The use of vertical electron density profiles to determine key parameters of the Chapman function for ionosphere modeling

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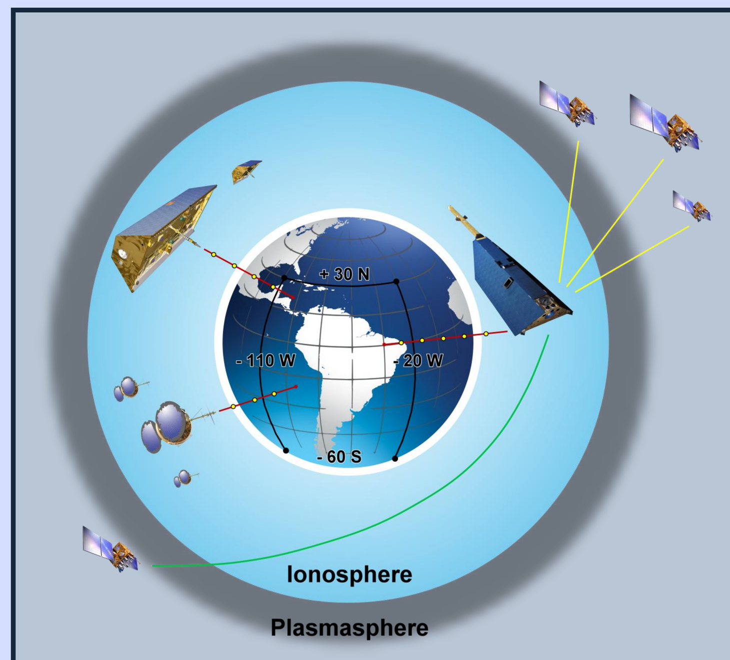
1. Motivation



Near real-time resolution and high precision ionosphere models are used for a large number of applications e.g. in navigation, positioning, telecommunications or astronautics. In order to understand the complex phenomena in the ionosphere an electron density (Ne) model based on the physics motivated Chapman function will be developed by a joint cooperation of IAPG, DGFI and DLR in the context of the project "**Mu**lti-**S**cale model of the **I**onosphere from the **C**ombination of modern space-geodetic satellite techniques", [1].

One task of the project concerns the implementation of **vertical electron density profiles** to overcome the insensitivity of ground-based GNSS to the radial geometry. Ne profiles are derived from radio-occultation measurements with respect to low-earth orbiter (LEO) satellites such as CHAMP, GRACE or COSMIC/FORMOSAT-3. The area of investigations is located in South America where exceptional phenomena like the equatorial anomaly can be observed.

Satellite images, Fig. (1):
http://www.pcrf.org
http://www.ucar.edu
http://grace.jpl.nasa.gov

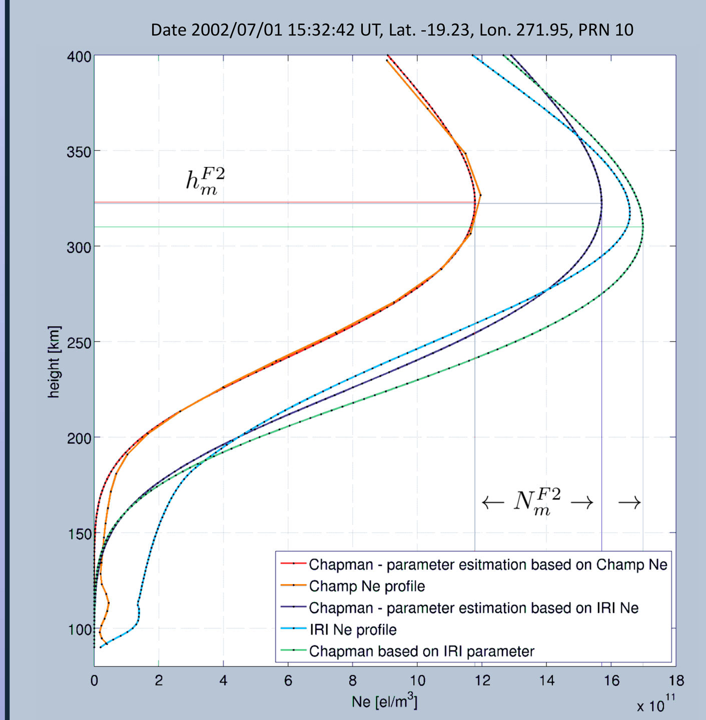
Fig. 1: Schematic representation of an observation scenario with CHAMP, GRACE and COSMIC/FORMOSAT-3. — GPS to LEO SST tracking, — Occultation measurement, — Vertical Ne bottomside profile, ∘ Ne measurement.

