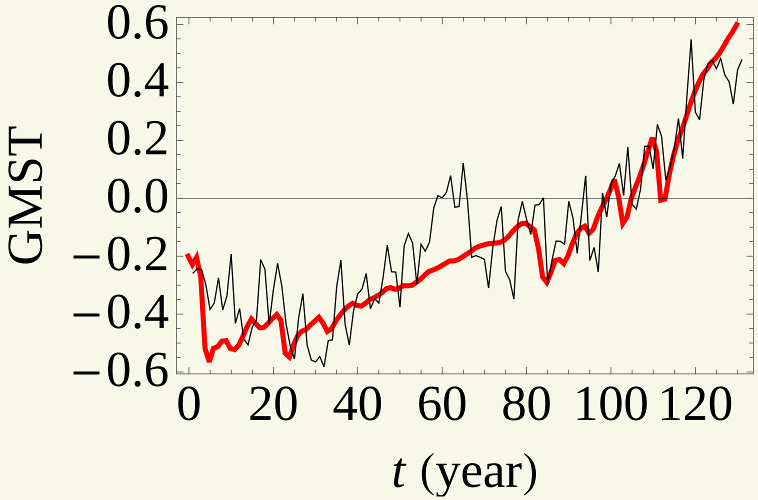
Exponential response: τ=20 years, S=0.75 K/Wm^{-2}, σ=0 Wm^{-2}



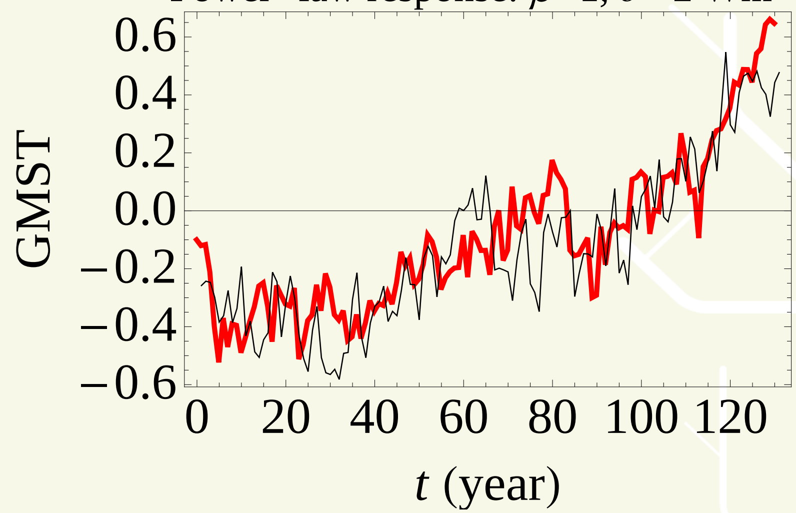
Exponential response: τ=20 years, S=0.75 K/Wm^{-2}, σ=8 Wm^{-2}



