Drought assessment in the Duero basin (Central Spain) by means of multivariate extreme value statistics

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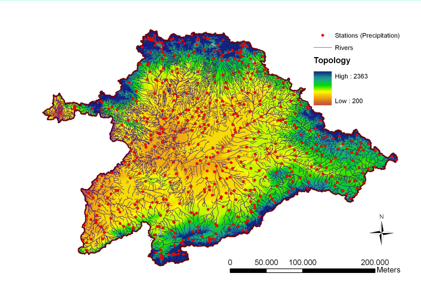
The Duero basin

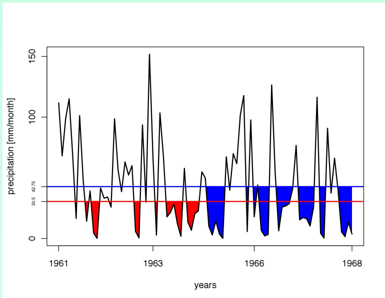


•Arid /semi-arid region. On average 53mm/month (1.72mm/day) precipitation.

•Most precipitation in the area of surrounding mountains.

Most agriculture in topographic depression in the center (mostly non-irrigated). Irrigation season: Mai-October.

Drought definition



Droughts are defined by excesses of run-sum deficits of monthly precipitation.

The data are stemming from the MOPREDAS database [González-Hidalgo et al., 2010]. Time: 1946-2005.

30.5mm/month (1mm/day) and 42.7mm/month (1.4mm/day) are critical levels for agriculture and are therefore chosen to define droughts.

Table 1. Drought Definition Levels and according Drought Characteristics on Average (Minimum – Maximum) over all Stations and Years.

Level [mm/month]	Drought Length [month]	Drought Number	Drought Deficit [mm/drought length]
30.5 (all year)	**2.00** (1.3 – 3.7)	**147** (60 – 180, 1.0 – 3.0 per year)	**33.73** (17.74 – 68.13)
30.5 (irrigation period)	**2.40** (1.3 – 3.7)	**68** (48 – 79, 0.8 – 1.3 per period)	**43.59** (18.05 – 101.93)
42.7 (all year)	**2.73** (1.4 – 5.5)	**147** (83 – 172, 1.3 – 2.8 per year)	**64.21** (27.13 – 151.29)
42.7 (irrigation period)	**3.10** (1.5 – 5.7)	**67** (46 – 81, 0.8 – 1.3 per period)	**84.10** (31.37 – 217.20)

Multivariate extremes

•Point process framework for threshold excesses (20%, 50%, 100% of drought data).

$$\mathcal{P}_k = \left\{\left(\frac{X_{1_i}}{b_k},\ldots,\frac{X_{d_i}}{b_k}\right) : i = 1,\ldots,k\right\}$$

$$k\bar{F}(b_k,\ldots,b_k) = 1$$

•Standard Frechet marginals.

$$\bar{F}(x_1,\ldots,x_d) = P(X_1 > x_1,\ldots,X_d > x_d) = \frac{\mathcal{L}(x_1,\ldots,x_d)}{(x_1\cdots x_d)^{1/(d\eta)}}$$

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The η–asymmetric logistic model is chosen as dependence measure $H_\eta(\omega)$, which leads to the measure density $h_\eta(\omega)$ with parameters α, η and $\zeta_1,\ldots,\zeta_{d-1}$ [Ramos and Ledford, 2009].

$$h_\eta(\omega) = \frac{\prod_{i=1}^{d-1}(i\eta-\alpha)}{\eta^d\alpha^{d-1}N_\zeta}\left\{\sum_{i=1}^{d}\left(\frac{\omega_i}{\zeta_i}\right)^{-1/\alpha}\right\}^{\alpha/\eta-d} \times \left(\prod_{i=1}^{d}\omega_i\right)^{-1/\alpha-1}$$

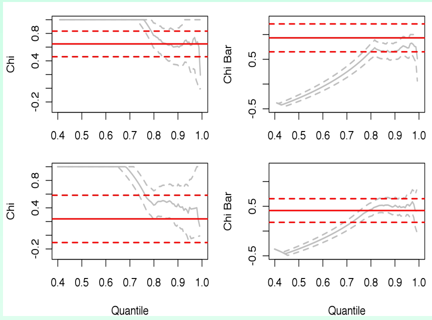
This model can capture the dependence structure of asymptotic dependent and asymptotic independent data.

The Fragility Index (FI)

Asymptotic dependent data: The *FI* represents the expected number of extremes given that one extreme already occurred.