2. Model description

The **Chapman layer** function is efficient for describing the vertical structure of the electron density. It is **physics-motivated** and primarily relies on the recombination process of free electrons with positive ions to atoms or neutral molecules. Combining a F2-Chapman layer with a plasmasphere profile yields the fundamental observation model

$$\begin{aligned} Ne(h) &= Ne^{F2}(h) + Ne^{P}(h) \\ &= N_m^{F2}\exp\left(\frac{1-z-exp(-z)}{2}\right) + N_0^P \exp\left(\frac{-|h-h_m^{F2}|}{H^P}\right) \end{aligned} \tag{1}$$

$$\text{with } z = \frac{h-h_m^{F2}}{H^{F2}} \text{ and } H^P = \begin{cases} 10^4\ km, & \text{if } h \geq h_m^{F2} \\ 10\ km, & \text{else} \end{cases} \tag{2}$$



wherein N_m^{F2} and h_m^{F2} are **F2-peak density** and **peak height** and H^{F2} indicates the **F2-scale height**. N_0^P and H^P substitute **plasmasphere basis density** and **scale height**, respectively.

Fig. 2: Comparison of a priori, measured and adjusted profiles for a particular day and location:
1.) Chapman profile based on the estimation of the three F2 key parameters using the CHAMP Ne profile (red);
2.) Radio occultation measurements GPS PRN10 – CHAMP (orange);
3.) Chapman profile based on the estimation of the three F2 key parameters using IRI Ne data (purple);
4.) IRI Ne profile (blue);
5.) Chapman profile based on IRI parameters (green)

Each of the five key parameters described in Eq. (1) is modeled in a (3D) series expansion in terms of **endpoint-interpolating polynomial B-splines**

$$\Theta_i = \sum_{k_1=1}^{K_1}\sum_{k_2=1}^{K_2}\sum_{k_3=1}^{K_3} d_{k_1,k_2,k_3}^{J_1,J_2,J_3}|_i \Phi_{k_1}^{J_1}(\lambda)\Phi_{k_2}^{J_2}(\varphi)\Phi_{k_3}^{J_3}(t) \tag{3}$$

depending on longitude λ, latitude φ and time t. $\Theta_i, (i=1,...,5)$, specifies the key parameter and $J_{1,2,3}$ gives the B-spline level indicating the number $K=2^J+2$ of **scaling coefficients** $d_{k_1,k_2,k_3}^{J_1,J_2,J_3}$, see [2] and [3].

3. Initialization

Caused by the exponential term of Eq. (1), linearization of the observation model is mandatory and consequently initial values for the scaling coefficients are required. The availability of a **background model**, e.g. IRI-2007, (indirectly) providing initial values for the key parameters enables the estimation of initial coefficients

$$\hat{\boldsymbol{d}}_{i,0} = \hat{\boldsymbol{d}}_{i,IRI} = (\boldsymbol{A}_{IRI}^T\boldsymbol{A}_{IRI})^{-1}\boldsymbol{A}_{IRI}^T\boldsymbol{l}_i\ . \tag{4}$$

$\boldsymbol{A}_{IRI}$ is the design matrix and $\boldsymbol{l}_i$ the observation vector covering initial values of specified key parameters derived from the background model.

