Simulation of poro-elastic seismic wave propagation in axis-symmetric open and cased boreholes

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Introduction

The comprehensive simulation of seismic wave propagation in realistic borehole environments represents a pertinent, but as of yet largely unresolved problem in exploration geophysics. Here, we present a method for the modeling of poro-elastic seismic wave propagation in 2D polar coordinates based on Biot's dynamic equations (Biot, 1962a,b) and its extension to cylindrical coordinates. The use of a poro-elastic approach is essential given that a key objective of borehole seismic experiments is the estimation of the governing hydraulic characteristics of the surrounding geological formations. The intermediate step of using 2D polar coordinates is motivated (i) by the inherent complexity of the derivation of the governing equations and the boundary conditions as well as their benchmarking and (ii) by the relative ease of the extension of a corresponding algorithm to cylindrical coordinates. For the sake of computational efficiency and simplicity of model parameterization, we then assume symmetry with respect to the vertical axis for the solution in cylindrical coordinates.

In the following, we first describe the design of the numerical algorithm. We then show benchmarks of the 2D polar problem, including comparisons to analytical and 2D numerical Cartesian solutions as well as the validation of the reciprocity principle. Finally, we apply the cylindrical solution to a borehole logging experiment and evaluate the impact of open- and closed-pore boundary conditions as well as of a PVC casing.

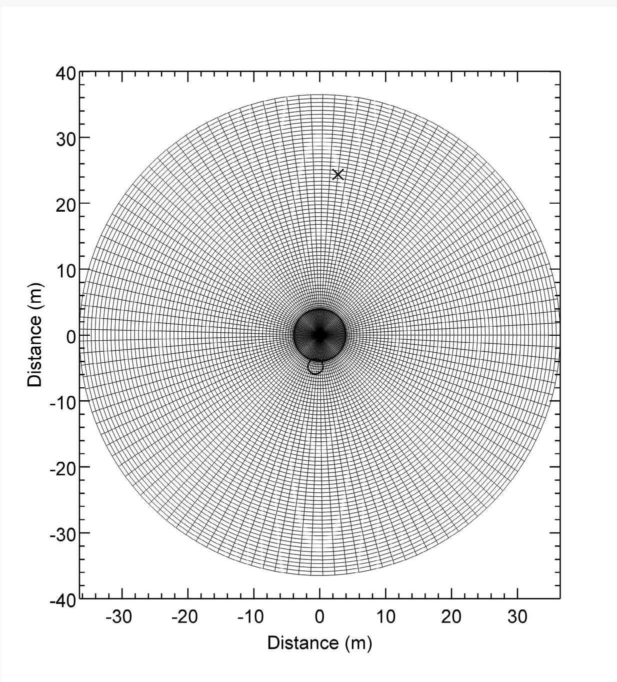


Figure 1: Polar geometry for the comparison of the numerical solution in 2D polar coordinates with solutions in Cartesian coordinates (Figures 2 and 3). X and O denote the source and receiver locations, respectively. The objective of this setup is to demonstrate that the wavefield is not distorted by the decomposition procedure at the interface. Note that, due to the inherent singularity at $r = 0$, the center of the innermost domain has a circular hole, which for the purpose of these tests must be chosen small enough so that the waves are not affected by its presence.

Numerical algorithm

Pseudo-spectral methods are efficient and highly accurate techniques for modeling complex wave propagation phenomena. They can be viewed as finite differences with infinite-order accuracy, as the lateral derivatives are calculated in the frequency domain using a forward and backward discrete Fourier transform. When there are physical boundary conditions to satisfy, the Fourier method is replaced by the Chebyshev method, which is not periodic and allows for an explicit boundary treatment by decomposing the wavefields using characteristic variables and applying appropriate boundary conditions. This allows, for example, to account for variable flow impedance at fluid/poro-elastic interfaces, which can be of the open-pore, closed-pore, or mixed-pore type (Deresiewicz & Skalak, 1963).

The presence of the slow diffusive compressional wave makes Biot's differential equations stiff. To overcome this problem, the corresponding equations are solved with the splitting scheme introduced by Carcione & Quiroga-Goode (1996) where the regular part is solved numerically and for each time step the stiff part of the equations is solved analytically. The time integration is performed with a 4th-order Runge-Kutta scheme.