Asymptotic independent data: The FI measures the speed of convergence towards independence (tail index η).

$$FI = \begin{cases} N = \lim_{\boldsymbol{q}\to\infty} E(\kappa_{\boldsymbol{q}}|\kappa_{\boldsymbol{q}} \geq 1) = \lim_{\boldsymbol{q}\to\infty} \frac{P(X_1>q_1)+\ldots+P(X_d>q_d)}{1-P(X_1\leq q_1,\ldots,X_d\leq q_d)} & \text{if } \eta = 1 \\ \eta = \frac{1}{d}\lim_{\boldsymbol{q}\to\infty} \frac{\log P(X_1>q_1)+\ldots+\log P(X_d>q_d)}{\log P(X_1>q_1,\ldots,X_d>q_d)} & \text{if } \eta < 1 \end{cases}$$

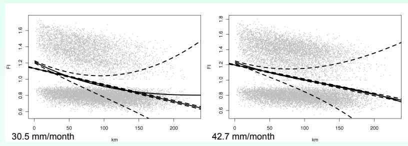


$$\chi = 2 - 2/N \qquad \bar{\chi} = 2\eta - 1$$

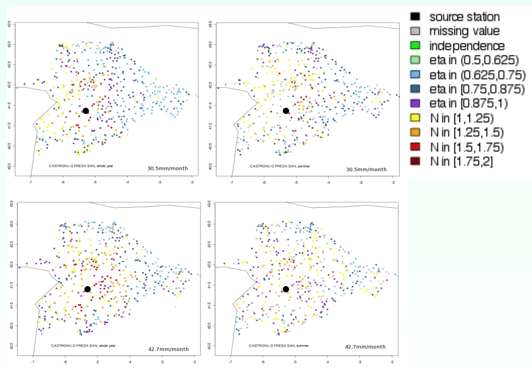
Upper line: Dependence for Aguas de Cabreiroa and Barxa. η: 0.967 (0.14), *N*: 1.48 (0.093), asymptotically dependent. Lower line: Dependence for Aguas de Cabreiroa and Cantimpalos. η: 0.7 (0.12), *N*: 1.13 (0.17), asymptotically independent.

X, $\overline{X}$ calculated from *FI* (red) and from empirical estimates (grey). Dashed lines: SE.

Results (bivariate)

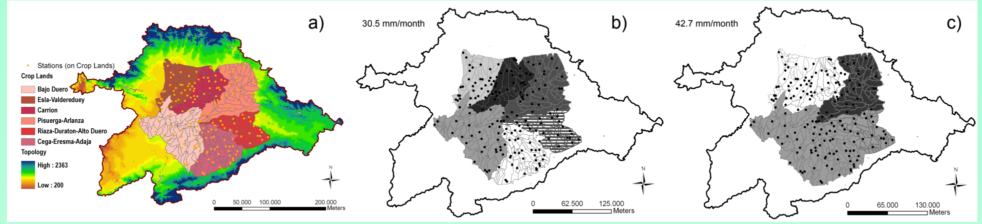


Dependence-Spatial Distance-relation: The FI decreases with spatial distance. However, it has a large variability over all distances.



Bivariate dependence to station Castronuño (black), which lies in the center of the sub-basin Bajo Duero, whose crop lands are affected by the most severe droughts. Strongest dependence to the Eastern part of the basin. In the irrigation period, extreme droughts at this station are strongly dependent with stations at larger distances, which indicates an allover dry situation. Here dependence is also weaker than for the whole year.

Results (regions)



6 regions are defined in the central plain of the Duero basin, where land use is predominantly agricultural [a)]. Sums of the station drought events in these regions are analyzed; they represent drought events, which happen anywhere in these regions. Strongly dependent regions (*FI* > 1.5) are shaded in the same color. When looking at the more severe droughts [b)], a strong connection between the Western regions is found. The crop lands of Riaza-Duraton (hatched in two colors) are strongly dependent with both neighboring sites, but the three regions together are not strongly dependent. For droughts defined with the 42.7mm/month level [c)], the Southern regions are strongly dependent.

Conclusions

•Topography and spatial distance impact the dependence of extreme droughts, but not in a generalizable manner.

• Comparison of any two stations in the basin shows that about two third of the stations are asymptotical independent.

•Droughts in the irrigation period do not exhibit as strong dependence as in the whole year, which supports the idea of a significant large-scale influence of precipitation in winter (the North Atlantic Oscillation).

•Shorter, more severe droughts show strong dependence in the Western regions of the Duero basin, whereas less severe droughts of the Southern regions are strongly connected.

•The MEVT (multivariate extreme value theory) model is suitable to analyze bivariate dependence of the stations and dependence between few regions. The results can be used to adapt short-term irrigation measures.

References

J. L. Geluk, L. de Haan, and C. G. de Vries. Weak & strong financial fragility. Technical Report 07-023/2,C6 G20, EconPapers, 2007.

J. C. González-Hidalgo, M. Brunetti, and M. de Luis. A new tool for monthly precipitation analysis in Spain: MOPREDAS database (monthly precipitation trends December 1945-November 2005). *International Journal of Climatology*, 2010. Doi: 10.1002/joc.2115.

A. Ramos and A. Ledford. A new class of models for bivariate joint tails. *Journal of the Royal Statistical Society B*, **71**(1):219-241, 2009.

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