Soundproof simulations of stratospheric gravity waves on unstructured meshes

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Aims:

a) highlight the progress with the development of a nonhydrostatic, soundproof, unstructured-mesh model for atmospheric flows;

b) assess the accuracy of unstructured-mesh discretization relative to equivalent structured-grid methods for wave dynamics;

c) assess relative merits of anelastic and pseudo-incompressible PDEs versus fully compressible PDEs, debated in the literature

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Nonoscillatory forward-in-time semi-implicit numerics (of EULAG) with flexible edge-based finite-volume (FV) spatial discretisation

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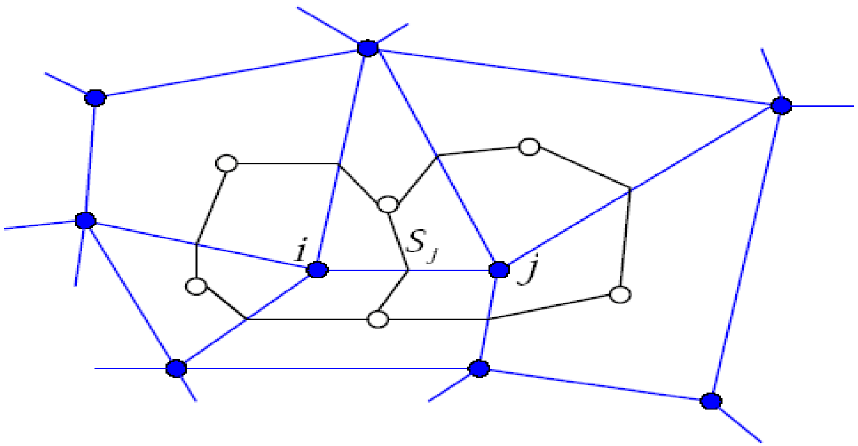
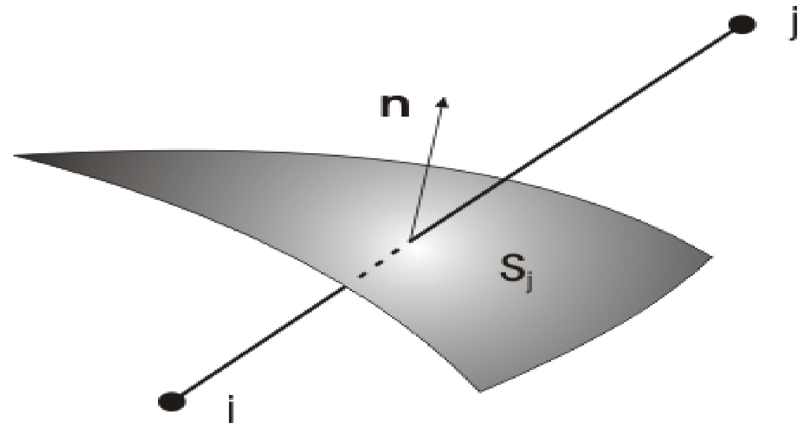
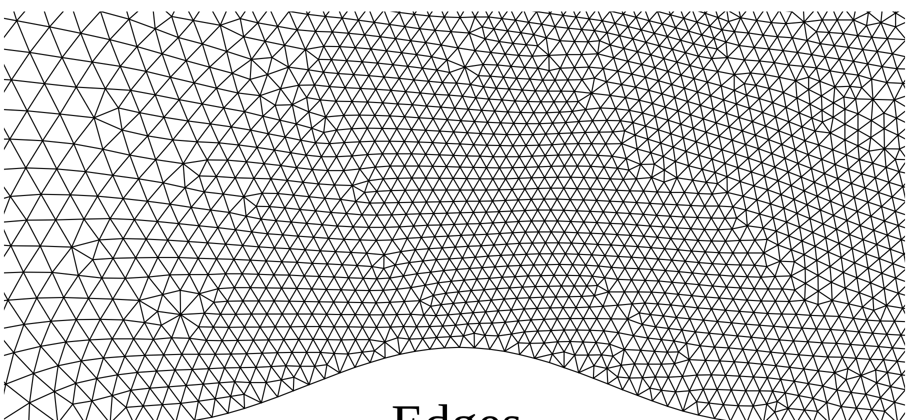
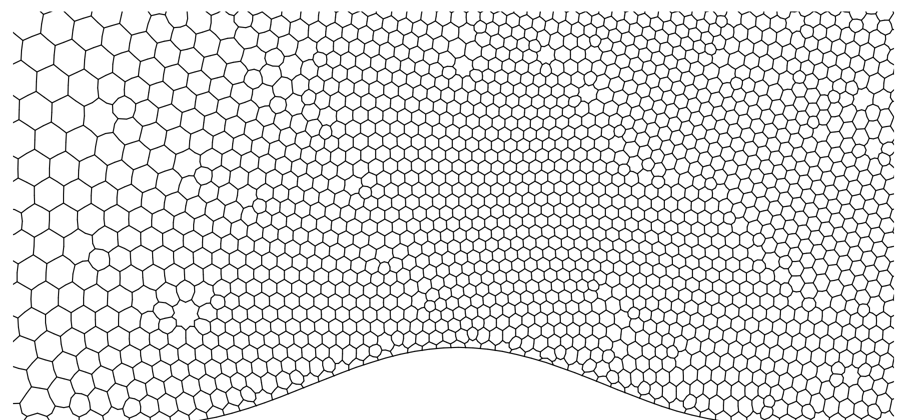






Edges



Dual mesh, FVs

See Szmelter & Smolarkiewicz, *Comp. Fluids* (2011), for our FV solutions to canonical problems of hydrostatic/nonhydrostatic, strongly/weakly stratified, and rapidly/slowly rotating mountain flows.