Verification in 2D polar coordinates

To test the viability and accuracy of the numerical approach described above, we compare the results of the 2D polar solution to the analytical solution for poro-acoustic media and to numerical wavefields obtained with a previously published pseudo-spectral method in 2D Cartesian coordinates (Sidler et al., 2010).

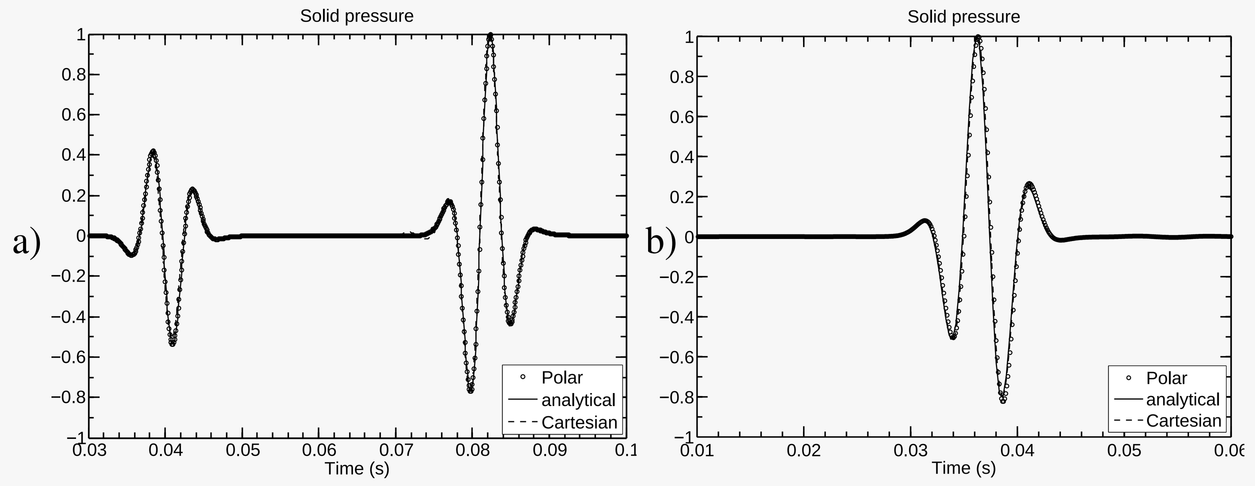
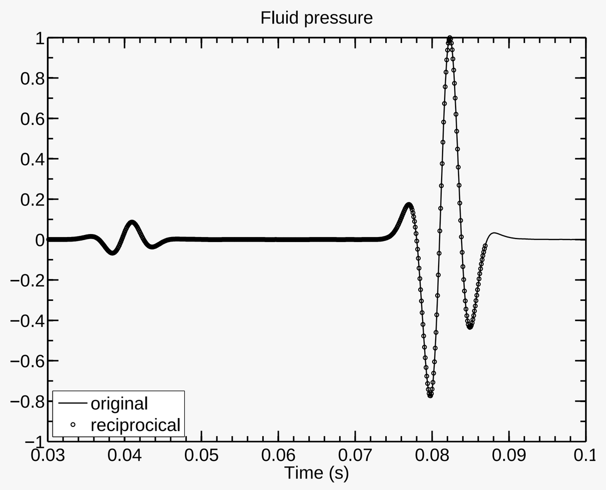


Figure 2: Seismograms of the solid pressure for the source and receiver locations shown in Figure 1. (a) Pore fluid is non-viscous and the fast as well as the slow waves are visible. (b) Pore fluid has the viscosity of water. In this case, the slow wave becomes dispersive and only the fast P-wave can be observed. The source mechanism is a fluid injection with the time history of a Ricker wavelet and a central frequency of 125 Hz.

Figure 3: Verification using the reciprocity principle. Inadvertent effects of grid design, notably with regard to the inherent singularity at $r = 0$, and boundary treatment could be identified with this test. The solid line corresponds to source and receiver positioned as shown in Figure 1 and the circles correspond to the corresponding reciprocal solution.