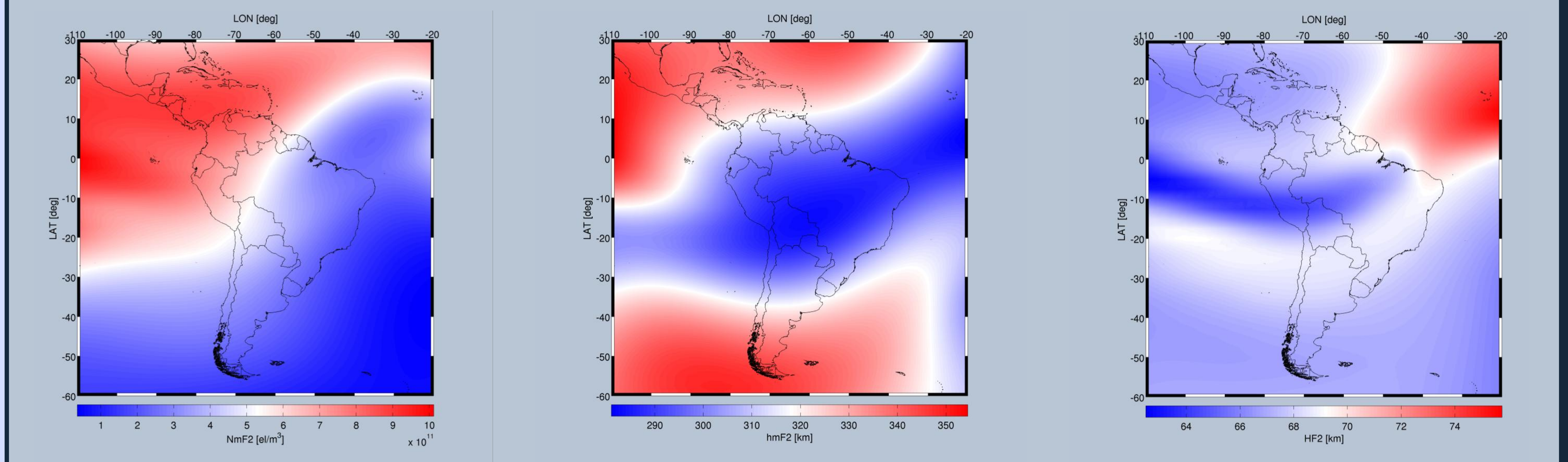


Fig. 3: $N_m^{F2}, h_m^{F2}, H^{F2}$ reconstructed for 2002/07/01 – 6:30 UT from scaling coefficients $\hat{\boldsymbol{d}}_0$ based on levels $J_1=4$ (longitude), $J_2=4$ (latitude) and $J_3=3$ (time) initially estimated from IRI model data(top).

4. Parameter estimation

Based on the set of initial coefficients, Eq. (4), simulated Ne observations are computed by $\boldsymbol{Ne}^{\text{sim}} = \boldsymbol{Ne}(\Theta(\boldsymbol{d}_0+\boldsymbol{d}_{\text{random}}))|_{\text{model}} + \text{noise}$ where randomly generated coefficients are added to simulate corrections. The aim of the simulation process is to recover the "true" values $\boldsymbol{d}_{true} = \boldsymbol{d}_0 + \boldsymbol{d}_{random}$.

The applied observation equation leads to the Gauss-Markov model

$$\boldsymbol{Ad} = \boldsymbol{l} + \boldsymbol{e} \quad \text{and} \quad \boldsymbol{D}(\boldsymbol{l}) = \sigma^2\boldsymbol{P}^{-1} \tag{5}$$

where $\boldsymbol{e}$ are observation errors, $\boldsymbol{d}$ indicates the vector of unknown scaling coefficients, σ^2 the unknown variance factor and $\boldsymbol{P}$ is the given positive definite weight matrix of the observations $\boldsymbol{l}$.

Linearization of the observation model, Eq. (1), is realized by a Taylor expansion

$$\boldsymbol{Ne}(h,\boldsymbol{d}_0+\boldsymbol{\Delta d}) = \boldsymbol{Ne}(h,\boldsymbol{d}_0) + \sum_{i=1}^{5}\left(\left[\frac{\partial \boldsymbol{Ne}(h)}{\partial\Theta_i}\bigg|_{\Theta_i=\Theta_{i,0}}\right]\left[\frac{\partial\Theta_i}{\partial \boldsymbol{d}_i}\bigg|_{\boldsymbol{d}_i=\boldsymbol{d}_{i,0}}\right]\right)\boldsymbol{\Delta d}_i\ . \tag{6}$$

The estimation of corrections for the initial coefficients yields

$$\boldsymbol{\Delta}\hat{\boldsymbol{d}} = (\boldsymbol{A}^T\boldsymbol{PA}+\lambda\boldsymbol{P}_w)^{-1}(\boldsymbol{A}^T\boldsymbol{PL}+\lambda\boldsymbol{P}_w\boldsymbol{\mu})\ . \tag{7}$$