Nonhydrostatic Boussinesq mountain wave

Szmelter & Smolarkiewicz , *Comp. Fluids*, 2011

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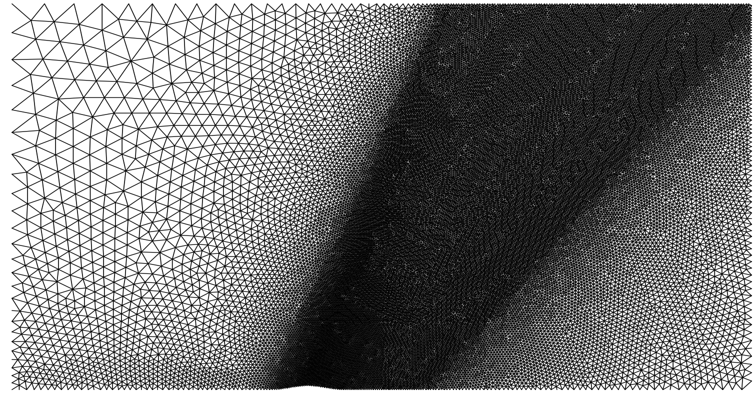
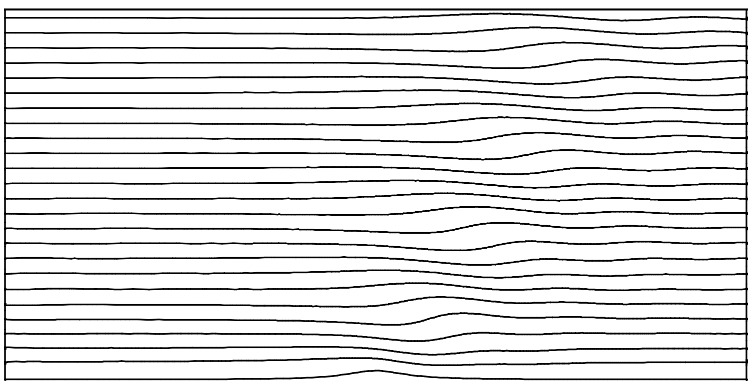
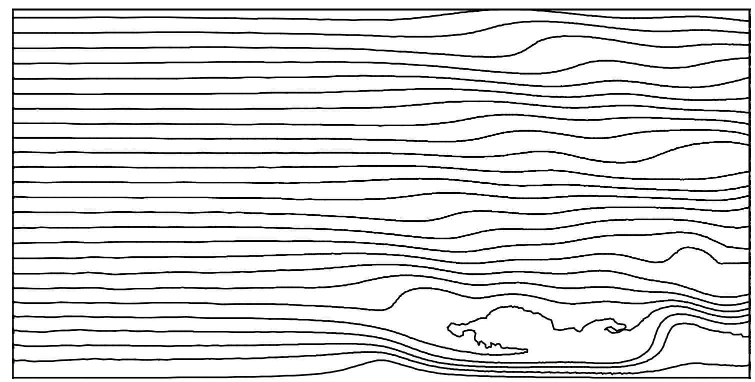
$$\nabla \bullet (\mathbf{V}\rho_o) = 0 \ ,$$

$$\frac{\partial \rho_o V^I}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o V^I) = -\rho_o \frac{\partial \tilde{p}}{\partial x^I} + g\rho_o \frac{\theta'}{\theta_o}\delta_{I2}$$

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o \theta) = 0 \ .$$







$Fr \lesssim 2$ $NL/U_o = 2.4$ $Fr \lesssim 1$;

Comparison with the EULAG's results and the linear theories (Smith 1979, Durran 2003): 3% in wavelength; 8% in propagation angle; wave amplitude loss 7% over 7 wavelenghts