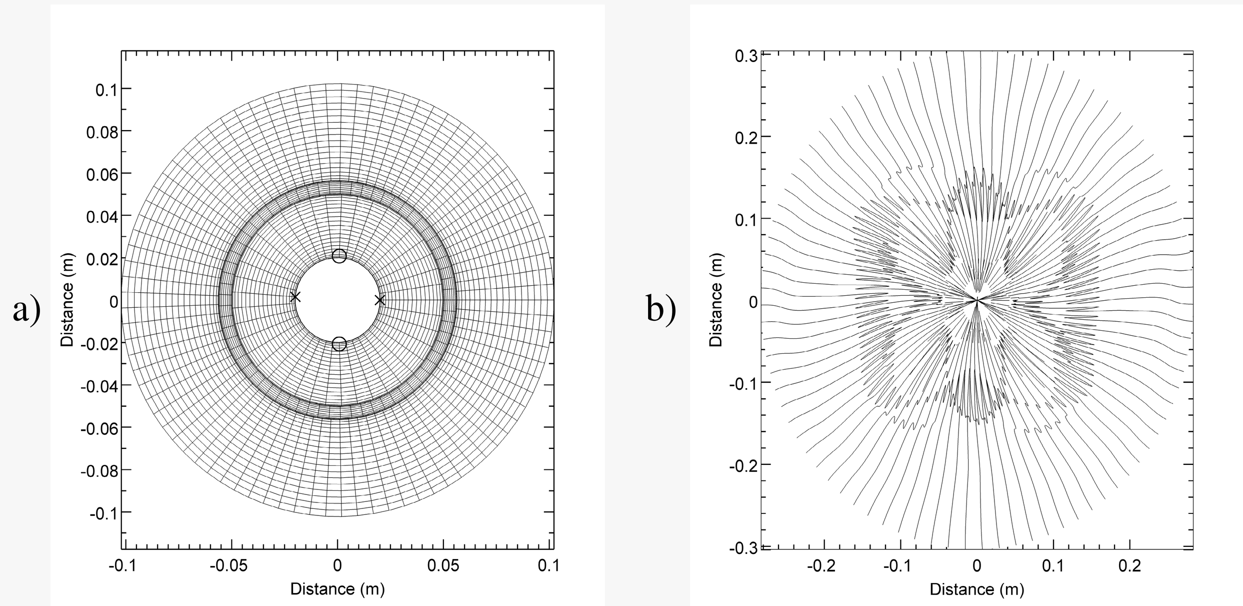


Figure 4: (a) Geometrical setup of a borehole-type experiment. The dipole-type source (x-x) acts in the horizontal direction and is located together with the receivers (O) on the surface of the perfectly rigid logging tool. (b) Snapshot of the pressure field after a propagation time of 300 μs for the model shown in (a) with no casing and open-pore boundary conditions.

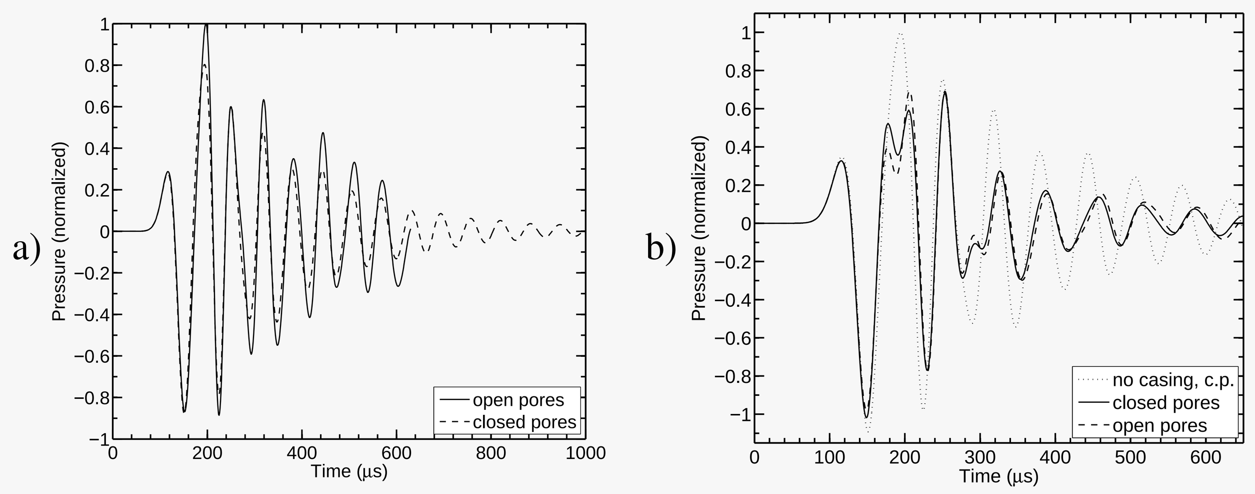


Figure 5: Comparison of the fluid pressure recordings (a) in the absence and (b) in the presence of a casing (Figure 4a, Table 1).