Power−law response: β=1, σ=0 Wm^{-2}

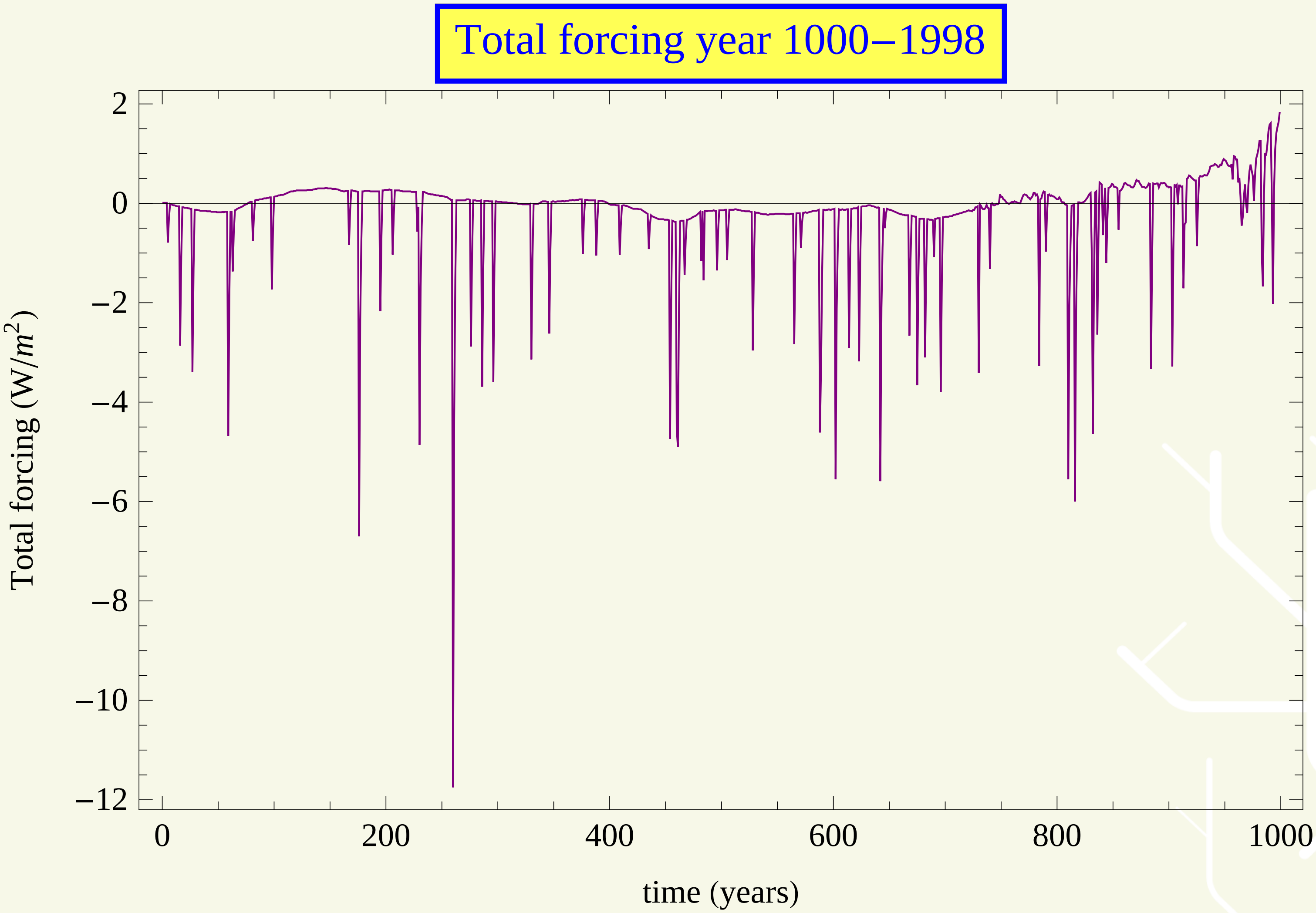


Power−law response: β=1, σ=2 Wm^{-2}



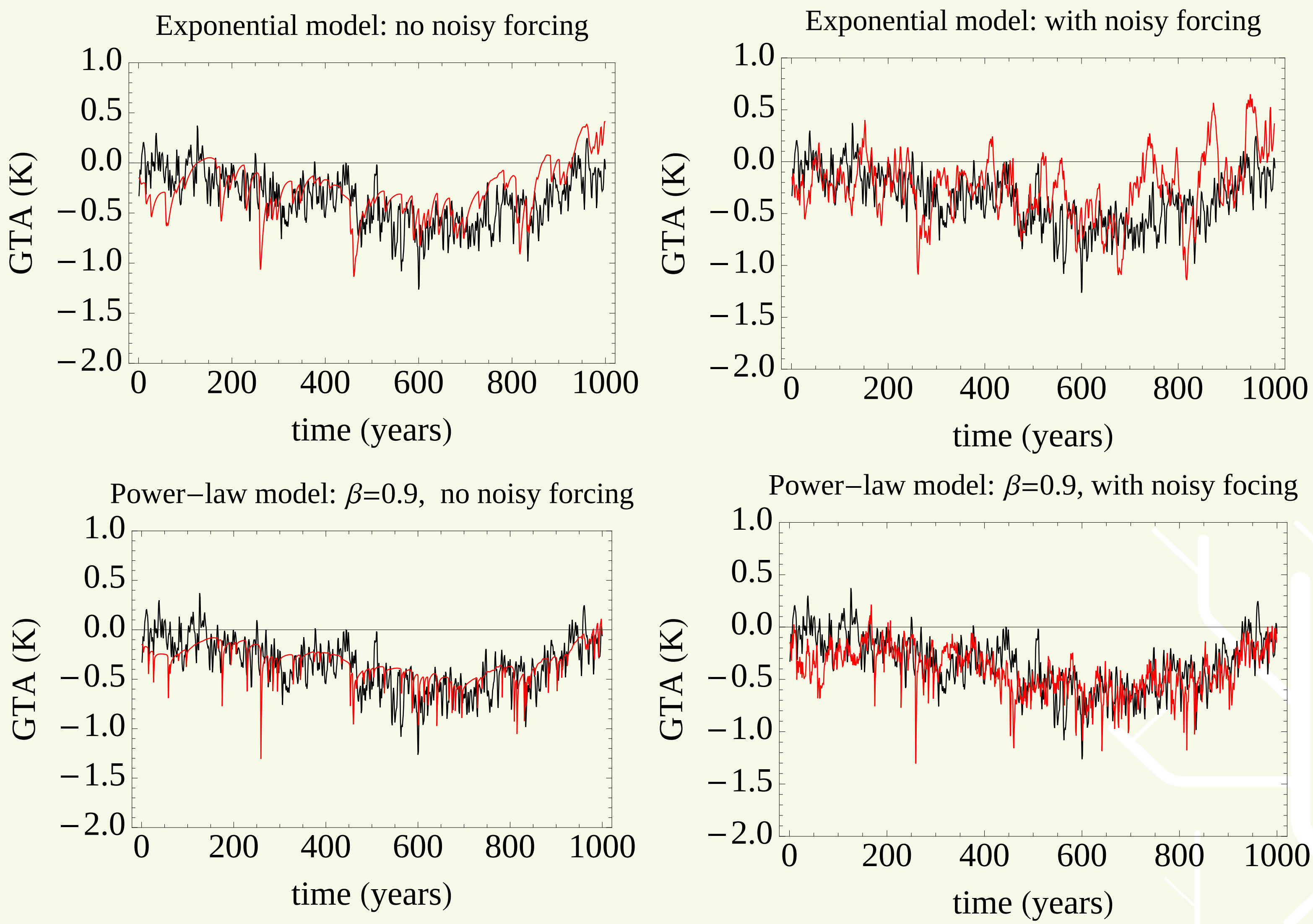
Forcing last millennium

Forcing from Crowley (2000):