Non-Boussinesq amplification and breaking of deep stratospheric gravity wave

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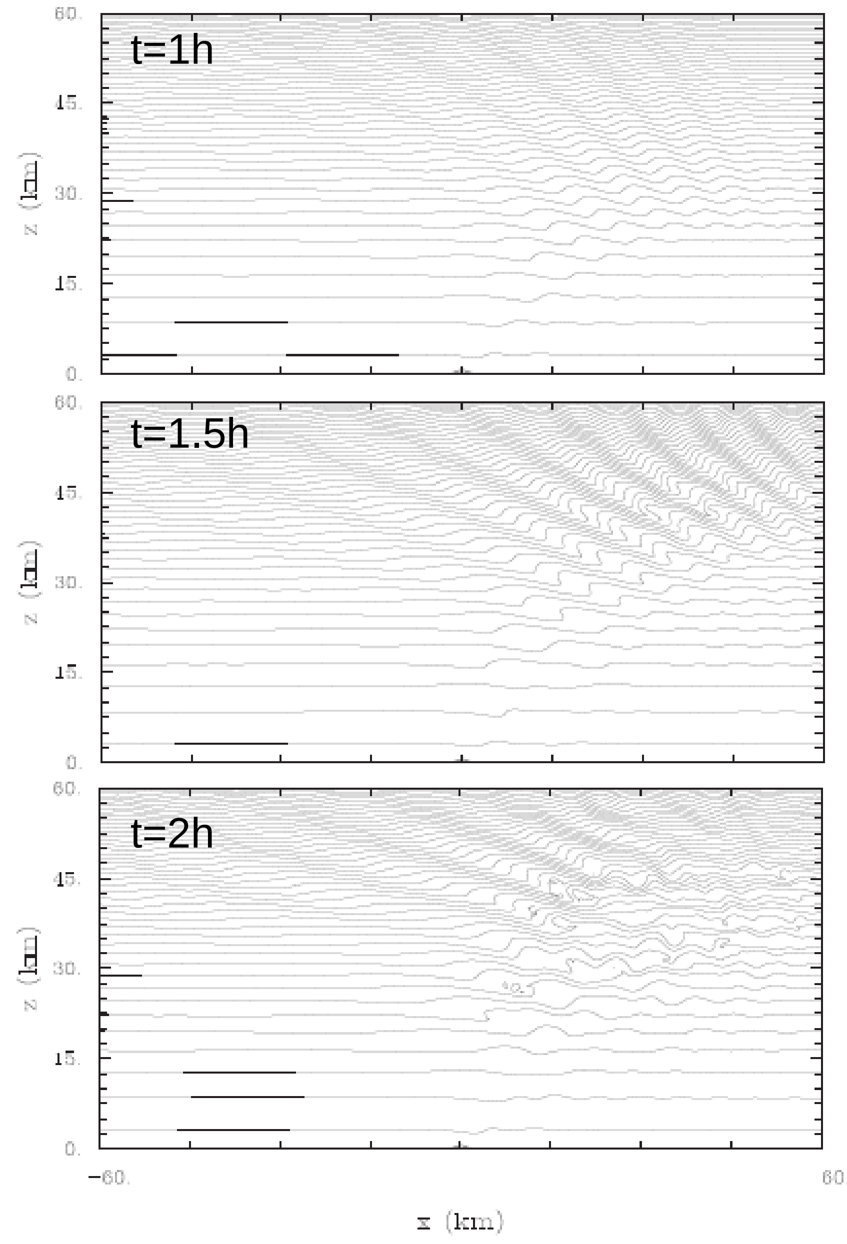
isothermal reference profiles ; $H_\theta = 3.5\, H_\rho$

$NL/U_o \approx 1\,,\;\; Fr \approx 1.6;$
$\lambda_o = 2\pi\ \text{km} \lesssim H_\rho \;\Rightarrow$
$A(H/2) = 10h_o = \lambda_o$

EULAG “reference” solution using terrain-following coordinates ➔



Prusa et al., *JAS* 1996;
Smolarkiewicz & Margolin, *Atmos. Ocean*,1997;
Klein, *Ann. Rev. Fluid Dyn.*, 2010

Soundproof generalizations

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Smolarkiewicz & Szmelter, *Acta Geophysica* (2011)

$$\nabla \cdot (\rho^* \mathbf{v}) = 0\,, \quad \frac{D\theta'}{Dt} = -\mathbf{v} \cdot \nabla \theta_e\,, \quad \frac{D\mathbf{v}}{Dt} = -\Theta \nabla \phi' - \mathbf{g}\Upsilon \frac{\theta'}{\bar{\theta}}$$

For [anelastic, pseudo-incompressible]:

$$\rho^* = [\bar{\rho},\ \bar{\rho}\bar{\theta}/\theta_o];\ \Theta = [1,\ \theta/\theta_o];\ \text{and}\ \Upsilon = [1,\ \bar{\theta}/\theta_e]$$

$$\theta' = \theta - \theta_e \qquad \phi' = [(p - p_e)/\bar{\rho},\ c_p(\pi - \pi_e)\theta_o]$$

$$0 = -\nabla \frac{p_e - \bar{p}}{\bar{\rho}} - \mathbf{g}\frac{\theta_e - \bar{\theta}}{\bar{\theta}} \qquad 0 = -c_p \theta_e \nabla(\pi_e - \bar{\pi}) - \mathbf{g}\frac{\theta_e - \bar{\theta}}{\bar{\theta}}$$

Numerics:

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$$\frac{\partial \rho^* \psi}{\partial t} + \nabla \cdot (\rho^* \mathbf{v} \psi) = \rho^* R \;\Rightarrow\; \psi_i^{n+1} = \mathcal{A}_i(\tilde{\psi}, \mathbf{v}^{n+1/2}, \rho^*) + 0.5\delta t R_i^{n+1} \;\equiv\; \widehat{\psi}_i + 0.5\delta t R_i^{n+1}$$

$$u = \hat{u} - \delta_h t\, \Theta\, \partial_x \phi' - \delta_h t\, \alpha(u - u_e)$$

$$w = \hat{w} - \delta_h t\, \Theta\, \partial_z \phi' + \delta_h t\, \beta \theta' - \delta_h t\, \alpha(w - w_e)$$

$$\theta' = \hat{\theta}' - \delta_h t\, w\, \partial_z \theta_e - \delta_h t\, \alpha' \theta' ,$$

$$u = \hat{\hat{u}} - C^{xx} \partial_x \phi' \qquad \theta' = \frac{\hat{\theta}' - \delta_h t\, w\, \partial_z \theta_e}{1 + \delta_h t\, \alpha'}$$

$$w = \hat{\hat{w}} - C^{zz} \partial_z \phi'$$

$$\hat{\hat{u}} = \frac{\hat{u} + \delta_h t\, \alpha u_e}{1 + \delta_h t\, \alpha}, \quad \hat{\hat{w}} = \frac{\hat{w} + \delta_h t\, \alpha w_e + \delta_h t\, \beta\, \hat{\theta}' \,(1 + \delta_h t \alpha')^{-1}}{(1 + \delta_h t\, \alpha)(1 + \delta_h t\, \alpha') + (\delta_h t)^2 \beta\, \partial_z \theta_e \,(1 + \delta_h t \alpha')^{-1}}$$

$$C^{xx} = \frac{\delta_h t\, \breve{\Theta}}{(1 + \delta_h t\, \alpha)}, \quad C^{zz} = \frac{\delta_h t\, \breve{\Theta}}{(1 + \delta_h t\, \alpha)(1 + \delta_h t\, \alpha') + (\delta_h t)^2 \beta\, \partial_z \theta_e \,(1 + \delta_h t \alpha')^{-1}}$$

$$\frac{1}{\rho^*}\left[\partial_x \rho^* (\hat{\hat{u}} - C^{xx} \partial_x \phi') + \partial_z \rho^* (\hat{\hat{w}} - C^{zz} \partial_z \phi')\right] \equiv -(\mathcal{L}\phi' - \mathcal{R}) = 0$$

unstructured-mesh "EULAG" solution, using mesh (left) mimicking terrain-following coordinates, and (right) a fully unstructured mesh