Matrix	bulk modulus, K_m	1.36 GPa
	shear modulus, μ_m	1.86 GPa
	porosity, ϕ	0.38
	permeability, κ	28.3 D
	tortuosity, T	1.8
Grain	bulk modulus, K_s	32 GPa
	shear modulus, μ_s	44 GPa
	density, ρ_s	2690 kg/m^3
Fluid	density, ρ_f	1090 kg/m^3
	viscosity, η	0.00105 Pa s
	bulk modulus, K	2.25 GPa

Table 1:
Material properties of unconsolidated sand as given by Jackson & Richardson (2007).

Cylindrical solutions for a dipole source

In the following simulations, we assume symmetry with regard to the vertical axis and hence replace the derivative with respect to the azimuthal direction by a multiplication with a constant factor obtained by a Fourier analysis (Randall et al., 1991). A borehole logging tool in a fluid filled borehole is simulated and the fluid pressure is recorded along the logging tool (Figure 6). The properties of the surrounding formation are given in Table 1 and are the same for all three examples.

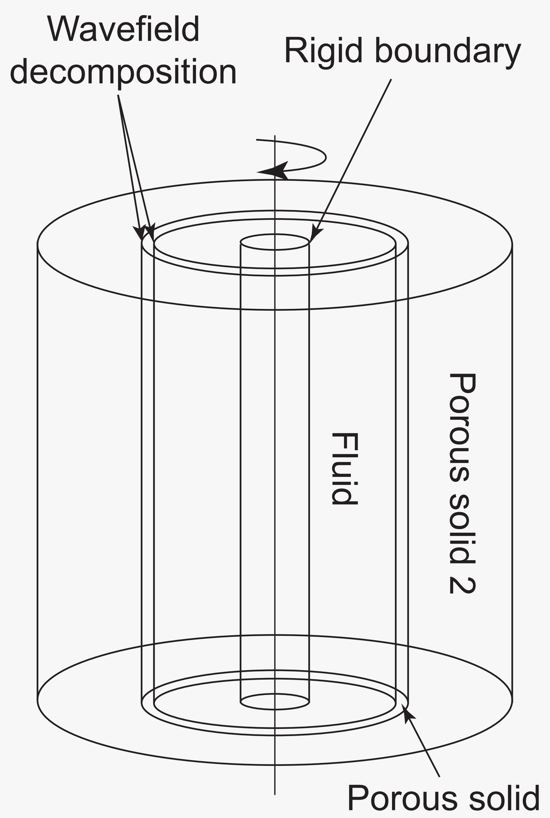


Figure 6: Cylindrical borehole geometry with angular symmetry. The fluid filling the borehole and the pores of the surrounding formation is water. Porous solid 2 corresponds to an unconsolidated sand (Table 1). The porous solid 1 can represent a casing between the fluid-filled borehole and the porous formation. The radius of the borehole is 125 mm. The hole in the center of the domain has a radius of 50 mm and represents a perfectly rigid borehole logging tool. The outer radius of the first of the two porous domains is 135 mm.

Figures 7 and 8 show examples for the use of the modeling algorithm. In Figure 7 the boundaries at the fluid-solid interface are varied between open- and closed-pores. The closed-pore case can, for example, be used to represent the presence of a thin, impermeable mud cake. In Figure 8 a PVC casing used to stabilize boreholes in unconsolidated surficial sediments is added between the fluid and the porous solid. The thickness of this casing is 1 cm. The material properties of PVC are adopted from Bakulin et al. (2008) and we use a porosity of 4 % and a permeability of 1900 D.

