

# New Cointegration Methods for Detection and Attribution of Climate Trends

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## Detection

IPCC defined **detection** of climate change as a process of demonstrating that climate or a system affected by climate has changed in some defined statistical sense without providing a reason for that change

## Attribution

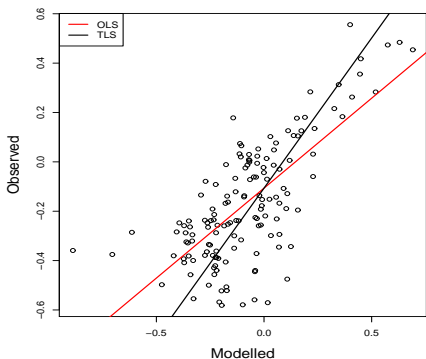
**Attribution** is a process of evaluating the relative contributions of multiple causal factors to a change or event with an assignment of statistical confidence.

Reliable detection and attribution of changes in climate is fundamental in enabling decision makers to manage climate-related risk (Hegerl et al., 2010).

## Existing regression approach

The ordinary least squares (OLS) and/or total least squares (TLS) regression methods have been employed in detection-attribution studies (Allen and Stott, 2003; Stott et al., 2003).

The OLS and TLS fits



$$\left. \begin{array}{l} \text{OLS : } \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}_t \\ \text{TLS : } \mathbf{y} = (\mathbf{X} - \mathbf{w})\boldsymbol{\beta} + \boldsymbol{\epsilon}_t \end{array} \right\} \begin{array}{l} \mathbf{y} - \text{observed, } \mathbf{X} - \text{modelled,} \\ \boldsymbol{\epsilon} - \text{climate noise \& } \mathbf{w} - \text{model uncertainty} \end{array}$$

Detection: reject  $H_0 : \boldsymbol{\beta} = \mathbf{0}$

Attribution: reject  $H_0 : \beta_f = 0$  for a particular forcing  $f$

## Why New Method?

- Most climatic time series are non-stationary.
- Non-stationary time series can lead to spurious regressions (Granger and Newbold, 1974).
- Better to see if one time series  $x$  can be used to detrend another time series  $y$
- Vector autoregressive (VAR) models are useful in capturing the linear interdependencies among multiple time series.

## Cointegrating time series model: VAR(2) model

### VAR(2)

For a bivariate vector  $\mathbf{z}_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}$ , we use a VAR(2) model:

$$\mathbf{z}_t = \mathbf{\Pi}_1 \mathbf{z}_{t-1} + \mathbf{\Pi}_2 \mathbf{z}_{t-2} + \boldsymbol{\epsilon}_t, \quad t = 1, 2, \dots, T$$

### VECM

The vector error correction model, VECM

$$\Delta \mathbf{z}_t = \mathbf{\Pi} \mathbf{z}_{t-1} + \mathbf{\Gamma} \Delta \mathbf{z}_{t-1} + \boldsymbol{\epsilon}_t$$

is estimated using Johansen's maximum likelihood method.

Given the rank of  $\mathbf{\Pi}$  matrix is 1, the long-run coefficient  $\boldsymbol{\beta}$  can be estimated from

$$\mathbf{\Pi} = \boldsymbol{\alpha} \boldsymbol{\beta}'$$



## Synthetic and Real Data

Simple “toy” models are used to generate synthetic data.

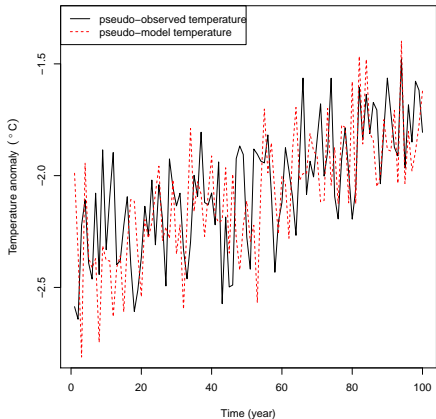
$$\left. \begin{aligned} c_t &= e + c_{t-1} + \varepsilon_t \\ y_t &= \alpha_0 + \alpha_1 c_t + \xi_t \\ x_t &= \gamma_0 + \gamma_1 c_t + \nu_t \end{aligned} \right\} \begin{aligned} c_t &- CO_2 \text{ in year } t, e - \text{emissions}, \\ y_t &- \text{observed temperature}, x_t - \text{modelled temp} \end{aligned}$$

Parameters estimated using the 20<sup>th</sup> century annual observation from HadCRUT3 and GISS model simulations.