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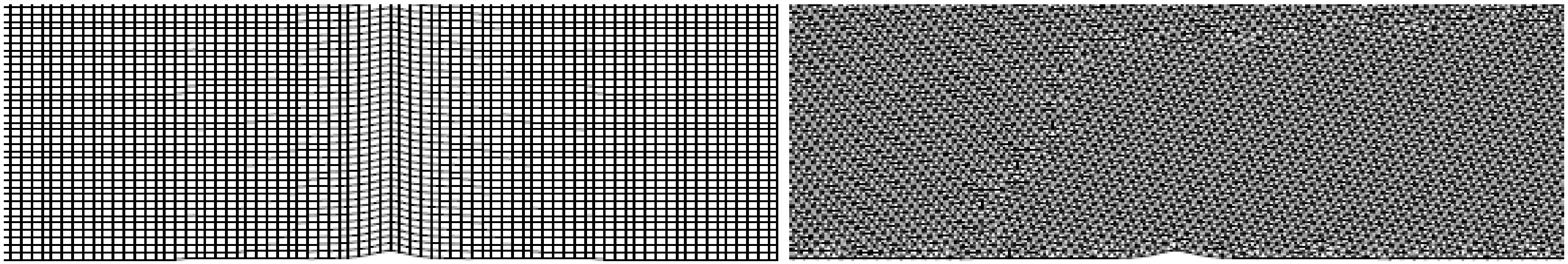
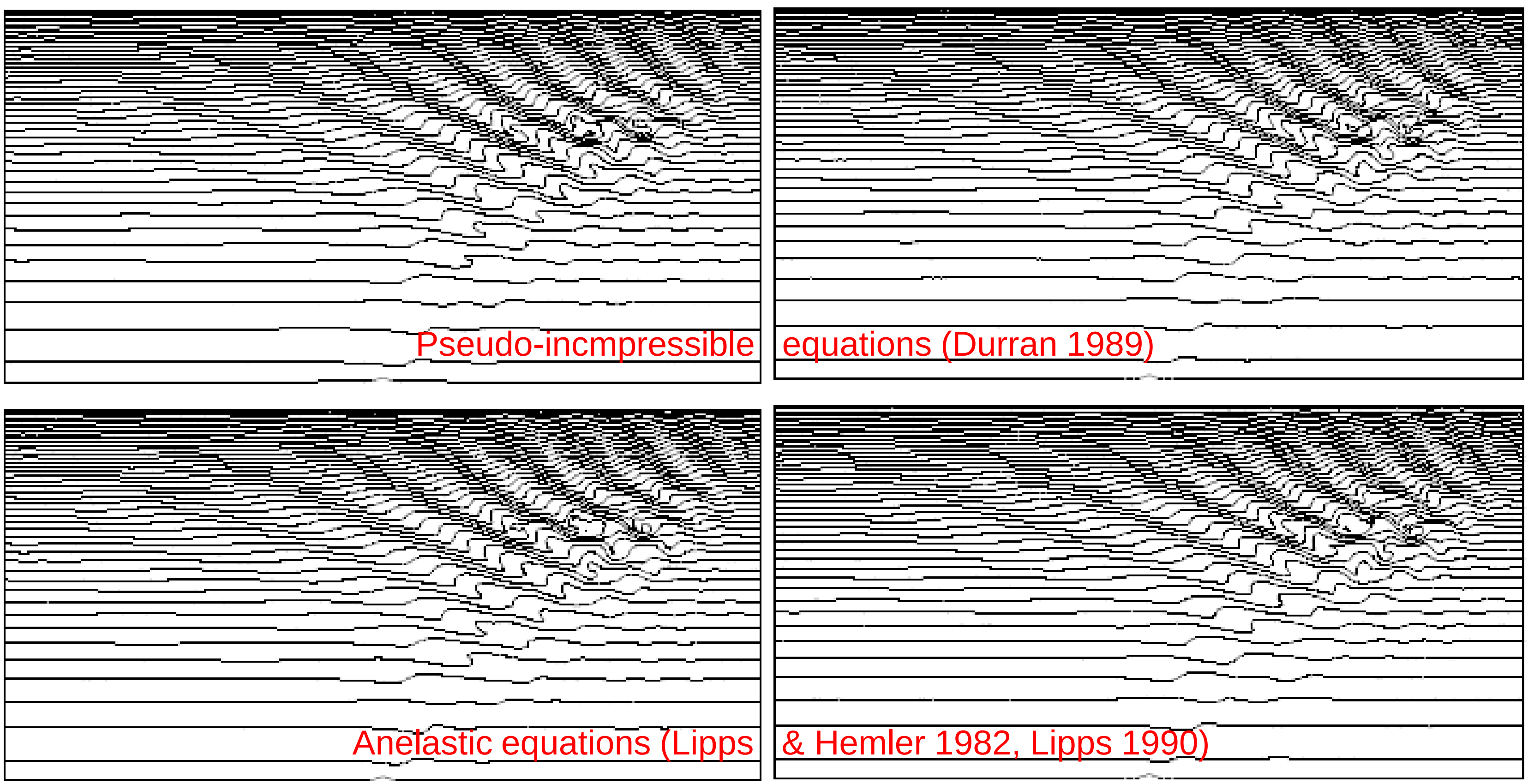


t=1.5h



$H_\theta \lesssim H_\rho$ (RE: Achatz et al., *JFM*, 2010)