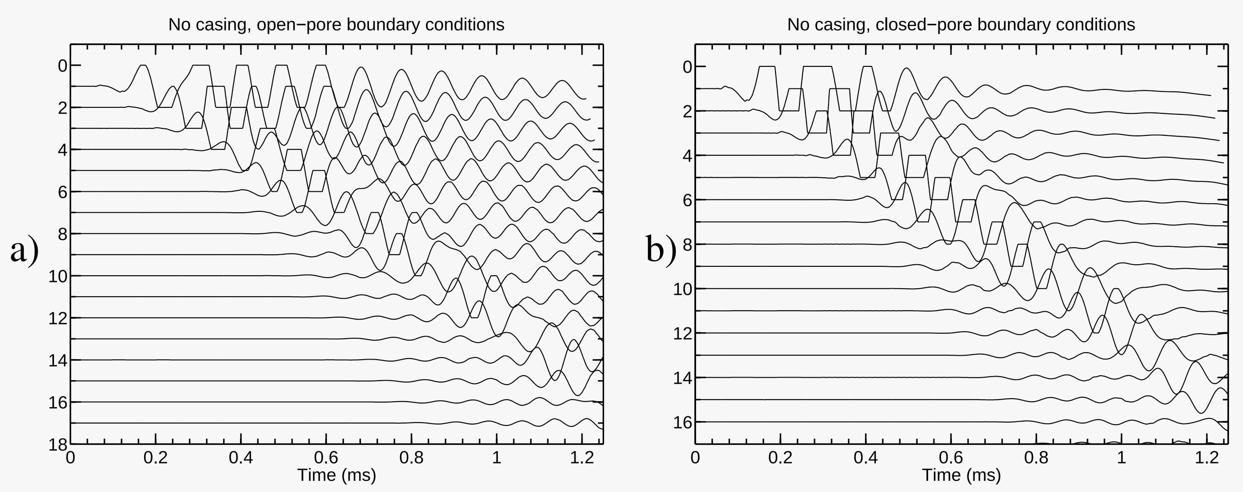


Figure 7: Waveforms of the fluid pressure for an uncased borehole with (a) open-pore and (b) closed-pore boundary conditions (Figure 6). The receiver spacing is 10 cm and the material properties are given in Table 1.

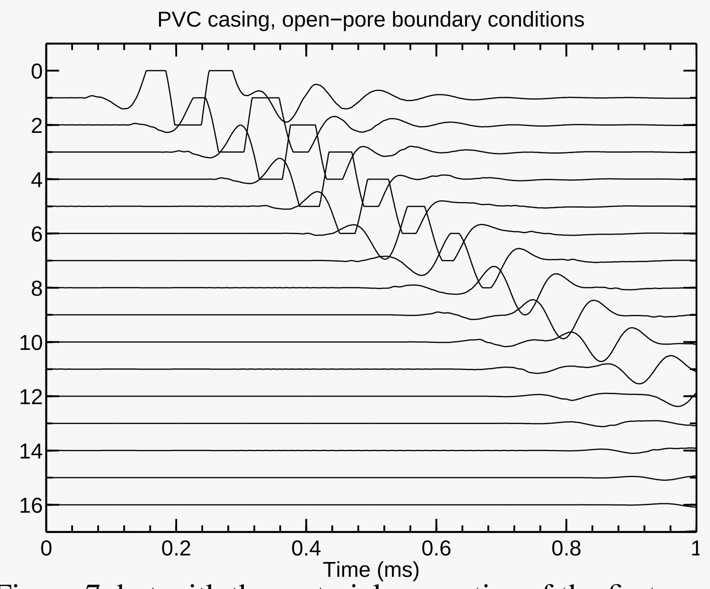


Figure 8: Same as Figure 7, but with the material properties of the first porous solid corresponding to a PVC casing (Figure 6).

The resulting synthetic seismograms show a large sensitivity to the boundary conditions as well as to the presence or absence of a casing, which in turn illustrates both the inherent complexity of the underlying physical problem as well as the need for realistic numerical simulations.

Conclusions

We have developed a pseudo-spectral numerical solution of the poro-elastic equations in cylindrical coordinates to simulate the propagation of seismic waves in complex borehole-type environments. Using a domain-decomposition technique based on the method of characteristics allows for splitting the numerical grid into several concentric sub-domains and to satisfy the complex and variable physical boundary conditions. The resulting numerical approach has been rigorously tested in 2D polar coordinates before being expanded to axis-symmetric cylindrical coordinates. Several examples involving fluid-filled boreholes, the presence or absence of a casing as well as open- and closed-pore boundary conditions demonstrate the potential of the numerical approach for the realistic modeling of complex seismic wave phenomena in heterogeneous borehole environments. The viability and flexibility of this approach will be further enhanced by its pending extension to azimuthally heterogeneous media.

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Acknowledgments

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