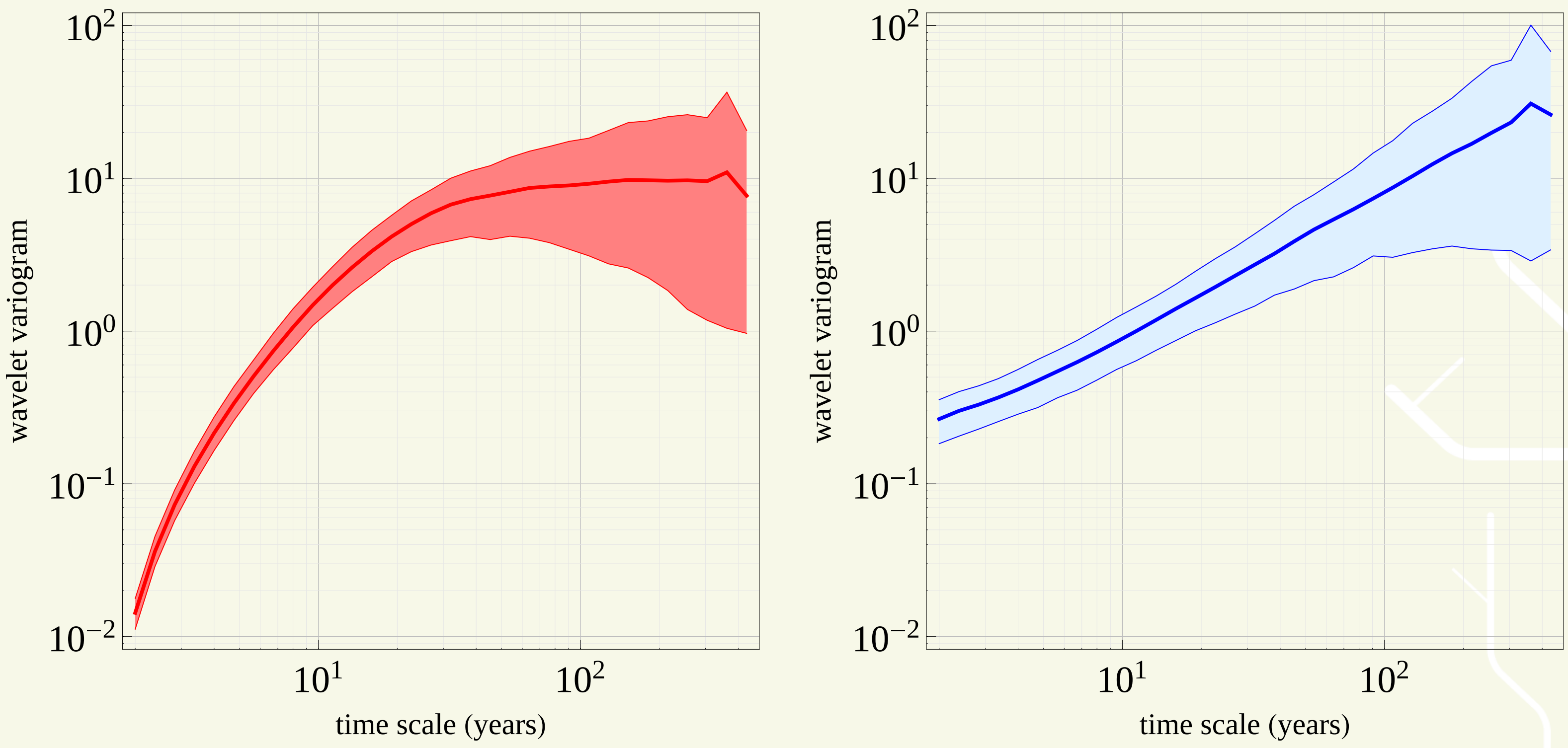
Model realizations last millennium

Out[436]=



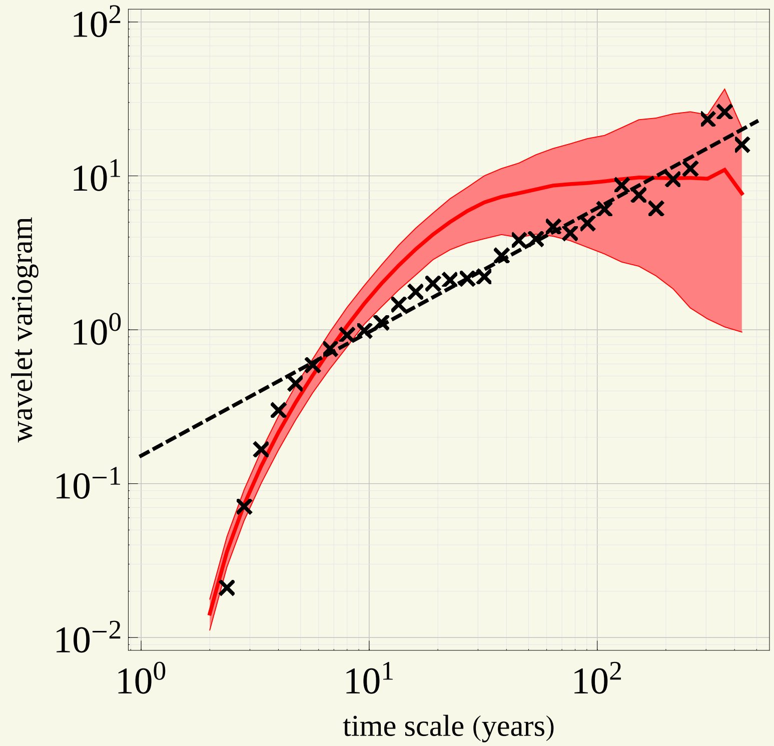
Wavelet variance of model realizations

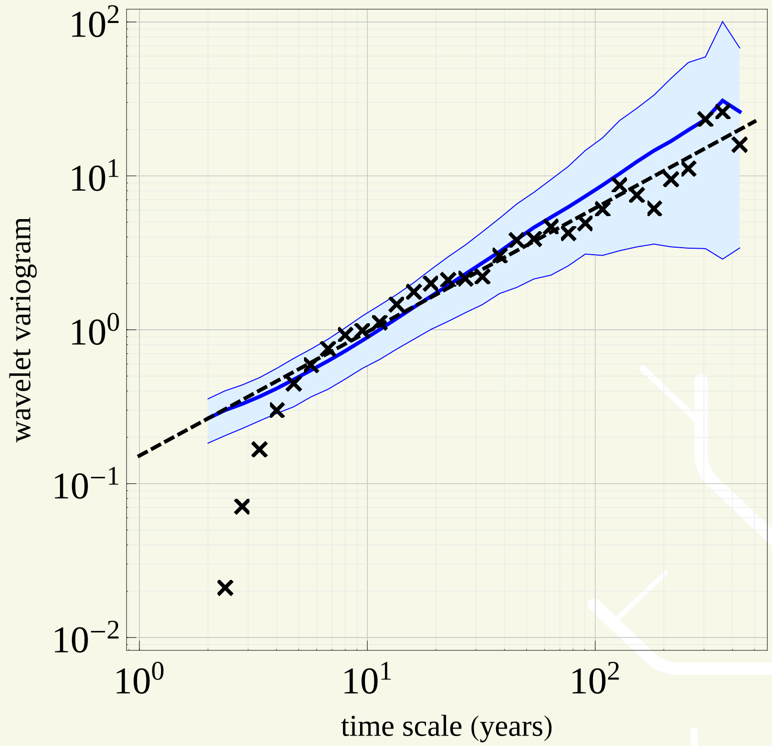
- Another method to detect power-law scaling and estimate Hurst exponents:
- Left: Range of variance of wavelet-coefficients versus scale for exponential response. *No scaling.*
- Left: Range of variance of wavelet-coefficients versus scale for power-law response. *Good scaling with $H \approx 0.9$.*



Comparison with Moberg data

- Power-law scaling consistent with Moberg data.





A global warming time bomb?

- Long-range memory implies more “warming in the pipeline”