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t=0.75h

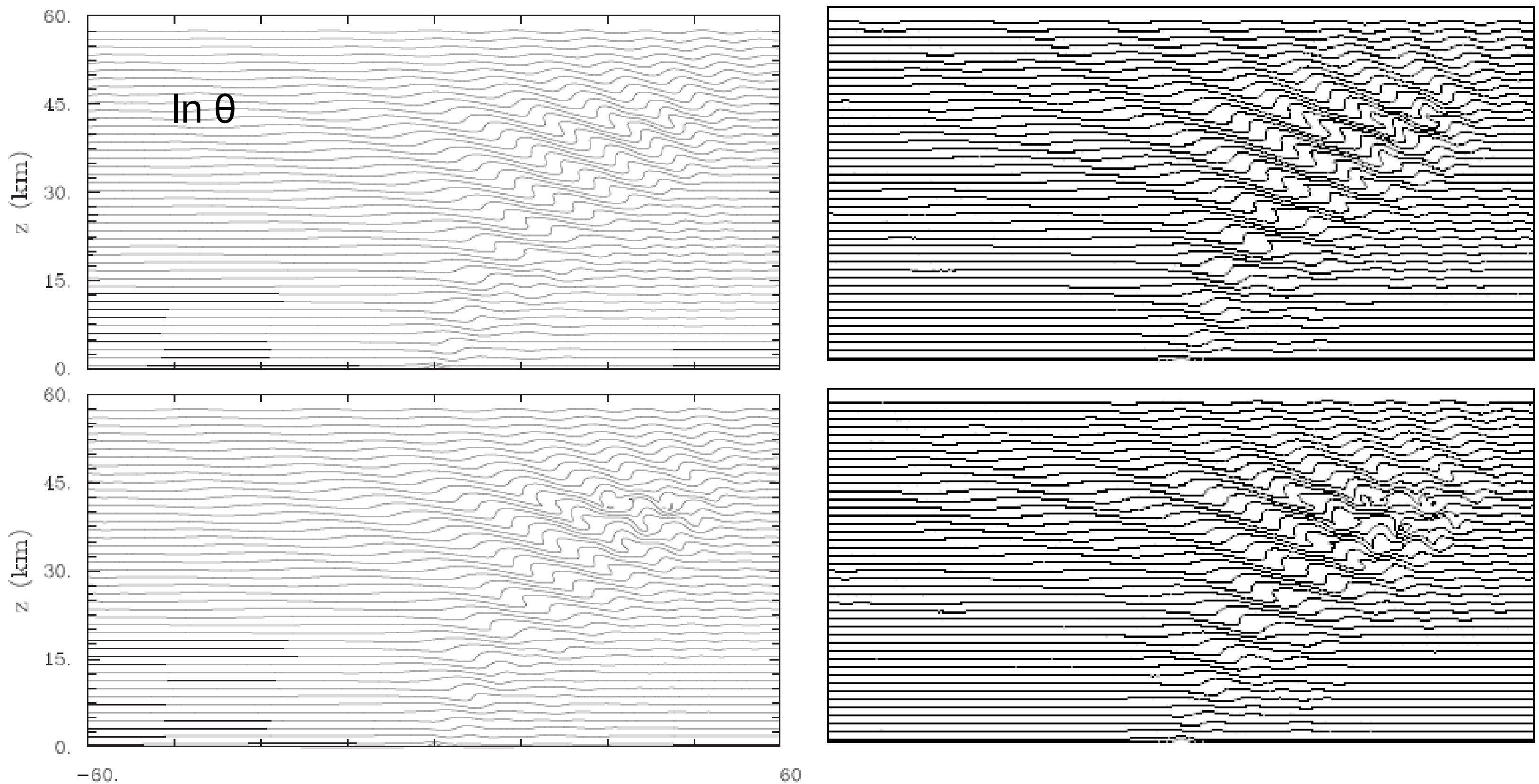

Table 1: Normalized vorticity: maximum, minimum, mean and standard deviation.

eqs.	*numerics*	$\max(\omega)$	$\min(\omega)$	$\overline{\omega}$	$\overline{(\omega-\overline{\omega})^2}^{1/2}$
PSI	CV/grid	0.17	-0.21	$1.6 \cdot 10^{-4}$	$3.3 \cdot 10^{-2}$
ANL	CV/grid	0.27	-0.41	$6.4 \cdot 10^{-5}$	$3.5 \cdot 10^{-2}$
PSI	CV/mesh	0.28	-0.24	$2.0 \cdot 10^{-4}$	$3.7 \cdot 10^{-2}$
ANL	CV/mesh	0.24	-0.36	$9.5 \cdot 10^{-5}$	$3.6 \cdot 10^{-2}$
PSI	SL/grid	0.28	-0.30	$2.1 \cdot 10^{-4}$	$3.1 \cdot 10^{-2}$
ANL	SL/grid	0.18	-0.24	$7.2 \cdot 10^{-5}$	$3.0 \cdot 10^{-2}$

Unstructured-mesh framework for atmospheric flows

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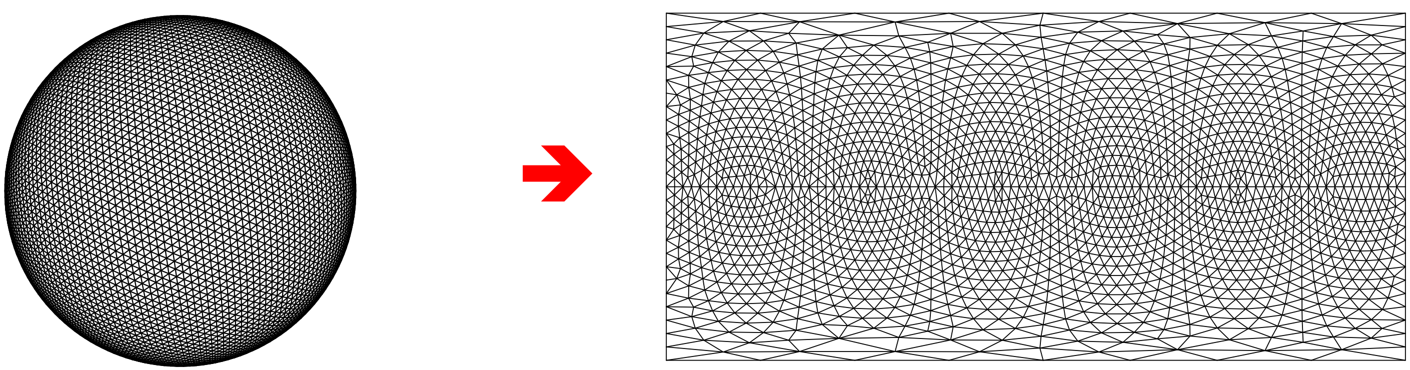
Smolarkiewicz ↻ Szmelter, pubs in *JCP, IJNMF,Comp. Fluids*, 2005-2011

- Differential manifolds formulation $$\frac{\partial G\boldsymbol{\Phi}}{\partial t} + \nabla \cdot (\mathrm{V}\boldsymbol{\Phi}) = G\boldsymbol{\mathcal{R}}\ , \quad \mathrm{V}(\mathrm{x},t) := G\dot{\mathrm{x}}$$