$\boldsymbol{L} = \boldsymbol{Ne}(h)|_{\text{observed}} - \boldsymbol{Ne}(h,\boldsymbol{d}_0)|_{\text{modeled}}$ is the reduced observation vector and $\boldsymbol{P}_w$ is the weighting matrix of the unknowns where $\boldsymbol{w}$ indicates the prior information. To bridge data gaps a regularization parameter λ is introduced, [4]. To constrain the solve-for coefficients to the original a priori coefficients prior information has to be adapted in each iteration step by $\boldsymbol{\mu} = \boldsymbol{w} - \sum_{q=0}^{q_I-1}\boldsymbol{\Delta}\hat{\boldsymbol{d}}_q$ where q_I substitutes the current iteration step. Assuming Q to be the total number of iterations a solution for the set of final coefficients can be found with

$$\hat{\boldsymbol{d}} = \boldsymbol{d}_Q + \boldsymbol{\Delta}\hat{\boldsymbol{d}}_Q = \boldsymbol{d}_0 + \textstyle\sum_{q=0}^{Q}\boldsymbol{\Delta}\hat{\boldsymbol{d}}_q\ . \tag{8}$$

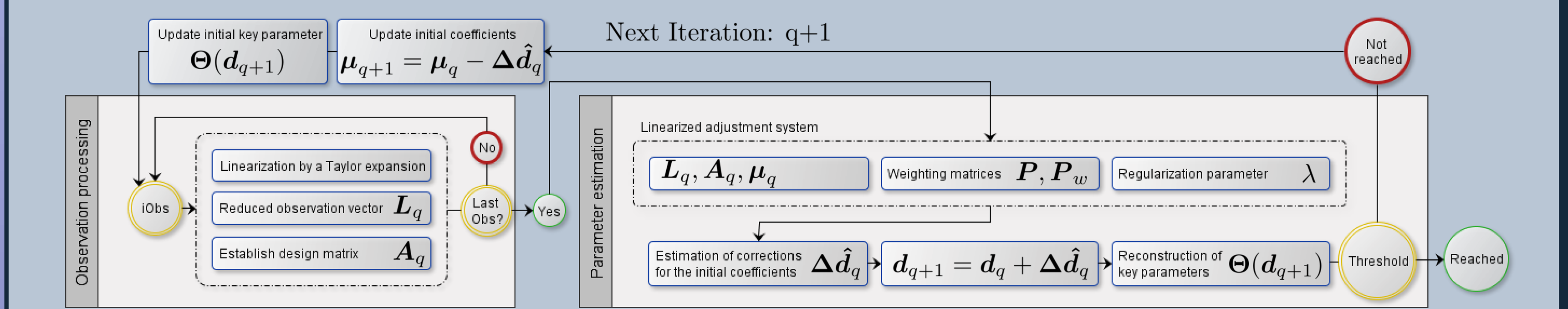


Fig. 4: Flowchart of the iterative estimation of scaling coefficients.

5. Simulation

The following results are based on simulated Ne measurements homogeneously distributed on a grid characterized by

- [250° : 5° : 340°] longitude,
- [-60° : 5° : 30°] latitude and
- [6:00 UT : 6 min : 7:00 UT] time

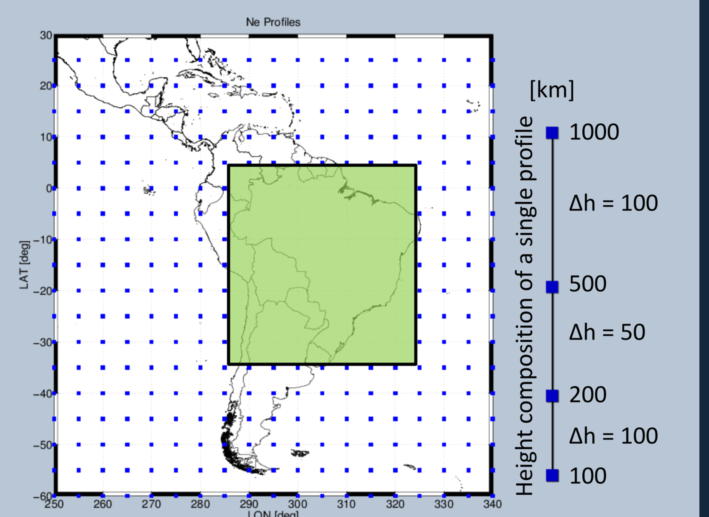


Fig. 5: Scenario with observation gap.

resolution with varying height levels, Fig (5).
An exclusion zone without any observations at no time is included (green area).
B-spline levels $J_1=4,\ J_2=4$ and $J_3=3$ have been taken into account. As preliminary results only scaling coefficients for N_m^{F2} and H^{F2} have been estimated. All results visualized in Fig. (6-8) are exemplary for H^{F2} dated to a specific epoch.