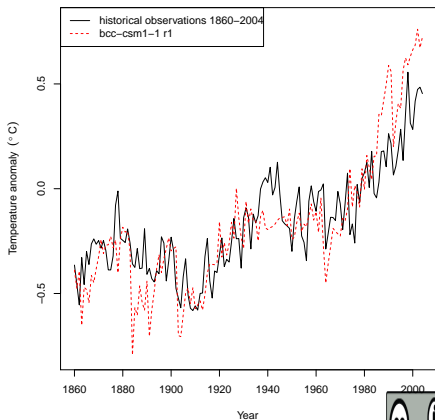
Historical near surface temperature (Brohan et al., 2006) and 16 of the models included in CMIP5 simulations are also used.

# Time series of synthetic and real data

## Example of synthetic data



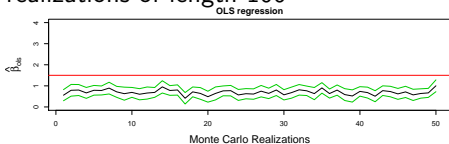
## Historical global mean temperature and example of CMIP5 simulation



⇒ Non-stationary

# Estimates for 50 MC realizations

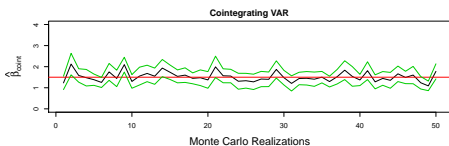
Estimates for 50 Monte Carlo realizations of length 100



Negatively biased?



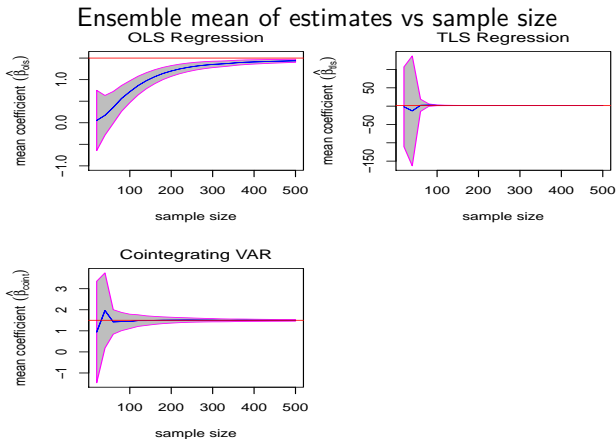
Less efficient?



Less biased and more efficient?



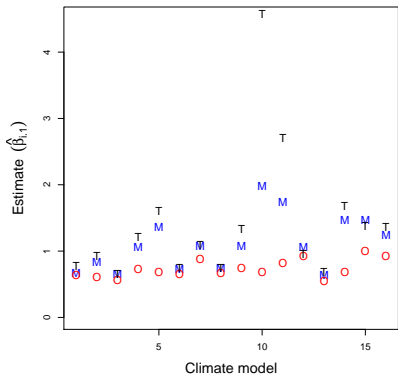
## Sampling properties of the estimators



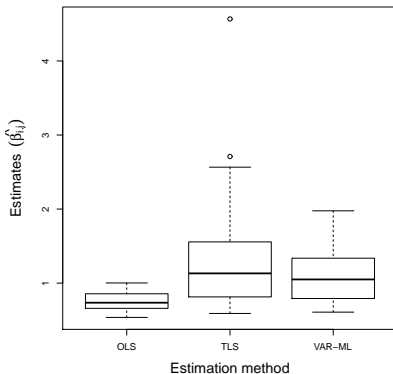
⇒ OLS-biased, TLS-very uncertain, VAR-ML-better than the two

## Distribution of estimates: using real observations and CMIP5 simulations

Estimates—one run from each model: O-OLS, M-VAR-ML, and T-TLS



Distribution of estimates: for all realizations



- ⇒ The cointegrating VAR estimates are not biased to either extremes.
- ⇒ The pattern/distribution of estimates do back results from synthetic data.

## Summary

- The static regression methods could end-up with spurious results for non-stationary climatic variables- high risk of misleading policy makers.
- The dynamic VAR based MLE are less biased and more efficient than those of the OLS and TLS estimates of static regression.
- We can do better with cointegrating VAR method in detection and attribution studies.



End

# Thank you

## Questions?

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