- Finite-volume NFT numerics with a fully unstructured spatial discretization, heritage of EULAG and its predecessors *(A = MPDATA)*

$$\boldsymbol{\Phi}_i^{n+1} = \mathcal{A}_i(\boldsymbol{\Phi}^n + 0.5\delta t\, \boldsymbol{\mathcal{R}}^n,\ \mathrm{V}^{n+1/2}, G) + 0.5\delta t\, \boldsymbol{\mathcal{R}}_i^{n+1}$$



- Focus (so far) on wave phenomena across a range of scales and Mach, Froude & Rossby numbers

Stratified (mesoscale) flow past an isolated hill on a reduced planet

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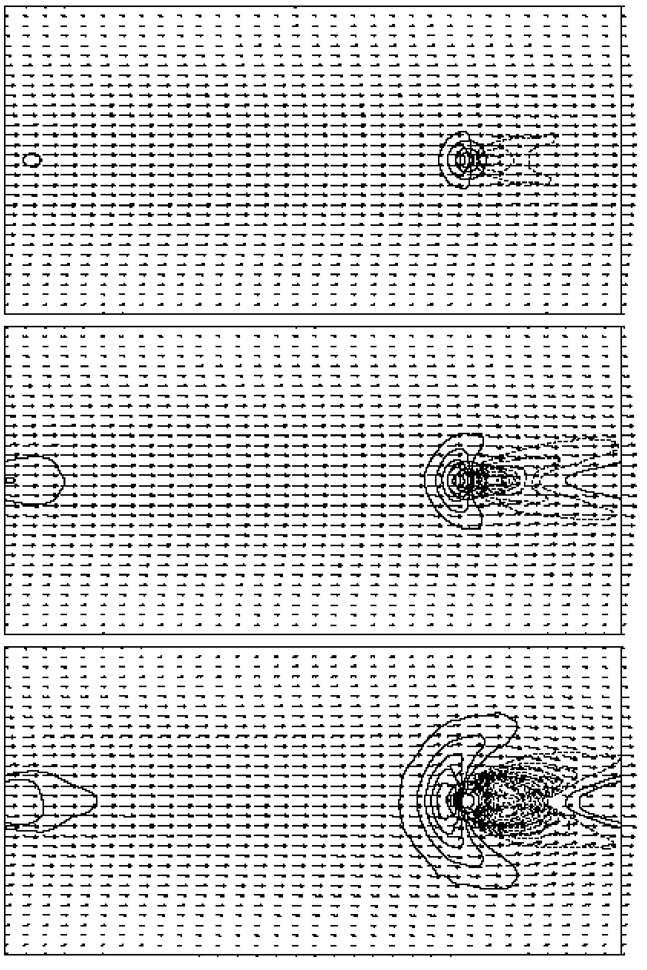
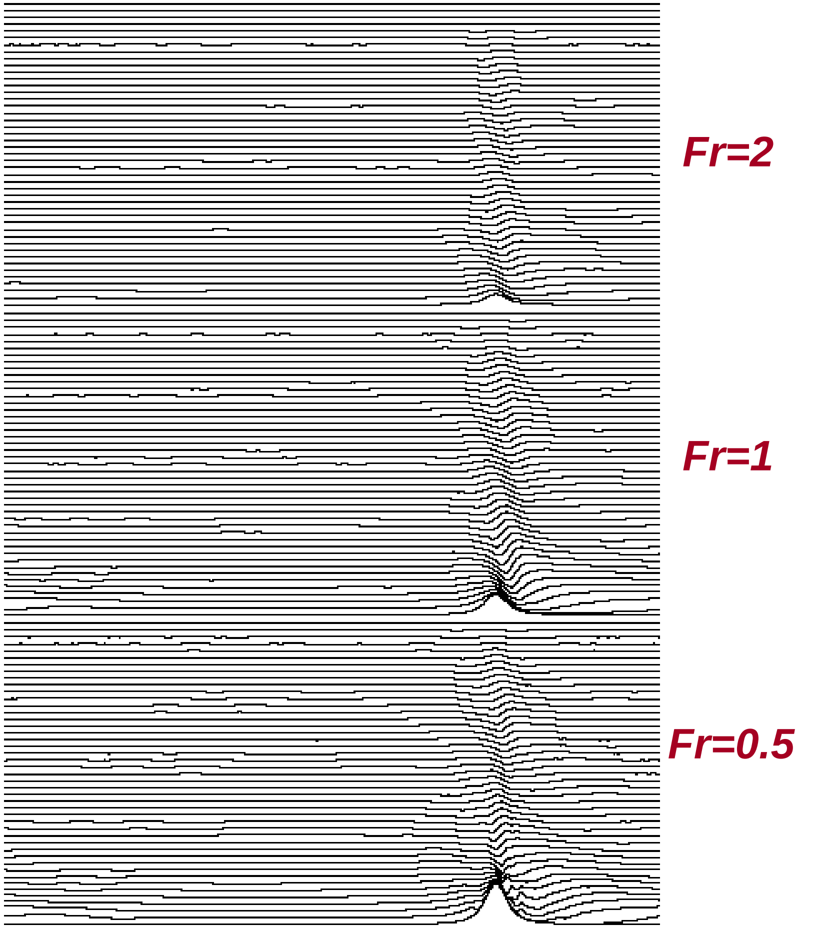
4 hours