ΔHF2 rms (Fig. 6)	8.165 km
ΔNmF2 rms	$8.672 \cdot 10^9$ el/m³

ΔHF2 rms (Fig. 8)	0.193 km
ΔNmF2 rms	$2.266 \cdot 10^8$ el/m³

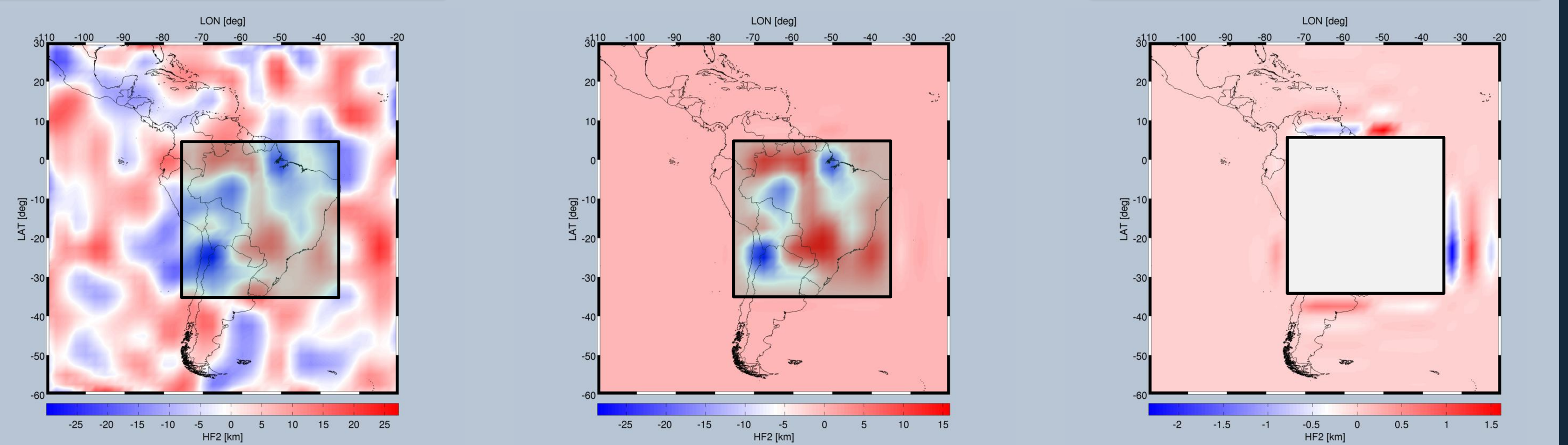


Fig. 6: Simulated „true" deviation from initial state of H^{F2}; Fig. 7: „Truth" – Estimation after iteration q = 3; Fig. 8: „Truth" – Estimation after iteration q = 3, supported coefficients only.

6. Outlook

So far, the parameter determination relies on the estimation of coefficients related to N_m^{F2} and H^{F2}. The upcoming steps will focus on the **determination of all five key parameters.** The use of slant total electron content (STEC) measurements with Ne profiles and **combination of different observation techniques** such as GNSS, altimetry and radio occultation will help to improve the system stability.

References: **[1]** Schmidt M., Hugentobler U., Jakowski N., Dettmering D., Limberger M., Liang W., Wilken V., Gerzen T., Hoque M., Berdermann J.: Multi-scale model of the ionosphere from the combination of modern space-geodetic satellite techniques – project status and first results; EGU 2012. **[2]** Schmidt M.: Wavelet modelling in support of IRI; Adv. in Space Res. V 39 I 5; 2007. **[3]** Schmidt M., Dettmering D., Mößmer M., Wank Y., Zhang J.: Comparison of spherical harmonic and B-spline models for the vertical total electron content; Radio Science V 46; 2011. **[4]** Koch K.-R., Kusche J.: Regularization of geopotential determination from satellite data by variance components; Journal of Geodesy ; 2002.

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