$Fr = U_0/Nh$



Hunt & Snyder J. Fluid Mech. 1980; Smolar. & Rotunno, J. Atmos. Sci. 1989 ; Wedi & Smolar., QJR 2009

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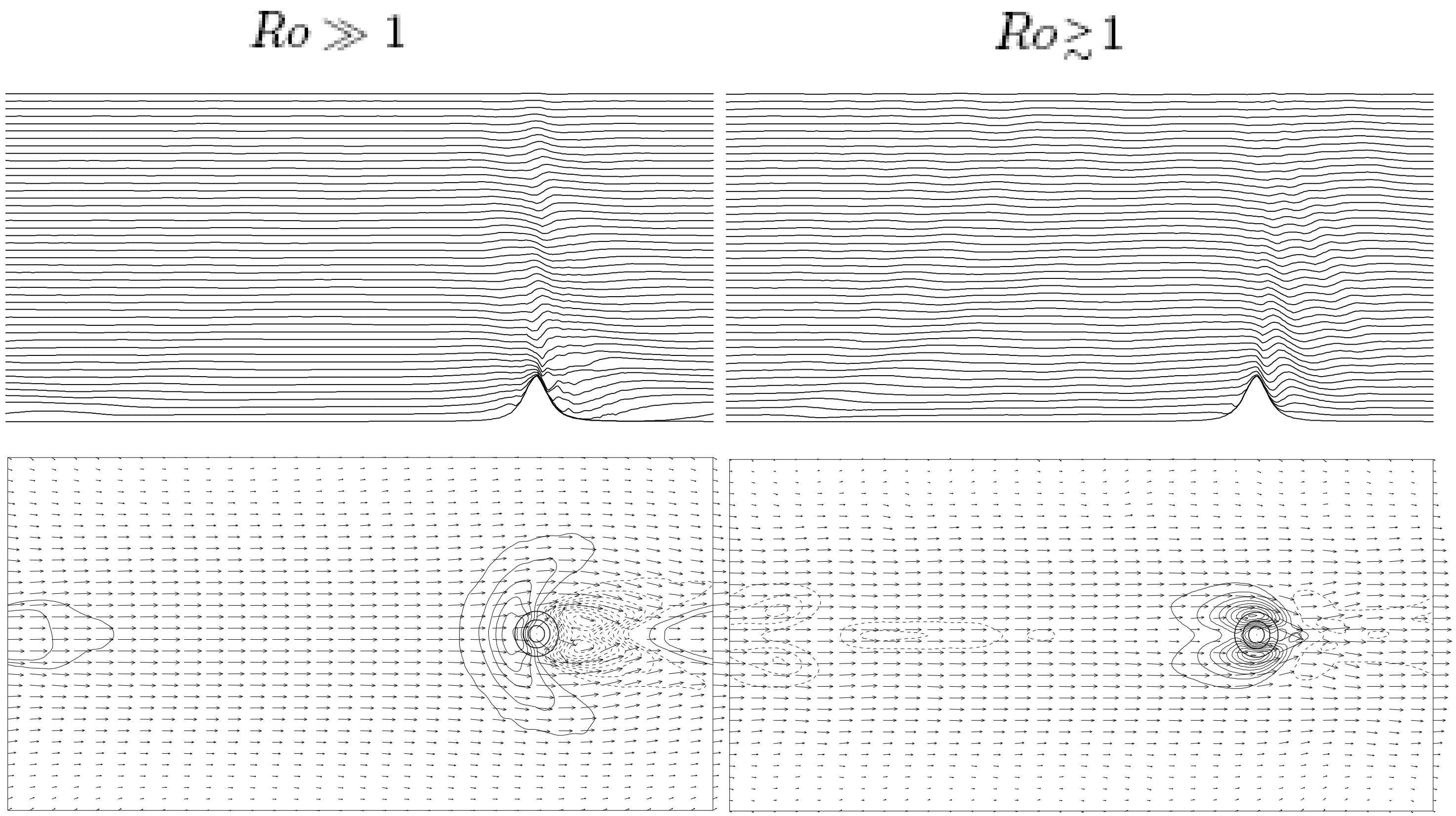
$Fr=0.5$



$h_\zeta=8^{-1}\lambda_o$

Smith, Advances in Geophys 1979; Hunt, Olafsson & Bougeault, QJR 2001

Conclusions:

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Unstructured-mesh discretization sustains the accuracy of structured-grid discretization and offers full flexibility in spatial resolution.

Soundproof models appear effective for a range of atmospheric flows much broader than thought the decade ago, while having considerable advantages over fully compressible models.